Quantitative methods for controlled variables selection

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Thesis outline

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Ch. 2. Brief overview of control structure design and methods

Ch. 3. Convex formulations for optimal CV using MIQP

Ch. 4. Convex approximations for optimal CV with structured H

Ch. 5. Quantitative methods for regulatory layer selection

Ch. 6. Dynamic simulations with self-optimizing CV

Ch. 7. Conclusions and future work

Appendices A - E

CV - Controlled Variables
MIQP - Mixed Integer Quadratic Programming
Presentation outline

- Plantwide control: Self optimizing control formulation for CV, $c = Hy$ - Chapter 2
- Convex formulation for CV with full H - Chapter 3
  - Convex formulation
  - Globally optimal MIQP formulations
  - Case studies
- Convex approximation methods for CV with structured H - Chapter 4
  - Convex approximations
  - MIQP formulations for structured H with measurement subsets
  - Case studies
- Regulatory control layer selection - Chapter 5
  - Problem definition
  - Regulatory control layer selection with state drift minimization
  - Case studies
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Plantwide control: Hierarchical decomposition

- Each layer operates at different time scales
- The decisions are cascaded from top to bottom
- Top layer provides set points to the bottom layer
- Scope of the thesis: Optimal operation constituting optimization layer and control layers
- Assumption: Economics are primarily decided by steady-state
- Focus is on the selection of controlled variables $CV_1$ and $CV_2$
Optimal operation

Real time optimization

Real Time Optimization (RTO)

Controller K

Plant \((G^y, G_d^y)\)

\(c_s\)

\(u\)

\(d\)

\(y\)

\(+n^y\)

Closed loop implementation with a separate control layer

Real Time Optimization (RTO)

Controller K

H

Plant \((G^y, G_d^y)\)

\(c_s\)

\(u\)

\(d\)

\(y\)

\(+n^y\)

Ref: Kassidas et al., 2000
Engell, 2007
Self-optimizing control is said to occur when we can achieve an acceptable loss (in comparison with truly optimal operation) with constant setpoint values for the controlled variables without the need to reoptimize when disturbances occur.

Problem Formulation, \( c = Hy \)

Assumptions:
1. Active constraints are controlled
2. Quadratic nature of \( J \) around \( u_{opt}(d) \)
3. Active constraints remain same throughout the analysis

Optimal steady-state operation

\[
\begin{align*}
\min_u J(u,d) \\
J(u,d) &= J(u_{opt}(d),d) + J_u (u-u_{opt}(d)) + \frac{1}{2} (u-u_{opt}(d))^T J_{uu} (u-u_{opt}(d)) + \xi^3 \\
L &= J(u,d) - J_{opt}(u_{opt}(d),d)
\end{align*}
\]

\[
L = \frac{1}{2} (u-u_{opt}(d))^T J_{uu} (u-u_{opt}(d))
\]
Problem Formulation, \( c = Hy \)

\[ L_{\text{avg}} = \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F^2 \quad \forall \left[ \begin{bmatrix} d' \\ n^y ' \end{bmatrix} \right] \in \square (0,1) \]

\[ Y = \left[ (G^y J_{uu}^{-1} J_{ud} - G_d^y) W_d \ W_n \right] \]

Kariwala et al. I&ECR, 2008

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CV - Controlled Variables
MIQP - Mixed Integer Quadratic Programming
Convex formulation (full $H$)

$$\min_H \left\| J_{uu}^{1/2} (H G_y)^{-1} H Y \right\|_F$$

$D$: any non-singular matrix

$H_1 = DH$

$$(H_G y)^{-1} H_1 = (DH G_y)^{-1} DH = (H G_y)^{-1} D^{-1} DH = (H G_y)^{-1} H$$

Objective function unaffected by $D$.
So can choose $H G_y$ freely.

$H$ is made unique by adding a constraint as

$$\min_H \| H Y \|_F$$

subject to $H G_y = J_{uu}^{1/2}$

Problem is convex in decision matrix $H$

Ref: Alstad 2009
Vectorization

\[
\begin{aligned}
\min_H \|HY\|_F \\
\text{subject to } HG^y = J_{uu}^{1/2}
\end{aligned}
\]

\[H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}
\]

is vectorized along the rows of H to form

\[h_\delta = \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{nu*ny} \end{bmatrix}_{(nu*ny) \times 1}
\]

\[
\begin{aligned}
\min_{h_\delta} \quad h_\delta^T F_\delta X_\delta \\
\text{st. } G_\delta^T X_\delta = J_\delta
\end{aligned}
\]

Problem is convex QP in decision vector \(h_\delta\)

\[F_\delta = Y_\delta Y_\delta^T\]
Controlled variable selection

Minimize the average loss by selecting H and CVs as

(i) best individual measurements

(ii) best combinations of all measurements

(iii) best combinations with few measurements
MIQP formulation (full H)

\[
H = \begin{bmatrix}
\sigma_1 & \sigma_2 & \cdots & \sigma_{ny} \\
h_{11} & h_{12} & \cdots & h_{1ny} \\
h_{21} & h_{22} & \cdots & h_{2ny} \\
\vdots & \vdots & \ddots & \vdots \\
h_{nu1} & h_{nu2} & \cdots & h_{nu*ny}
\end{bmatrix}_{nu \times ny}
\]

is vectorized along the rows of H to form

\[
\sigma_i \in \{0, 1\} \\
i = 1, 2, \cdots, ny
\]

\[
h_\delta = \begin{bmatrix}
h_{11} \\
h_{12} \\
\vdots \\
h_{nu*ny}
\end{bmatrix}_{(nu*ny) \times 1}
\]

\[
\sigma_\delta = \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_{ny}
\end{bmatrix}_{ny \times 1}
\]
**MIQP formulation**

### Big-m method

\[
\begin{align*}
\min_{x_\delta, \sigma_\delta} & \quad h_\delta^T F_\delta h_\delta \\
\text{st.} & \quad G_\delta^y h_\delta = J_\delta \\
& \quad P \sigma_\delta = n \\
& \quad \begin{bmatrix}
-m \\
-m \\
\vdots \\
-m
\end{bmatrix}
\begin{bmatrix}
h_{i1} \\
h_{i2} \\
\vdots \\
h_{ni}
\end{bmatrix}
\leq
\begin{bmatrix}
m \\
m \\
\vdots \\
m
\end{bmatrix}
\sigma_i \\
\forall i = 1, 2, \ldots, ny
\end{align*}
\]

Selection of appropriate \( m \) is an iterative method and can increase the computational requirements.

### Indicator constraint method

\[
\begin{align*}
\min_{x_\delta, \sigma_\delta} & \quad h_\delta^T F_\delta h_\delta \\
\text{st.} & \quad G_\delta^y h_\delta = J_\delta \\
& \quad P \sigma_\delta = n \\
& \quad \sigma_i = 0 \Rightarrow \begin{bmatrix}
h_{i1} \\
h_{i2} \\
\vdots \\
h_{ni}
\end{bmatrix} = 0_{n \times 1} \\
& \quad \forall i = 1, 2, \ldots, ny
\end{align*}
\]
Case Study: Distillation Column

Binary Distillation Column
LV configuration
(methanol & n-propanol)

41 Trays

Level loops closed with D,B

2 MVs - L,V
41 Measurements - \( T_1, T_2, T_3, \ldots, T_{41} \)
3 DVs - \( F, ZF, qF \)

*Compositions are indirectly controlled by controlling the tray temperatures

\[
J = \left( \frac{y_D - y_{D,s}}{y_{D,s}} \right)^2 + \left( \frac{x_B - x_{B,s}}{x_{B,s}} \right)^2
\]
Distillation Column: Full H

\[ C = H y \]

\[ c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \quad y = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{41} \end{bmatrix} \]

\[ c_1 = h_{11} T_1 + h_{12} T_2 + \cdots + h_{141} T_{41} \]

\[ c_2 = h_{21} T_1 + h_{22} T_2 + \cdots + h_{241} T_{41} \]

\[ H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{120} & \cdots & h_{130} & \cdots & h_{141} \\ h_{21} & h_{22} & \cdots & h_{220} & \cdots & h_{230} & \cdots & h_{241} \end{bmatrix} \]

Find \( H \) that minimizes

\[ L_{avg} = \| J_{uu}^{1/2} (H G^y)^{-1} HY \|_F \]
Case Study : Distillation Column

\[ L_{avg} = \frac{1}{2} \|(J_{uu}^{1/2} (HG_y)^{-1} HY)\|_F^2 \]

\[ Y = [FW_d \ W_n] \]

\[ F = G_y J_{uu}^{-1} J_{ud} - G_d^y \]

Data

\[ G_y \in \mathbb{R}^{41 \times 2}; G_d^y \in \mathbb{R}^{41 \times 3}; J_{uu} \in S^2; J_{ud} \in \mathbb{R}^{2 \times 3}; W_d \in \mathbb{R}^{3 \times 3}; W_n \in \mathbb{R}^{41 \times 41} \]

\[ G_y = \begin{bmatrix} 10.83 & -10.96 \\ 15.36 & -15.55 \\ \vdots & \vdots \\ 13.01 & -12.81 \\ 8.76 & -8.62 \end{bmatrix}; G_d^y = \begin{bmatrix} 5.85 & 11.17 & 10.90 \\ 8.30 & 15.86 & 15.47 \\ \vdots & \vdots & \vdots \\ 5.85 & 13.10 & 12.90 \\ 3.94 & 8.82 & 8.68 \end{bmatrix} \]

\[ J_{uu} = \begin{bmatrix} 3.88 & -3.88 \\ -3.89 & 3.90 \end{bmatrix}; J_{ud} = \begin{bmatrix} 1.96 & 3.96 & 3.88 \\ -1.97 & -3.97 & -3.89 \end{bmatrix}; \]

\[ W_d = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}; W_n = \text{diag}(0.5*\text{ones}(41,1)) \]
## Distillation Column Full H : Result

<table>
<thead>
<tr>
<th>No. Measurement</th>
<th>( c's ) as combinations of measurements</th>
<th>Loss ( \frac{1}{2} | M |_F^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( c_1 = T_{12} ) &lt;br&gt;( c_2 = T_{30} )</td>
<td>0.5477</td>
</tr>
<tr>
<td>3</td>
<td>( c_1 = T_{12} + 0.0446 T_{31} ) &lt;br&gt;( c_2 = T_{30} + 1.0216 T_{31} )</td>
<td>0.4425</td>
</tr>
<tr>
<td>4</td>
<td>( c_1 = 1.0316 T_{11} + T_{12} + 0.0993 T_{31} ) &lt;br&gt;( c_2 = 0.0891 T_{11} + T_{30} + 1.0263 T_{31} )</td>
<td>0.3436</td>
</tr>
<tr>
<td>41</td>
<td>( c_1 = f(T_1, T_2, \ldots, T_{41}) ) &lt;br&gt;( c_2 = f(T_1, T_2, \ldots, T_{41}) )</td>
<td>0.0813</td>
</tr>
</tbody>
</table>
Distillation Column Full H : Result

Comparison with customized Branch And Bound (BAB)

- MIQP is computationally more intensive than Branch And Bound (BAB) methods (Note that computational time is not very important as control structure selection is an offline method)

- MIQP formulations are intuitive and easy to solve

* Kariwala and Cao, 2010
Other case studies

• Toy example
  - 4 measurements, 2 inputs, 1 disturbance

• Evaporator system
  - 10 measurements, 2 inputs, 3 disturbances

• Kaibel distillation column
  - 71 measurements, 4 inputs, 7 disturbances
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Convex approximation methods for structured H

Structured H will have some zero elements in H

Example:
decentralized H
(block-diagonal H)

\[
H_{BD} = \begin{bmatrix}
H_1 & 0 & \cdots & 0 \\
0 & H_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & H_{n_{iu}}
\end{bmatrix}
\]

\[
H_T = \begin{bmatrix}
H_{11} & H_{12} & \cdots & H_{1n_{iu}} \\
0 & H_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & H_{n_{iu}n_{iu}}
\end{bmatrix}
\]

triangular H
Convex approximations for Structured $H$

$$\min_H \left\| J_{uu}^{1/2} (HG_y)^{-1} HY \right\|_F$$

$D$ : any non-singular matrix

$$H_1 = DH \quad (H_1G_y)^{-1}H_1 = (DHG_y)^{-1}DH = (HG_y)^{-1}D^{-1}DH = (HG_y)^{-1}H$$

For a structured $H$ like

$$H_{BD} = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{n_{iu}} \end{bmatrix}$$

or

$$H_T = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n_{iu}} \\ 0 & H_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{n_{iu}n_{iu}} \end{bmatrix}$$

only a block diagonal

$$D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{n_{iu}} \end{bmatrix}$$

or triangular

$$D = \begin{bmatrix} D_{11} & D_{12} & \cdots & \vdots \\ 0 & D_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{n_{iu}n_{iu}} \end{bmatrix}$$

preserves the structure in $H$ and $H_i = DH$ and the degrees of freedom in $D$ is used to arrive at convex approximation methods.
CVs with structural constraints (structured H) : Convex upper bound (structured H)

Examples 1 :

Full H

\[ H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, \quad H_1 = DH \]

Decentralized H

\[ H = \begin{bmatrix} h_{11} & h_{12} & 0 & 0 \\ 0 & 0 & h_{23} & h_{24} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}, \quad H_1 = DH \]

Triangular H

\[ H = \begin{bmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix}, \quad H_1 = DH \]

For structured H, less degrees of freedom in D result in convex upper bound

\[ HG^y \neq J_{uu}^{1/2} \]
Convex approximation methods for structured $H$

Convex approximation method 1:
matching elements in $HG_y$ to $J_{uu}^{1/2}$

$$\min_{h_\delta, \beta_\delta} h_\delta^T F_\delta h_\delta$$

s.t.

$$-b(1 - \beta_l) \leq (G_\delta^y)^T h_\delta - j_\delta) |t \leq b(1 - \beta_l), \forall l = 1, 2, \ldots, n_u n_u$$

$$n_u \leq \sum_{l=1}^{n_u n_u} \beta_l \leq n_{nz} \quad \beta_l \in \{0, 1\}$$

$$n_{uk} \leq \sum_{p=0}^{n_u-1} \sum_{j=1}^{n_{uk}} \beta_{u_p+j} \leq n_{nzk}, \forall k = 1, 2, \ldots, \text{number of blocks}$$

$$h_\delta(\text{ind}) = 0, \text{ind is for 0 in particular structure } H$$

Convex approximation method 2:
Relaxing the equality constraint to inequality constraint

$$\min_{h_\delta} h_\delta^T F_\delta h_\delta$$

s.t. $G_\delta^y h_\delta \leq j_\delta$

$h_\delta(\text{ind}) = 0, \text{ind is for 0 in particular structure } H$
Controlled variable selection with structured H

Optimization problem:

Minimize the average loss by selecting a structured H and CVs as

(i) best individual measurements

(ii) best combinations of all measurements

(iii) best combinations with few measurements
structured H with optimal measurement subsets

Convex approximation method 1: matching elements of $HG^y$ to $J_{uu}^{1/2}$

\[
\begin{align*}
\min_{h_\delta, j_\delta} & \quad h_\delta^T F_\delta h_\delta \\
\text{s.t.} & \quad -b(1 - \beta_l) \leq (G_\delta^T h_\delta - j_\delta)_l \leq b(1 - \beta_l), \ \forall l = 1, 2, \ldots, n_u n_u \\
& \quad n_u \leq \sum_{l=1}^{n_u n_u} \beta_l \leq n_z \\
& \quad \beta_l \in \{0, 1\}
\end{align*}
\]

\[
\begin{align*}
\sum_{p=0}^{n_u-1} \sum_{j=\sum_{n_u k} + 1}^{n_u k + \beta_{n_u k} + j} \beta_{n_u k + j} \leq n_{n_z}, \ \forall k = 1, 2, \ldots, \text{number of } b_{n_u n_u}
\end{align*}
\]

$h_\delta(\text{ind}) = 0$, ind is for 0 in particular structure $H$

Convex approximation method 2: relaxing equality constraint to inequality constraint

\[
\begin{align*}
\min_{h_\delta} & \quad h_\delta^T F_\delta h_\delta \\
\text{s.t.} & \quad G_\delta^y h_\delta \leq j_\delta \\
& \quad h_\delta(\text{ind}) = 0, \ \text{ind is for 0 in particular structure } H \\
& \quad \mathbf{P} \mathbf{\sigma_\delta} = \mathbf{s}
\end{align*}
\]

\[
\begin{bmatrix}
-m \\
-m \\
\vdots \\
-m \\
\end{bmatrix} \leq \begin{bmatrix}
h_{1j} \\
h_{2j} \\
\vdots \\
h_{n_u j} \\
\end{bmatrix} \leq \begin{bmatrix}
m \\
m \\
\vdots \\
m \\
\end{bmatrix} \mathbf{\sigma_j}, \ \forall j \in 1, 2, \ldots, n_y
\]

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Ramprasad Yelchuru, Quantitative methods for controlled variables selection, 28
Distillation column: Decentralized H

\[ c_1 = h_{11}T_1 + h_{12}T_2 + \ldots + h_{120}T_{20} \]
\[ c_2 = h_{21}T_{21} + h_{22}T_{22} + \ldots + h_{241}T_{41} \]

\[ H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{120} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & h_{221} & h_{222} & \cdots & h_{241} \end{bmatrix} \]

Decentralized structure

Binary distillation column
**Distillation Column : Results**

<table>
<thead>
<tr>
<th>Meas</th>
<th>CV</th>
<th>Full H</th>
<th>Block diagonal H</th>
<th>Convex approximation method 1</th>
<th>Convex approximation method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>CV</td>
<td>$c_1 = T_{12}$</td>
<td>$c_1 = T_{12}$</td>
<td>$c_1 = T_{12}$</td>
<td>$c_1 = T_{12}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_2 = T_{30}$</td>
<td>$c_2 = T_{29}$</td>
<td>$c_2 = T_{29}$</td>
<td>$c_2 = T_{29}$</td>
</tr>
<tr>
<td>Loss $|\mathbf{M}|_F^2$</td>
<td>0.548</td>
<td>0.553*</td>
<td>0.553*</td>
<td>0.553*</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>CV</td>
<td>$c_1 = -0.0369T_{12} + 0.6449T_{30} + 0.6572T_{31}$</td>
<td>$c_1 = 0.63T_{30} + 0.6229T_{31}$</td>
<td>$c_1 = 0.63T_{30} + 0.6229T_{31}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_2 = -1.2500T_{12} + 0.2051T_{30} + 0.1537T_{31}$</td>
<td>$c_2 = 0.9675T_{12}$</td>
<td>$c_2 = 0.9675T_{12}$</td>
<td></td>
</tr>
<tr>
<td>Loss $|\mathbf{M}|_F^2$</td>
<td>0.443</td>
<td>0.443**</td>
<td>0.443**</td>
<td>0.443**</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>CV</td>
<td>$c_1 = 0.01T_{11} - 0.0460T_{12} + 0.6450T_{30} + 0.6574T_{31}$</td>
<td>$c_1 = 0.63T_{30} + 0.6229T_{31}$</td>
<td>$c_1 = 0.63T_{30} + 0.6229T_{31}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_2 = -0.6576T_{11} - 0.6548T_{12} + 0.2017T_{30} + 0.1413T_{31}$</td>
<td>$c_2 = -0.5151T_{11} - 0.5110T_{12}$</td>
<td>$c_2 = -0.5151T_{11} - 0.5110T_{12}$</td>
<td></td>
</tr>
<tr>
<td>Loss $|\mathbf{M}|_F^2$</td>
<td>0.344</td>
<td>0.344</td>
<td>0.344</td>
<td>0.344</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>CV</td>
<td>$c_1 = f(T_{11}, T_{21}, \ldots , T_{41})$</td>
<td>$c_1 = f(T_{21}, T_{22}, \ldots , T_{41})$</td>
<td>$c_1 = f(T_{21}, T_{22}, \ldots , T_{41})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_2 = f(T_{11}, T_{21}, \ldots , T_{41})$</td>
<td>$c_2 = f(T_{11}, T_{21}, \ldots , T_{41})$</td>
<td>$c_2 = f(T_{11}, T_{21}, \ldots , T_{41})$</td>
<td></td>
</tr>
<tr>
<td>Loss $|\mathbf{M}|_F^2$</td>
<td>0.081</td>
<td>0.105†</td>
<td>0.127†</td>
<td>0.127†</td>
<td></td>
</tr>
</tbody>
</table>

*clearly not optimal as the solutions must be same with CVs as individual measurements
† small differences in the optimal solution in convex approximation methods 1 and 2 for triangular H and block diagonal H
Decentralized H: Result

- The proposed methods are not exact (Loss should be same for H full and H disjoint for individual measurements)
- Proposed method provide good upper bounds for the distillation case
Distillation column: Triangular H

$$c_1 = h_{121}T_{21} + h_{122}T_{22} + \cdots + h_{141}T_{41}$$
$$c_2 = h_{21}T_1 + h_{22}T_2 + \cdots + h_{241}T_{41}$$

$$H = \begin{bmatrix}
0 & 0 & \cdots & 0 & h_{121} & h_{122} & \cdots & h_{141} \\
h_{21} & h_{22} & \cdots & h_{220} & h_{221} & h_{222} & \cdots & h_{241}
\end{bmatrix}$$

Triangular structure
**clearly not optimal as triangular H must at least be as good as H disjoint**

† small differences in the optimal solution in convex approximation methods 1 and 2 for triangular H and block diagonal H
The proposed methods are not exact (Loss should be same for full H, triangular H for individual measurements)

- Proposed method provide good upper bounds for the distillation case
- In convex approximation methods we are minimizing $\|HY\|_F$ and $\|HY\|_F$ smaller for $n = 5$ than $n = 4$, but the loss $\left\|F^{1/2}(HG)^{-1}HY\right\|_F$ is higher for $n = 5$ than $n = 4$ and causes irregular behavior
Presentation outline

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CV - Controlled Variables
MIQP - Mixed Integer Quadratic Programming
Control system hierarchy for plantwide control

- Scheduling (weeks)
- Site-wide optimization (day)
- Local optimization (hour)
  - Control layer
    - Supervisory control (minutes)
      - \( CV_1 = H_1 y \)
    - Regulatory control (seconds)
      - \( CV_2 = H_2 y \)

Self optimizing control

Regulatory control
Regulatory control layer: Objectives

Regulatory layer should

(1) facilitate stable operation
    regulate the process
    operate the plant in a linear operating region

(2) be simple

(3) avoid control loop reconfiguration

How to quantify?
Regulatory control layer: Objectives

(1) Minimize state drift \( J(\omega) = \|Wx(j\omega)\|_2^2 \)  \( W \) : state weighting matrix

(2) Simple: Close minimum number of loops

(3) Avoid control loop reconfiguration

Quantified the regulatory layer objectives
Regulatory control layer: Justification to use steady state analysis

Typical frequency dependency plot

Steady state based state drift is fairly good over a frequency bandwidth
Regulatory control layer: Problem Formulation

\[ L = J(u, d) - J_{opt}(u_{opt}(d), d) \]
\[ = \|Wx\|^2 - \|Wx_{opt}(d)\|^2 \]

\[ L_{avg} = \left\| J_{2_{uu}}^{1/2} (H_2 G^y)^{-1} H_2 Y_2 \right\|^2_F \]

Loss is due to
(i) Varying disturbances
(ii) Implementation error in controlling \( c \) at set point \( c_s \)

\[ Y_2 = [(G^y J_{2_{uu}}^{-1} J_{2_{ud}} - G_d^y)W_d \ W_n] \]
\[ = [F_2 W_d \ W_n] \]

Kariwala et al. I&ECR, 2008
Problem formulation

\[ c = H_2[y_m \ u_0] \]

- \( n_{ym} \) number of \( y_m \)
- \( n_{u0} \) number of physical valves
- \( n_c = \) number of CVs = \( n_u \)

P1. Close 0 loops: Select \( n_{c} \) variables from \( u_0 \)
or (0 variables from \( y_m \))

P2. Close 1 loops: Select 1 variables from \( y_m \)

P3. Close 2 loops: Select 2 variables from \( y_m \)

P4. Close \( k \) loops: Select \( k \) variables from \( y_m \)

P5. Close \( n_c \) loops: Select \( n_c \) variables from \( y_m \)

Example

\[ y_m \] \[ [H_y \ H_u] \]

\[ H_2 = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{14} \\ h_{15} & h_{16} & \cdots & h_{18} \\ h_{21} & h_{22} & \cdots & h_{24} \\ h_{25} & h_{26} & \cdots & h_{28} \end{bmatrix} \]

- \( n_{ym} = 4 \)
- \( n_{u0} = 4 \)
- \( n_c = 2 = n_u \)

\[ \implies \text{Pick } n_c \text{ columns in } H_u \]

\[ \implies \text{Pick 1 column in } H_y \text{ and } n_c - 1 \text{ columns in } H_u \]

\[ \implies \text{Pick 2 columns in } H_y \text{ and } n_c - 2 \text{ columns in } H_u \]

\[ \implies \text{Pick } k \text{ columns in } H_y \text{ and } n_c - k \text{ columns in } H_u \]

\[ \implies \text{Pick } n_c \text{ columns in } H_y \text{ and 0 columns in } H_u \]
MIQP formulation

\[ H_2 = \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_{ny} \\ h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny} \]

is vectorized along the rows of \( H \) to form

\[ \sigma_i \in \{0, 1\} \]

\[ i = 1, 2, \cdots, ny \]

\[ h_\delta = \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{nu*ny} \end{bmatrix}_{(nu*ny) \times 1} \quad \sigma_\delta = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{ny} \end{bmatrix}_{ny \times 1} \]
Regulatory layer selection: Solution approach

**MIQP formulation**

\[
\begin{aligned}
\min_{x_\delta, \sigma_\delta} & \quad h_\delta^T F_\delta h_\delta \\
\text{st.} & \quad G_\delta^T h_\delta = J_\delta \\
& \quad P \sigma_\delta = n \\
& \quad \begin{bmatrix}
- m \\
- m \\
\vdots \\
- m \\
\end{bmatrix} \leq \\
& \quad \begin{bmatrix}
h_{1i} \\
h_{2i} \\
\vdots \\
h_{nui} \\
\end{bmatrix} \leq \\
& \quad \begin{bmatrix}
m \\
m \\
\vdots \\
m \\
\end{bmatrix} \\
\forall i = 1, 2, \ldots, ny
\end{aligned}
\]
Case Study: Distillation Column

Binary Distillation Column
LV configuration

41 Trays

Level loops closed with D, B

2 MVs - L, V
41 Measurements - T₁, T₂, T₃,..., T₄₁
3 DVs - F, ZF, qF

*Compositions are indirectly controlled by controlling the tray temperatures

\[ J = \| W \Delta x \|_2^2 \]
Case Study: Distillation Column

\[ L_{\text{avg}} = \left\| J_{2_{2u}}^{1/2}(H_2 G^y)^{-1} H_2 Y_2 \right\|_F^2 \]

\[ Y_2 = (G^y J_{2_{2u}}^{-1} J_{2_{2d}} - G_d^y) W_d \ W_n \]

Data

\[ G^y \in \mathbb{R}^{41 \times 2}; \quad G_d^y \in \mathbb{R}^{41 \times 3}; \quad J_{2_{2u}} \in \mathbb{S}^2; \quad J_{2_{2d}} \in \mathbb{S}^{2 \times 3}; \quad W_d \in \mathbb{R}^{3 \times 3}; \quad W_n \in \mathbb{R}^{41 \times 41} \]

\[ G^y = \begin{bmatrix} 10.83 & -10.96 \\ 15.36 & -15.55 \\ \vdots & \vdots \\ 13.01 & -12.81 \\ 8.76 & -8.62 \end{bmatrix}; \quad G_d^y = \begin{bmatrix} \vdots & \vdots & \vdots \\ 5.85 & 11.17 & 10.90 \\ 8.30 & 15.86 & 15.47 \end{bmatrix} \]

\[ W_d = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}; \quad W_n = \text{diag}(0.5 \times \text{ones}(41,1)) \]
Regulatory control layer

CVs \( c = H_2 y \) as individual measurements

\[
c = H_2 [y_m \ u_0]
\]

- \( n_{ym} \) number of \( y_m \)
- \( n_{u0} \) number of physical valves
- \( n_c = \) number of CVs = \( n_u \)

\[
P1. \text{ Close 0 loops: Select (2 variables from } u_0) \quad \Rightarrow \quad \text{Pick 2 columns in } H_u
\]

\[
P2. \text{ Close 1 loops: Select 1 variables from } y_m \quad \Rightarrow \quad \text{Pick 1 column in } H_y \text{ and 1 column in } H_u
\]

\[
P3. \text{ Close 2 loops: Select 2 variables from } y_m \quad \Rightarrow \quad \text{Pick 2 columns in } H_y \text{ and 0 column in } H_u
\]

Total \( n_u + 1 = 3 \) MIQP problems
### Regulatory control layer: Result

Table 5.1: Distillation column case study: the self optimizing variables $c$’s as combinations of 2, 3, 4, 5, 41 measurements with their associated losses in state drift

<table>
<thead>
<tr>
<th>No. of loops closed</th>
<th>No. of meso. used</th>
<th>Optimal meso.</th>
<th>$c_2 = Y$</th>
<th>$c_2 = B$</th>
<th>$c_1 = L$</th>
<th>$c_2 = T_{15}$</th>
<th>$c_2 = T_{26}$</th>
<th>$c_2 = T_{38}$</th>
<th>$c_2 = T_{39}$</th>
<th>Loss ($J - J_{OPT}(d)$)</th>
<th>$|M_2|$</th>
<th>$J - |W_2|$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>$[T_{15} \ T_{26} \ L]$</td>
<td>$c_1 = L$</td>
<td>$c_2 = 3.512T_{15}$</td>
<td>0.026</td>
<td>0.017</td>
<td>0.031</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>$[T_{15} \ T_{16} \ T_{27} \ L]$</td>
<td>$c_1 = L$</td>
<td>$c_2 = 3.512T_{15}$</td>
<td>0.026</td>
<td>0.017</td>
<td>0.031</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>$[T_{15} \ T_{16} \ T_{27} \ L]$</td>
<td>$c_1 = L$</td>
<td>$c_2 = 3.512T_{15}$</td>
<td>0.026</td>
<td>0.017</td>
<td>0.031</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† In addition to two closed level loops

The loss is minimized to obtain $H_2$

The optimal state drift $J_{OPT}(d) = 0.0204$

1 loop closed: 1 $c$ from $y_m$, 1 $c$ from $u_0$

2 loops closed: 2 $c$ from $y_m$

The loss is minimized to obtain $H_2$

†† Such a high value is not physical, but it follows because our linear analysis is not appropriate when we close 0 loops

* used partial control idea to find optimal $H_2$ in two step approach
Regulatory control layer results

Figure 5.5: Distillation column state drift in the presence of disturbances $F, z_F, q_F$: (a) optimal policy (minimum achievable state drift), (b) optimal zero-loop policy, (c) optimal one-loop policy, (d) optimal two-loop policy. Effect of a measurement noise on state drift is shown with + in subplots (b), (c) and (d).
Regulatory control layer result

Figure 5.6: Distillation case study: The reduction in loss in state drift vs number of used measurements, top: loss with one loop closed, bottom: loss with two loops closed.
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Conclusions and Future work

Concluding remarks

- Controlled variables selection formulation in the self-optimizing control framework is presented
- Using steady state economics, the optimal controlled variables, c = Hy, are obtained as
  - optimal individual measurements
  - optimal combinations of ’n’ measurements
    for full H using MIQP based formulations.
- Controlled variables c = Hy, are obtained with a structured H. The proposed convex approximation methods are not exact for structured H, but provide good upper bounds.
- Extended the self-optimizing control concepts to find regulatory layer control variables (CV₂) that minimize the state drift.

Future work:

- Robust optimal controlled variable selection methods
- Fixed CV for all active constraint regions
- Economic optimal CV selection based on dynamics

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Publications

Chapter 3


Chapter 4


Chapter 5

Thank You