Economic Model Predictive Control – Historical Perspective and Recent Developments and Industrial Examples

Public Trial Lecture

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Presentation outline

- Introduction and basic concepts
- Historical overview
- Recent developments
- Industrial examples
- Final thoughts
Introduction

Plant operation objectives

- Maximize the economic operating value of the plant
- Achieve environment, health and safety targets (regulations)

Need tight integration between

- Plant management → Economics
- Process operation → Control
Traditional paradigm: Two layer structure

- Upper layer: steady state optimization - Real Time Optimization (RTO)
- RTO provides setpoints to a lower (dynamic) control layer
- Control layer follows setpoints
  - linear model predictive controllers (MPC) often employed

Traditional paradigm: Two layer structure

Advantages of two layers approach

- Simpler sub-problems
- Reduced complexity
- Clear separation between economic and control objectives (based on time-scale separation)

But every advantage is also a disadvantage*

- Delay in the optimization (need to wait for steady state)
- Time-scale separation may not hold

(*) Johan Cruyff
Current trend: Integrate control and economic optimization in one layer

Economic Model Predictive Control (EMPC)

- (Dynamic) optimization over a moving horizon of process economic performance
- Process constraints directly represented in the optimization problem
- Maximum freedom for optimization → better economic performance
Model Predictive Control (MPC)

Over 30 years of successful application in the industry
- Handles constrained multivariable processes

Successful application to large scale nonlinear processes (Seki 2001)
- Leveraged by great developments in the numerical solution strategies and increased computational power
Economic Model Predictive Control (EMPC)

\[
\begin{align*}
\text{minimize} & \quad \int_0^{\tau_N} l_e(\tilde{x}(t), u(t)) \, dt \\
\text{subject to} & \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\
& \quad \tilde{x}(0) = x(\tau_k) \\
& \quad g(\tilde{x}(t), u(t)) \leq 0, \quad \forall t \in [0, \tau_N]
\end{align*}
\]

- Economic cost (profit)
- Dynamic model
- Initial condition
- State and input constraints
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- Final thoughts (Open issues, etc)
Early beginning

- Application in the industry long before understanding its theoretical properties (‘Brave era’)  
  - First report of MPC application (Richalet et al. in 1976) on a Fluid catalytic cracking unit  
  - Dynamic Matrix Control at Shell (Cutler and Ramaker, 1980)

- 3rd Generation MPC → Use of state space model, Kalman filters, hard/soft constraint handling, etc (SMOC: Shell Multivariable Optimizing control)
Early beginning

- Model predictive control initially used for multivariable process regulation (setpoint control)

- Smart practitioners, however, use/used the same framework for performance optimization:

**Unreachable setpoint trick:**

\[
\text{minimize } \int_{0}^{\tau_N} \left( |y - y_{usp}|^2 + |u|^2 \right) dt
\]

- \(y\): variable we want to maximize/minimize
- \(y_{usp}\): unreachable setpoint
Integrate steady-state optimization into MPC

Intermediate optimization layer:

- Use info from RTO and lower MPC
- Computes setpoints for MPC → the best (dynamic) way to reach the RTO target

Very common in the industry (Morshed (1985), Nath (2002))
Integrate steady-state optimization into MPC

\[
\min_{\gamma_{\text{set}}, u_{\text{set}}} \left[ (\gamma_{\text{set}} - y^*)^T C_y (\gamma_{\text{set}} - y^*) + (u_{\text{set}} - u^*)^T C_u (u_{\text{set}} - u^*) \right] \\
+ c_y (\gamma_{\text{set}} - y^*) + c_u (u_{\text{set}} - u^*) \]

Subject to
\[
\gamma_{\text{set}} = A_s u_{\text{set}} + d(k), \\
d(k) = d(k - 1) + \Delta(k), \\
\gamma_{\text{min}} \leq \gamma_{\text{set}} \leq \gamma_{\text{max}}, \\
u_{\text{min}} \leq u_{\text{set}} \leq u_{\text{max}}
\]

- Model constraints: from lower MPC
- Weights for cost function: linearized from RTO
Add economic steady-state term to the cost function

Economic objective $f_{eco}$ computed using a nonlinear steady-state process model

\[
\begin{align*}
\min_{\Delta u(k+i); i=0,...,m-1} & \sum_{j=1}^{p} ||W_1(y(k+j) - r)||_2^2 + \sum_{i=0}^{m-1} ||W_2 \Delta u(k+i)||_2^2 \\
&+ W_3 f_{eco}(u(k+m-1)) + ||W_4(u(k+m-1) \\
&- u(k-1) - \Delta u(k)||_2^2 + W_5 [f_{eco}(u(k+m-1), y(k+\infty)) \\
&- f_{eco}(u(k), y'(k+\infty))]^2.
\end{align*}
\]
Integration of nonlinear steady-state optimization in the linear MPC controller

Industrial implementation in a refinery by Petrobras.

Objective is maximize production of LPG in a FCC unit

*Integrating real-time optimization into the model predictive Controller of the FCC system,* Zanin et al. (2002)
Economic model predictive control

- Branded as ‘Direct finite horizon optimizing control’ (Engell, 2007)
  → Reported application to a Simulated Moving Bed (SMB) process

- Putting Nonlinear Model Predictive Control into Use (Foss & Schei, 2007)
  → Several industrial applications


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Economic model predictive control is finally baptized

- Economic Model Predictive Control for Building Energy Systems (Ma, 2011)
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Presentation outline

Recent developments

- Infinite horizon EMPC
- Terminal cost/constraint EMPC
- Lyapunov based EMPC
- Closed-loop (economic) performance analysis
Economic Model Predictive Control (EMPC)

\[
\text{minimize} \quad \int_0^{\tau_N} l_e(\tilde{x}(t), u(t)) \, dt \\
\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\
\tilde{x}(0) = x(\tau_k) \\
g(\tilde{x}(t), u(t)) \leq 0, \quad \forall t \in [0, \tau_N]
\]

Replace the tracking cost function by an economic objective

There is no reference/setpoint to track
Recap: stability of tracking MPC

Nominal stability for finite horizon MPC

- Convergence to a desired equilibrium point

- Need to add terminal constraint set $X_f$ and terminal cost $V_N(x_N)$ to original problem. $X_f$ must be invariant with a local controller $k_f(x)$

- Optimal cost function is a Lyapunov function! $\rightarrow$ Monotonically decreasing
Challenges for the closed-loop stability analysis of EMPC

- There is no target to converge to
- Optimal cost is **not** a Lyapunov function for the closed-loop system
- Sequence of optimal costs is not monotone decreasing

Next we are going to see different EMPC formulations which tackle stability analysis in various ways
Infinite-horizon economic model predictive control

\[ L_e(x(t), u(t)) = - \int_0^\infty e^{-\rho t} l_e(x(t), u(t)) \, dt \]

\( \rho > 0 \): discount factor

Commonly used in economic growth theory

Stability follows from Bellmann’s optimality principle

But the problem is very hard to solve

Economic model predictive control with terminal constraints

\[
\begin{align*}
\text{minimize} & \quad \sum_{j=0}^{N-1} l_e(\hat{x}(j), u(j)) + V_f(\hat{x}(N)) \\
\text{subject to} & \quad \hat{x}(j + 1) = f_d(\hat{x}(j), u(j), 0) \\
& \quad \hat{x}(0) = x(k) \\
& \quad \hat{x}(N) \in X_f \\
& \quad (\hat{x}(j), u(j)) \in \mathbb{Z}, \quad \forall j \in \mathbb{I}_{0:N-1}
\end{align*}
\]

- \( V_f \rightarrow \text{final cost} 
- \( X_f \rightarrow \text{terminal constraint set} 

Commonly used:
\[
x(N) = x_s^*
\]

Optimal steady-state solution

Example of trajectory of EMPC with terminal constraint \( x(N) = x_s \)

This formulation guarantees boundedness of trajectory.

Not necessarily asymptotic stability.

But the extra transients can be beneficial for economics.

Dissipativity

\[ S(x(t_1)) \leq S(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t)) \, dt \]

- \( S \) is a storage function
- \( s \) is the supply rate

There can be no internal creation of energy; only internal **dissipation** of energy is possible.

Dissipativity

\[ S(x(t_1)) \leq S(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t)) \, dt \]

For linear systems, this is equivalent to Positive Realness
\( \rightarrow \) Nyquist plot \( G(jw) \) always on RHP
\( \rightarrow \) Any negative feedback can stabilize the system

EMPC stability based on dissipativity

(1) Assume weak controllability
(2) Assume the closed loop under EMPC with terminal constraint is strictly dissipative with supply rate

\[ s(x, u) = l_e(x, u) - l_e(x^*_s, u^*_s) \]

Then optimal steady state \( x^*_s \) is asymptotically stable

EMPC with Lyapunov-based constraints

Assume there exists a Lyapunov controller \( u = k(x) \)

- \( V(x) \rightarrow \) associated Lyapunov function
- \( \Omega_\rho \rightarrow \) stability region of the closed-loop under \( k(x) \)

A Lyapunov EMPC has two operating modes:

**Mode 1:** Ensures boundedness of state in \( \Omega_{\rho e} \subset \Omega_\rho \)

**Mode 2:** Ensures convergence to the origin \( (x_s^*) \)

Heidarinejad et al., Economic model predictive control of nonlinear process systems using Lyapunov techniques, (2012)
EMPC with Lyapunov-based constraints

\[
\begin{align*}
\text{minimize} & \quad L_e(\tilde{x}(t), u(t)) \\
\text{subject to} & \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\
& \quad \tilde{x}(0) = x(\tau_k) \\
& \quad u(t) \in U, \quad \forall \ t \in [0, \tau_N) \\
& \quad V(\tilde{x}(t)) \leq \rho_e, \quad \forall \ t \in [0, \tau_N) \\
& \quad \text{if } V(x(\tau_k)) < \rho_e \quad \text{and} \quad t < t_s \\
& \quad \frac{\partial V}{\partial x} f(x(\tau_k), u(\tau_k), 0) \leq \frac{\partial V}{\partial x} f(x(\tau_k), k(x(\tau_k)), 0) \\
& \quad \text{if } V(x(\tau_k)) \geq \rho_e \quad \text{or} \quad t \geq t_s
\end{align*}
\]

Mode 1

Mode 2

EMPC with Lyapunov-based constraints

✓ No need to modify economic cost
✓ Better feasibility and stability properties compared to end constraint EMPC
✓ Construction of controller \( u = k(x) \) and corresponding Lyapunov function \( V(x) \) for general constrained nonlinear systems is hard!

Closed-loop performance under EMPC

Two common methods to ensure performance:

✓ Use very large prediction horizon
✓ Use terminal constraint $x(N) = x_s^*$ (best steady state)

$$\limsup_{T \to \infty} \sum_{k=0}^{T} \frac{l_e(x(k), u(k))}{T + 1} \leq l_e(x_s^*, u_s^*)$$

Future directions

- Robustness → most of the results are based on nominal analysis
- Use of state-estimation → all EMPC schemes rely on state feedback
- Less conservative stability results → Only sufficient conditions so far
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Economic Model Predictive Control of Wastewater Treatment Processes

- 145 differential states
- 2 manipulated inputs \((Q_a \text{ and } K_L a)\)
- 2 controlled variables \((S_{NO,2} \text{ and } S_{O,5})\)

Zeng & Liu (2014)
Objective function

\[
\min_{u(\tau) \in \mathcal{S}(\Delta)} \sum_{j=k}^{j=k+N} l(\tilde{x}(t_j|t_k), u(t_j|t_k)) \, dt + c(\tilde{x}(t_{k+N}), N_h)
\]

s.t. \( \dot{x}(t) = f(x(t)) + g(x(t))u(t) \)

\( \dot{y}(t) = h(x(t)) \)

\( x(t_k) = x(t_k) \)

\( u(t) \in \mathcal{U} \)

\( y(t) \in \mathcal{Y} \)

Stage cost: effluent quality + operating cost

\[
l(x(t_k), u(t_k)) = w_{EQ}EQ(t_k) + w_{OCI}OCI(t_k)
\]

Zeng & Liu (2014)
Simulation results

Comparison with

- PI control
- tracking MPC
Performance comparison

7.4% Improvement over PI control and 5.8% over tracking MPC

Black: EMPC; Blue: PI; Red: tracking MPC
Process outputs (top) and manipulated variables (bottom)

- EMPC is not required to track setpoints
  - Great dynamic freedom for optimization
- PI and MPC may achieve similar performance by optimizing setpoints

Black: EMPC; Blue: PI; Red: MPC
Economic Model Predictive Control of a continuous catalytic distillation process

Reactants
- Acetic acid
- Methanol

Products
- Methyl acetate
- Water

Degrees of freedom
- Reboiler heat duty
- Reflux
- Inflow of reactants

Idris & Engell (2012)

581 differential states
Economic Model Predictive Control of a continuous catalytic distillation process

\[ \Psi(k) = \left( \dot{P}(k) \cdot C_P - \dot{H}(k) \cdot C_E - \sum_{j=1}^{N_f} \dot{R}_j(k) \cdot C_{R,j} \right) \]

Average quality constraint on the valuable product

\[ \sum_{i=1}^{P} \text{Purity}_{\text{MeAc},k+i} \frac{P}{P} \geq L_{\text{MeAcPurity}} \]

*Idris & Engell (2012)*
Economic Model Predictive Control of a continuous catalytic distillation process

\[
\Phi_{EOPC} = \sum_{b=1}^{R} \left( \sum_{j=1}^{M} \alpha_{b,j} \Delta u^2_{b}(k+j) \right) \\
- \left( \sum_{i=1}^{P} \beta_i \left( \dot{P}(k+i) \cdot C_P - \dot{H}(k+i) \cdot C_E - \sum_{j=1}^{N_f} \dot{R}_j(k+i) \cdot C_{R,j} \right) \right)
\]

s.t.
\[
\begin{align*}
\mathbf{x}_{(i+1)} &= f(\mathbf{x}_i, \mathbf{z}_i, \mathbf{u}_i, i), i = k, \ldots, k + P \\
0 &= g(\mathbf{x}_i, \mathbf{z}_i, \mathbf{u}_i, i), i = k, \ldots, k + P \\
\mathbf{u}_{\text{min}} &\leq \mathbf{u}(i) \leq \mathbf{u}_{\text{max}}, i = k, \ldots, k + M \\
-\Delta \mathbf{u}_{\text{min}} &\leq \Delta \mathbf{u}(i) \leq \Delta \mathbf{u}_{\text{max}}, i = k, \ldots, k + M \\
\mathbf{u}(i) &= \mathbf{u}(k + M), \forall i > k + M
\end{align*}
\]

Idris & Engell (2012)
Alternative I: tracking MPC with economic term

\[
\Phi_{EoTC} = \sum_{n=1}^{N} \left( \sum_{i=1}^{P} \gamma_{n,i} (y_{n,ref}(k+i) - y_n(k+i))^2 \right) + \sum_{b=1}^{R} \left( \sum_{j=1}^{M} \alpha_{b,j} \Delta u_b^2(k+j) \right) 
- \left( \sum_{i=1}^{P} \beta_i \left( \dot{P}(k+i) \cdot C_P - \dot{H}(k+i) \cdot C_E - \sum_{j=1}^{N_f} \dot{R}_j(k+i) \cdot C_{R,j} \right) \right),
\]

Idris & Engell (2012)
Alternative II: purely tracking MPC

\[ \Phi_{EoTC} = \sum_{n=1}^{N} \left( \sum_{i=1}^{P} \gamma_{n,i}(y_{n,ref}(k+i) - y_n(k+i))^2 \right) + \sum_{b=1}^{R} \left( \sum_{j=1}^{M} \alpha_{b,j} \Delta u_b^2(k+j) \right) \]

Idris & Engell (2012)
Cost comparison

8.2% improvement in profit over tracking MPC

Red: EMPC; Green: tracking MPC with economic term; Black: pure tracking MPC
### Computational time

<table>
<thead>
<tr>
<th>Method</th>
<th>Computational time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking MPC</td>
<td>3.5-3.9 min</td>
</tr>
<tr>
<td>Tracking MPC econ. term</td>
<td>4.1–5.0 min</td>
</tr>
<tr>
<td>EMPC</td>
<td>15–20 min</td>
</tr>
</tbody>
</table>

Apparently, the economic cost function is very flat → NLP solver has problems converging
Final thoughts

We have seen an overview of a method that combines economic optimization and control in one layer

*Economic Model Predictive Control*

Makes sense if there is no time scale separation → time constant of process is comparable to that of the economics (e.g. price variations)

Great research efforts in the latest years

Simulations suggest some economic benefit → not a lot of validation in practice
Limitations

Reliability → optimizer must converge or else...

Robustness → Some processes may be very hard to stabilize

Computational cost → although it’s becoming less of a problem due to improvements in computer power and solution approaches

Higher cost → implementation and maintenance
    (Often good performance is achievable with simpler methods)
Alternative: Hierarchical EMPC approach

\[ x(t_k) \]

\[ u_{1,E}^*(t|\hat{t}_k) \]

\[ u_{2,E}^*(\cdot|\hat{t}_k) \]

\[ x_E^*(\cdot|\hat{t}_k) \]

\[ w(t) \]

\[ x(t_j) \]

\[ u_2^*(t|t_j) \]

\[ \dot{x} = f(x, u_1, u_2, w) \]

EMPC

Tracking MPC

System
References

Thank you!

The end