Dynamic compensation of static estimators from Loss method

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**Motivation**

**Goal:** Correct the dynamic behaviour of a variable which is calculated by combining different measurements

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**Area of application**

1. **Self-optimizing control**
   - Active constraint \( c = Hy \) (\( c \) is a physical variable)
   - Combination of measurements \( c = Hy \) (\( c \) is not a physical variable)

2. **Static soft-sensor**
   - Estimation of a primary variable by combining different measurements with different weights \( \hat{y} = Hy \)

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1. Ghadr dan et al. (NTNU & SINTEF) Dynamic compensation of static estimators
1. Introduction
   - Loss method: closed-loop estimation
   - Distillation case-study
   - Results

2. Dynamic compensation of static estimators
   - Cascade control
   - Selection of subset of measurements
   - Filtering
     - Optimization of LP filter parameters
     - Explicit solution for the filter problem

3. Concluding remarks
OBJECTIVE
The main objective is to find a linear combination of measurements such that keeping these constant indirectly leads to nearly accurate estimation with a small loss $L$ in spite of unknown disturbances, $d$, and measurement noise, $n^x$.

$$\min_H \| e \|_2 = \| y - \hat{y} \|_2$$
Assumption: Linear models for the primary variables $y$, measurements $x$, and secondary variables $z$

\begin{align*}
y &= G_y u + G_y^d d \\
x &= G_x u + G_x^d d \\
z &= G_z u + G_z^d d \\
G_y &= \left( \frac{\partial y}{\partial u} \right)_d, \quad G_y^d = \left( \frac{\partial y}{\partial d} \right)_u \\
G_x &= \left( \frac{\partial x}{\partial u} \right)_d, \quad G_x^d = \left( \frac{\partial x}{\partial d} \right)_u \\
G_z &= \left( \frac{\partial z}{\partial u} \right)_d, \quad G_z^d = \left( \frac{\partial z}{\partial d} \right)_u
\end{align*}

The actual measurements $x_m$, containing measurement noise $n^x$ is

\[ x_m = x + n^x \]

The linear estimator is of the form

\[ \hat{y} = Hx_m \]
If $\tilde{F} = \begin{bmatrix} FW_d & W_{n^x} \end{bmatrix}$ is full rank, which is always the case if we include independent measurement noise, then

$$H_4^T = \left(\tilde{F}\tilde{F}^T\right)^{-1} G_x \left(G_x^T \left(\tilde{F}\tilde{F}^T\right)^{-1} G_x\right)^{-1} G_y$$

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3 Alstad et al. (2009), Optimal measurement combinations as controlled variables, J. Proc. Control, 19 (1), 138-148
Distillation case-study

- Components
  - A - Methanol
  - B - Ethanol
  - C - Propanol
  - D - n-Butanol

- 4-component system

- Thermodynamics: Wilson

- Objective: Estimate compositions from combination of temperature measurements
H values

\[
\begin{bmatrix}
\Delta \hat{y}_1 \\
\Delta \hat{y}_2
\end{bmatrix} = H
\begin{bmatrix}
\Delta T_5 \\
\Delta T_6 \\
\Delta T_7 \\
\vdots \\
\Delta T_{32}
\end{bmatrix}
\]
Monitoring the composition estimated when single temperature loops are closed ("Open-loop estimation (S1)")

![Top composition estimate with -1% change in boilup](image1)

![Bot composition estimate with -1% change in boilup](image2)

**Figure**: Top estimate with -1% change in boilup

**Figure**: Bot. estimate with -1% change in boilup
Monitoring the composition estimated when single temperature loops are closed ('Open-loop estimation (S3)')

**Feed disturbance:** +10%

**zF1 disturbance:** -4%

**zF4 disturbance:** -4%

Ghadrdan et al. (NTNU & SINTEF)
We studied 3 approaches:

- **Cascade Control:**
  The idea is to close a fast inner loop based on a measurement with no RHP-zero and adjust the setpoint on a time scale slower than the RHP-zero.

- **Use of measurements from the same section of the process:**
  It is less likely to get RHP-zero if the dynamic behavior of the measurements are similar. However, this gives a larger steady-state error.

- **Filters:**
  Low-pass filters will keep the system optimal at steady state. The filtered measurements are \( \hat{y} = H_H F u \)
Example

\[ G_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix} \]

and the optimal matrix \( H \) is

\[ H = \begin{bmatrix} 2 & -1 \end{bmatrix} \]

the transfer function from \( u \) to \( \hat{y} \) is

\[
G = HG_x = \frac{2}{3s+1} - \frac{1}{s+1} = \frac{1-s}{(3s+1)(s+1)} \approx \frac{e^{-1.5s}}{3.5s+1}
\]

**Figure**: Block diagram of the estimation
Theorem

*Cascade (inner-loop) can not move the zero of $HG_x$*

Proof.

The expression for the estimated primary variable is

$$\hat{y} = h_1 x_1 + h_2 x_2$$

where

$$x_1 = g_1 u, \quad x_2 = g_2 u$$

and $u = k(x_2 - x_2)$
So,

\[ x_1 = \frac{g_1}{g_2} x_2, \quad x_2 = \frac{kg_2}{1 + kg_2} x_{2s} \]

The transfer function from \( x_{2s} \) to \( \hat{y} \) is

\[ \hat{y} = \left( h_2 + h_1 \frac{g_1}{g_2} \right) \frac{kg_2}{1 + kg_2} x_{2s} \]

The term \( (h_1 g_1 + h_2 g_2) \), which includes the RHP zero, is unchanged.
To improve the dynamic controllability: Put structural constraints on the measurements \(^4\)

This is done to

- reduce the time delay between the MVs to CVs,
- have measurements of the same dynamics to avoid inverse response.

**In our example:** Choose measurement from one side of the column

**Drawback:** Less accurate compared to the option where we use all the measurements

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Filtering

Figure: Block diagram of the estimation system including filter ($H_F$)

$$H_F = \begin{bmatrix} \frac{1}{\tau_{F1}s+1} & 0 \\ 0 & \frac{1}{\tau_{F2}s+1} \end{bmatrix}$$

$H_F(0) = I$
Example

\[ G_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix} \]

and the optimal matrix \( H \) is

\[ H = \begin{bmatrix} 2 & -1 \end{bmatrix} \]

Some Filters:

\[ H_{F1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{3s+1} \end{bmatrix} \]

\[ H_{F2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{s+1}{3s+1} \end{bmatrix} \]

\[ H_{F3} = \begin{bmatrix} \frac{3s+1}{s+1} & 0 \\ 0 & 1 \end{bmatrix} \]

The filtered transfer function will be

\[ H_{dyn1}G_x = \frac{1}{(3s+1)(s+1)} \approx \frac{e^{-0.5s}}{3.5s+1} \]

\[ H_{dyn2}G_x = \frac{1}{3s+1} \]

\[ H_{dyn3}G_x = \frac{1}{s+1} \]
Example

\[
G_x = \begin{bmatrix}
\frac{1}{3s+1} \\
\frac{1}{1} \\
\frac{1}{s+1}
\end{bmatrix}
\]

and the optimal matrix \( H \) is

\[
H = \begin{bmatrix}
2 \\
-1
\end{bmatrix}
\]

Some Filters:

\[
H_{F4} = \begin{bmatrix}
1 \\
0 \\
\frac{1}{3s+1}
\end{bmatrix}
\]

The filtered transfer function will be

\[
H_{\text{dyn}4}G_x = \frac{2s + 1}{(3s + 1)(s + 1)} \approx \frac{0.83e^{-0.25s}}{1.25s + 1}
\]
Example

Using Lead-lag compensators, we can make the response as fast as we want.

**Figure**: Step response for different cases
**Distillation case-study**

**Figure**: \( \mathbf{H}_G \mathbf{x} (t) \) with -1% change in boilup and constant Reflux ratio

**Figure**: Estimated composition (\( t_f = \mathbf{H} \mathbf{G}_x \)) and filtered estimated composition (\( t_f = \mathbf{H} \mathbf{H}_F \mathbf{G}_x \)) where there are filters only on 6th, 16th and 17th measurements.
Optimizing filters

\[
\min_{H_F} \| G_{ref} - HH_F G_x \|
\]

Figure: Real composition, Estimated composition \((HG_x)\) and filtered estimated composition \((HH_F G_x)\) where filters are optimized for first 100 min. assuming \(G_{ref} = G_{u \rightarrow y_1}\).
Optimizing filters

Figure: Real composition, Estimated composition ($x_{C3}$) and filtered estimated composition ($\hat{x}_{C3}$) where filters are optimized for first 100 min. assuming $G_{ref} = G_u \rightarrow y$.
Explicit solution for the filter problem

Approach:
Convert the model matching problem to Nehari problem

\[ \| T_1 - T_2 Q \|_\infty \Rightarrow \| R - X \|_\infty = \| \Gamma_R \| \leq 1 \]

- An optimal \( Q \) exists if the ranks of the two matrices \( T_2(j\omega) \) and \( T_3(j\omega) \) are constant for all \( 0 < \omega < \infty \).

Our problem:

\[ \min_{H_F} \| G_{ref} - HH_F G_x \|_\infty \]

Define: \( H_{dyn} = HH_F \)

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Lemma

Let $U$ be an inner matrix and define $E = \begin{bmatrix} U^\sim & \_ \\ \_ & I - UU^\sim \end{bmatrix}$, then,

$$\|EG\|_\infty = \|G\|_\infty$$

Proof.

It suffices to show that $E^\sim E = I$.

Lemma

Suppose $F$ and $G$ are matrices with no poles on imaginary axis with equal number of columns. If $\| \begin{bmatrix} F \\ G \end{bmatrix} \|_\infty < \gamma$ then $\|G\|_\infty < \gamma$ and $\|FG_0^{-1}\|_\infty < 1$. 

Ghadradan et al. (NTNU & SINTEF)
Theorem

(i) \( \alpha = \inf \{ \gamma : \|Y\|_{\infty} < \gamma, \text{dist}(R, RH_{\infty}) < 1 \} \)

(ii) Suppose \( \gamma > \alpha \), \( G, X \in RH_{\infty} \)

\[ \|R - X\|_{\infty} \leq 1 \]

Then \( \|T_1 - T_2 Q\|_{\infty} \leq \gamma \)

Proof.

(i) Let

\[ \gamma_{inf} = \inf \{ \gamma : \|Y\|_{\infty} < \gamma, \text{dist}(R, RH_{\infty}) < 1 \} \]

choose \( \varepsilon > 0 \) and then choose \( \gamma \) such that \( \alpha + \varepsilon > \gamma > \alpha \). Then there exist \( Q \) in \( RH_{\infty} \) such that

\[ \|T_1 - T_2 Q\|_{\infty} < \gamma \]

From Lemma 1 we have:

\[ \left\| \begin{bmatrix} \hat{U}_i^T \\ I - \hat{U}_i \hat{U}_i^T \end{bmatrix} (T_1 - T_2 Q) \right\|_{\infty} \leq \gamma \]
This is equivalent to \[ \| \begin{bmatrix} U_i T_1 - U_o Q \\ Y \end{bmatrix} \|_\infty < \gamma \]

This implies from Lemma 2 that
\[ \| Y \|_\infty < \gamma \]
\[ \| U_i T_1 Y_o^{-1} - U_o Q Y_o^{-1} \|_\infty < 1 \]

The latter inequality implies \[ \text{dist} \left( R, U_o R H_\infty Y_o^{-1} \right) < 1 \]

\( U_o \) is right-invertible in \( R H_\infty \) and \( Y_o \) is invertible in \( R H_\infty \). So, (26) gives
\[ \text{dist} \left( R, H_\infty \right) < \text{dist} \left( R, R H_\infty \right) < 1 \]
The general algorithm to obtain $Q$ is as follows

**Step 1** Compute $Y$ and $\|Y\|_\infty$

**Step 2** Find an upper bound $\alpha_1$ for $\alpha$ ($\|T_1\|_\infty$ is the simplest choice)

**Step 3** Select a trial value for $\gamma$ in the interval $(\|Y\|_\infty, \alpha_1]$

**Step 4** Compute $R$ and $\|\Gamma_R\|$. Then $\|\Gamma_R\| < 1$ iff $\alpha < \gamma$. Change the value of $\gamma$ correspondingly to meet this criteria

**Step 5** Find a minimal realization of $R$: $R(s) = [A, B, C, 0]$

**Step 6** Solve the Lyapunov equations to find controllability and observability gramians and set $N = (I - L_0 L_c)^{-1}$
Step 7 Set

\[
L_1(s) = \begin{bmatrix} A & -L_c NC^T & C & I \end{bmatrix}
\]
\[
L_2(s) = \begin{bmatrix} A & N^T B & C & 0 \end{bmatrix}
\]
\[
L_3(s) = \begin{bmatrix} -A^T & NC^T & -B^T & 0 \end{bmatrix}
\]
\[
L_4(s) = \begin{bmatrix} -A^T & NL_0 B^T & B^T & I \end{bmatrix}
\]

Step 8 Select \( Y \) in \( RH_\infty \) with \( \| Y \|_\infty \leq 1 \) (for example \( Y = 0 \)) and set

\[
X = R - (L_1 Y + L_2)(L_3 Y + L_4)
\]
Example

\[ G_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix}, \quad G_{\text{ref}} = \frac{1}{0.5s + 1} \]

and the optimal matrix \( H \) is

\[ H = \begin{bmatrix} 2 & -1 \end{bmatrix} \]

\[ H_F = \begin{bmatrix} \frac{2.011s^5 + 5.971s^4 + 0.5466s^3 - 7.412s^2 - 1.321s + 0.2051}{s^6 + 3.699s^5 - 0.418s^4 - 13.02s^3 - 7.559s^2 + 1.578s + 0.4668} \\ \frac{3.017s^4 + 11.27s^3 + 9.929s^2 - 1.597s - 1.026}{s^6 + 3.699s^5 - 0.418s^4 - 13.02s^3 - 7.559s^2 + 1.578s + 0.4668} \end{bmatrix} \]

- A weighting transfer function should be included to make \( H_F(0) = I \)
What should $G_{ref}$ be?

In the case of estimation

- First option: actual values from simulation
- The estimate can be even faster (since $\hat{y}$ is being controlled)

One idea is to specify a first-order transfer function with the smallest time constant in the process as the desired transfer function from inputs to the estimates.

For our case: $G_{ref} = \frac{1}{\tau_{int}s+1}$

Internal time constants can be found from changing the two inputs boilup and reflux rate at the same time such that the external flows remain constant.

\[
\Delta y_D = \left(\frac{-0.06e^{-2s}}{740s + 1}\right)\Delta V
\]

\[
\Delta x_B = \left(\frac{-0.067e^{-0.33s}}{137s + 1}\right)\Delta V
\]
Concluding remarks

- Extra dynamic compensation is necessary when measurements with different dynamics are combined
  - Cascade will not remove the RHP zero, but helps with rejecting disturbance
  - Choosing measurements with similar dynamics might help to avoid dynamic problems
  - Filtering fast dynamic measurements will help remove the inverse response
- Explicit solution comes from converting model matching problem to Nehari problem
- The filter matrix gets big as the number of measurements increase.
- A weight function should be considered to weaken the effect of filter at steady-state
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Thanks for your attention