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Tuning PI controllers based on $\mathcal{H}_\infty$ Weighted Sensitivity

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***Abstract*— $\mathcal{H}_\infty$ control theory is usually associated with high-order controllers. In this paper, simple tuning rules (extending the well-known SIMC) for the Proportional-Integral (PI) algorithm are derived analytically based on $\mathcal{H}_\infty$ Weighted Sensitivity. The presented approach deals with stable, integrating and unstable plants in a unified way, avoiding any notion of coprime factorization. The final tuning involves two adjustable parameters, λ and γ, with clear engineering meaning: λ selects the *Robustness/Performance* compromise, whereas γ allows to balance the *Servo/Regulator* performance. Based on this, several application scenarios are considered. First, a One-Degree-Of-Freedom (1-DOF) PI controller is assumed. Second, 2-DOF and switched implementations are investigated.**

I. INTRODUCTION

The most widely used control algorithm in industry still corresponds to the Proportional-Integral-Derivative (PID) controller, commonly simplified in practice to the Proportional-Integral (PI) type. On the other hand, the modern control literature has been dominated by optimal and robust control theories, including the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimization-based paradigms [1], [2], [3]. Influenced by the control theory mainstream, PID design methods have received a revived interest during the last two decades [4], resulting into a considerable number of analytically derived tuning rules as for example [5], [6], [7], [8], [9], [10], to cite just a few.

For stable plants, PID tuning rules based on Internal Model Control (IMC) [1] appeared in [5], and have since then become very popular. However, for lag-dominant or integrating processes, the associated input disturbance response is sluggish. To circumvent this problem, Horn and coworkers [6] proposed alternative IMC filters. A similar idea was applied later in [7]. As the overshoot after a set-point change in the revised methods [6], [7] can be considerably high, a reference prefilter is recommended to avoid an excessive overshoot.

Adopting a different viewpoint, Skogestad [9] suggested to augment the integral gain in the original IMC-based tuning rules [5], and obtained remarkably simple and effective expressions for the PI(D) parameters with a good tradeoff between *Servo* and *Regulator* operation. One limitation of the so-called SIMC rule in [9] is that it does not consider unstable plants, as recently reported in the closely-related work [11].

In this communication, we focus on PI control and look for a simple tuning of the controller along the lines of [9], but also including unstable plants. The tuning rules are obtained analytically using $\mathcal{H}_\infty$ optimization, and represent both an extension and a simplification of those in the previous works [10], [12], where the full PID structure is used to deal with stable plants only. A previous $\mathcal{H}_\infty$ approach to PI control can be consulted in [13], where, however, no tuning rules are finally provided.

The presented approach stems from considering the $\mathcal{H}_\infty$ Weighted Sensitivity Problem [14], [3], posed in terms of a two-parameter weight: the first parameter (λ) adjusts the *Robustness/Performance* tradeoff, whereas the second one (γ) is used to balance the performance between the set-point and disturbance responses. A distinguishing feature of our $\mathcal{H}_\infty$ method is the usage of a possibly unstable weight, which unifies the treatment of the stable/unstable plants, avoiding any notion of coprime factorization [15], [3].

After deriving the tuning rules, the SIMC rule is revisited to fix γ for balanced *Servo/Regulator* operation. In addition, as extreme values of γ yield *Servo*-type and *Regulator*-type tunings, a switched scheme to integrate them is investigated as an alternative to the more common 2-DOF PI controller.

The organization for the rest of the paper is as follows. Section II reviews the $\mathcal{H}_\infty$ Weighted Sensitivity Problem. Afterwards, the analytical solution is obtained based on the usage of a possibly unstable weight. Section III applies the latter result to PI control, and tuning rules are obtained for the simplest stable, integrating and unstable models. Section IV gives tuning guidelines for linear and switched implementations. Section V evaluates the presented design through simulation examples. To conclude the paper, in Section VI we summarize the main ideas.

Notation:

$\mathcal{RH}_\infty$ is the set of stable transfer functions.

$\mathcal{C}$ is the set of internally stabilizing controllers K. This means that, for any $K \in \mathcal{C}$, all the closed-loop transfer functions in Fig. 1 are stable [1], [3].

$\mathcal{Q}$ is the set of internally stabilizing Q's, assuming the parameterization $K = \frac{Q}{1-PQ}$.

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II. $\mathcal{H}_\infty$ WEIGHTED SENSITIVITY

Consider the basic unity feedback configuration depicted in Fig. 1: P and K are the plant and the controller, respectively, and r, y, u, e, d denote (in the same order) the reference, output, control, error and input (or load) disturbance signals. A basic problem in $\mathcal{H}_\infty$ control is the weighted sensitivity problem [14], [16], [3]:

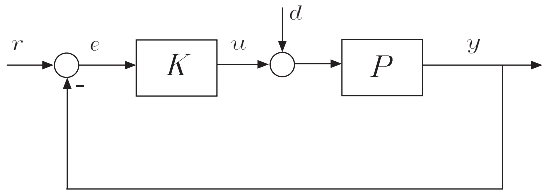

Fig. 1. Basic feedback configuration.

$$\min_{K\in\mathcal{C}} \|\mathcal{N}\|_\infty = \min_{K\in\mathcal{C}} \|WS\|_\infty \tag{1}$$

where

- $\|\mathcal{N}\|_\infty \doteq \sup_\omega |\mathcal{N}(j\omega)|$
- S is the sensitivity function: $S = \frac{1}{1+PK}$.
- W is a Minimum-Phase (MP) weight responsible for the shaping of S (*the design key point*).

As explained in [3], $\mathcal{H}_\infty$ controllers are usually found numerically by posing the problem in the generalized framework of Fig. 2. Within this control setup, G denotes the generalized plant and the objective is to minimize the norm from w to z. Mathematically, this is expressed as

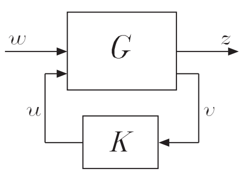

Fig. 2. Generalized control setup.

$$\min_{K\in\mathcal{C}} \|T_{zw}\|_\infty = \min_{K\in\mathcal{C}} \|\mathcal{F}_l(G,K)\|_\infty \tag{2}$$

where $\mathcal{F}_l(G,K)$ is the Lower Fractional Transformation (LFT):

$$\mathcal{F}_l(G,K) \doteq G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{3}$$

For the case at hand,

$$G = \left[\begin{array}{c:c} W & -WP \\ \hdashline 1 & -P \end{array}\right] \tag{4}$$

results into $T_{zw} = \mathcal{F}_l(G,K) = WS$, making (1) equivalent to (2). Note, however, that this numerical approach is not suitable for the purpose of obtaining tuning rules. A more convenient analytical solution is investigated next. The celebrated Youla-Kucera parameterization [17] constitutes an important result for optimization-based controller synthesis, showing that any $K \in \mathcal{C}$ can be parameterized as

$$K = \frac{Y + MQ}{X - NQ} \tag{5}$$

where $Q \in \mathcal{RH}_\infty$ is a free parameter and $X, Y, M, N \in \mathcal{RH}_\infty$ form a coprime factorization of P [15], [3], implying that $P = NM^{-1}$ and $XM + YN = 1$. The above result allows to express any closed-loop transfer function affinely in Q. Therefore, the $\mathcal{H}_\infty$ synthesis problem (2) turns out to be a Model Matching Problem [2], [10]:

$$\min_{K\in\mathcal{C}} \|\mathcal{N}\|_\infty = \min_{Q\in\mathcal{RH}_\infty} \|T_1 - T_2Q\|_\infty \tag{6}$$

where $T_1, T_2 \in \mathcal{RH}_\infty$. For example, the sensitivity function can be parameterized as $S = M(Y - NQ)$, and the weighted sensitivity problem (1) corresponds to $T_1 = WM, T_2 = WMN$, where $T_1, T_2 \in \mathcal{RH}_\infty$ as long as $W \in \mathcal{RH}_\infty$. This procedure is the common approach found in [16], [2]. The main problem with the Youla-Kucera parameterization is the need of calculating a coprime factorization. For our purposes, we want to avoid this computation. The following lemma shows that using a possibly unstable weight does the trick:

Lemma 2.1: Assume that P is purely rational (i.e., there is no time delay in P) and has at least one RHP zero. Take W as a MP weight *including the unstable poles of* P. Then, the optimal weighted sensitivity subject to internal stability ($K \in \mathcal{C}$) is given by

$$\mathcal{N}^o = \rho\frac{q(-s)}{q(s)} \tag{7}$$

where ρ and $q = 1 + q_1 s + \cdots + q_{\nu-1}s^{\nu-1}$ (Hurwitz) are uniquely determined by the interpolation constraints:

$$W(z_i) = \mathcal{N}^o(z_i) \qquad i = 1\ldots\nu, \tag{8}$$

being $z_1 \ldots z_\nu$ ($\nu \geq 1$) the RHP zeros of P.

Proof: Let us parameterize K as follows

$$K = \frac{Q}{1 - PQ} \tag{9}$$

As shown in [1], internal stability is then equivalent to

- $Q \in \mathcal{RH}_\infty$
- $S = 1 - PQ$ has zeros at the unstable poles of P

The weighted sensitivity $WS = W(1 - PQ) = \mathcal{N}^o$ in (7) is achieved by

$$Q_0 = P^{-1}(1 - \mathcal{N}^o W^{-1}) \tag{10}$$

First, we must verify that $Q_0 \in \mathcal{Q}$ (i.e., internal stability):

- That $Q_0 \in \mathcal{RH}_\infty$ follows from the interpolation constraints (8) [14], [2], [10].
- On the other hand, $S = 1 - PQ_0 = W^{-1}\mathcal{N}^o$ is such that $S = 0$ at the unstable poles of P (because W contains them by assumption).

Now that internal stability has been verified, it remains to be proved that Q_0 is optimal. In [1], it is shown that any $Q \in \mathcal{Q}$ has the form $Q_0 + \Upsilon Q_1$, where $Q_1 \in \mathcal{RH}_\infty$ and $\Upsilon \in \mathcal{RH}_\infty$ has (exclusively) two zeros at each closed RHP pole of P (the exact shape of Υ is not necessary for the proof). Hence, any admissible weighted sensitivity can be expressed as

$$\begin{aligned} W(1 - PQ) &= W(1 - P[Q_0 + \Upsilon Q_1]) \\ &= W(1 - PQ_0) - WP\Upsilon Q_1 \\ &= \mathcal{N}^o - WP\Upsilon Q_1 \end{aligned}$$

Minimizing $\|\mathcal{N}^o - WP\Upsilon Q_1\|_\infty$ is a standard Model Matching Problem [16], [2], [10] with: $T_1 = \mathcal{N}^o, T_2 = WP\Upsilon, Q_1 \in \mathcal{RH}_\infty$. The optimal error $\tilde{\mathcal{N}}^o = T_1 - T_2 Q_1$ is well-known to be all-pass and completely determined by the RHP zeros of T_2, which are those of P. More concretely, for each RHP zero of P, we have the interpolation constraint $\tilde{\mathcal{N}}^o(z_i) = \mathcal{N}^o(z_i)$. Obviously, this implies that $\tilde{\mathcal{N}}^o = \mathcal{N}^o$. Equivalently, the optimal solution is achieved for $Q_1 = 0$, showing that Q_0 is indeed optimal. ■

Remark 2.1: Note that the assumption $\nu \geq 1$ is not restrictive because many processes exhibit inverse response characteristics or are affected by time delay, which can be easily approximated by a Non-Minimum Phase rational term. Furthermore, if $\nu = 0$, then the optimal weighted sensitivity is identically zero regardless of W.

Once $\mathcal{N}^o = WS$ has been computed using Lemma 2.1, the associated feedback controller is

$$K = \frac{1-S}{S}P^{-1} = \left(\frac{1-\mathcal{N}^o W^{-1}}{\mathcal{N}^o W^{-1}}\right)P^{-1} \qquad (11)$$

III. GETTING THE PI TUNING RULES

In order to obtain PI tuning rules using Lemma 2.1, we need to specify:

- A simple model for P (this is done in subsection A).
- A simple weight W (this is done in subsection B).

After that, the final tuning rules are obtained in subsection C.

A. Model

We take a First Order plus Time Delay[1] (FOPTD) model [9], [4]:

$$P = K_g \frac{e^{-sh}}{\tau s + 1} \qquad (12)$$

where

- K_g is the (zero frequency) gain.
- h is the (effective) time delay.
- τ is the time constant of the process.

In particular, τ may be negative to account for unstable plants. We will also assume that P is lag-dominant ($h > |\tau|$). Note that integrating plants can be treated considering the limit $\tau \to \infty$, as it will be seen later on.

B. Weight

The following *possibly unstable* weight is proposed

$$W = \frac{(\lambda s + 1)(\gamma s + 1)}{s(\tau s + 1)} \qquad (13)$$

where $\lambda > 0$ and $\gamma \in [\lambda, |\tau|]$ are used as tuning parameters.

The rationale behind the choice is explained below. Start considering $\lambda = 0$, then

- If $\gamma = |\tau|$, $|W| = |1/s|$, and the optimization problem (1), taking P as in (12), is equivalent to $\min_{K \in \mathcal{C}} \|S\|_\infty$ subject to integral action. As $e = Sr$, this choice of γ yields good results for set-point tracking.
- If $\gamma = \lambda$, $|W| = \frac{1}{|K_g|}|P/s|$. Since the constant $\frac{1}{|K_g|}$ plays no role, the optimization problem (1) is now equivalent to $\min_{K \in \mathcal{C}} \|PS\|_\infty$ subject to integral action. As $e = -PSd$, this choice of γ yields the best disturbance attenuation.
- For intermediate values of γ, a *Servo/Regulator* balance is obtained.

As we increase λ, the minimization of $|S|$ at higher frequencies is emphasized, preventing large peaks on S at the expense of closed-loop bandwidth. Thus, once γ is fixed, λ can be used to select a compromise between robustness and performance as in the IMC procedure [1], [10].

C. Solution

In order to apply Lemma 2.1, we need first to approximate the time delay in the FOPTD model (12). For simplicity, we use a first order Taylor approximation:

$$P \approx K_g \frac{-sh + 1}{\tau s + 1} \qquad (14)$$

Now, applying Lemma 2.1 with P and W as in (14) and (13), respectively, the optimal weighted sensitivity function is given by the constant

$$\mathcal{N}^o = \rho = \frac{(\lambda + h)(\gamma + h)}{\tau + h} \qquad (15)$$

From (11) and (14), the corresponding feedback controller is

$$K = \frac{\chi}{K_g \rho s}, \; \chi = 1 + \frac{\tau(h + \lambda + \gamma) - \lambda\gamma}{\tau + h}s \qquad (16)$$

which is of PI type

$$K = K_c\left(1 + \frac{1}{T_i s}\right) \qquad (17)$$

The expressions for the PI parameters can be consulted in Table I. Note that the tuning rule for the integrating

TABLE I
PI TUNING RULE FOR THE MODEL $P = K_g \frac{e^{-sh}}{\tau s + 1}$.

K_c	T_i	
$\frac{1}{K_g}\frac{T_i}{\lambda+\gamma+h-T_i}$	$\frac{\tau(h+\lambda+\gamma)-\lambda\gamma}{\tau+h}$	$\lambda > 0, \gamma \in [\lambda, \|\tau\|]$

model $P = K_g \frac{e^{-sh}}{s}$ can be obtained as a particular case by considering the approximation $K_g \frac{e^{-sh}}{s} \approx K_g \frac{e^{-sh}}{s + 1/\tau} = K_g \tau \frac{e^{-sh}}{\tau s + 1}$ in Table I, and taking the limit $\tau \to \infty$. The final tuning rule is given in Table II. Thus, as the FOPTD model *contains* the Integrating Plus Time Delay (IPTD) one, in the remaining of this paper we will only consider the FOPTD representation.

TABLE II
PI TUNING RULE FOR THE MODEL $P = K_g \frac{e^{-sh}}{s}$.

K_c	T_i	
$\frac{1}{K_g}\frac{T_i}{\lambda\gamma+hT_i}$	$h + \lambda + \gamma$	$\lambda > 0, \gamma \in [\lambda, \infty)$

[1]The time delay will be eventually approximated in subsection C.

IV. EXTENSIONS BASED ON γ-AUTOTUNING

This section outlines possible applications of the tuning rule in Table I. Although we concentrate on how to fix γ, tuning guidelines for λ are also given.

A. (Linear) PI Control

We know from Section III.B that if set-point tracking is the major concern, then $\gamma = |\tau|$ yields the best results. In particular, note that when $\tau > 0$ (stable case): $K_c = \frac{\tau}{K_g(\lambda+h)}, T_i = \tau$ (the well-known IMC rule [5], [1], [9]). On the other hand, if disturbance rejection is the most important thing, the recommended value is $\gamma = \lambda$. The corresponding extreme tuning rules have been collected in Table III. It can happen that both set-point and disturbance responses are important. In a 1-DOF setting, we are forced to seek for a *Servo/Regulator* tradeoff. In such a case, the guidelines below are given:

- If $h/|\tau| \geq 1$, choose $\gamma = |\tau|$
- As $h/|\tau| \to 0$, select $\gamma \to \lambda$ (otherwise the disturbance response would be very sluggish)

For lag-dominant plants, one possibility is to mimmic SIMC [9] by selecting $T_i = 4(\lambda + h)$. This yields:

$$\gamma = (3\tau + 4h)\frac{\lambda + h}{\tau - \lambda} \tag{18}$$

The resulting tuning behaves approximately like SIMC for stable plants but also applies to slow unstable processes. Consequently, the third row of Table III can be regarded as an extended SIMC, or ESIMC for short. Once γ has been fixed, λ can be adjusted online. The larger the value of λ, the smoother the control. A good starting point, based on [9], is to select $\lambda \approx h$ for tight control. If the resulting controller is too aggressive, then increase λ according to [18].

The use of 2-DOF controllers is the most common option to decouple set-point tracking and disturbance rejection. For example, the following PI control law is commercially available

$$u(t) = K_c\left(br(t) - y(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right) \tag{19}$$

where b is the so-called set-point weight. If $b = 1$, (19) reduces to the conventional 1-DOF PI controller (17). In general, the closed-loop system is described by the equation

$$y = \frac{K_2 P}{1 + K_1 P}r + \frac{P}{1 + K_1 P}d \tag{20}$$

where

$$K_1 = K_c\left(1 + \frac{1}{T_i s}\right) \qquad K_2 = K_c\left(b + \frac{1}{T_i s}\right) \tag{21}$$

This corresponds to the block diagram of Fig. 3.

As disturbance rejection only depends on the feedback (or internal) block K_1, the following tuning procedure arises naturally:

- Select K_c and T_i as in the second row of Table III (corresponding to $\gamma = \lambda$), and adjust λ according to the guidelines given in Section IV.

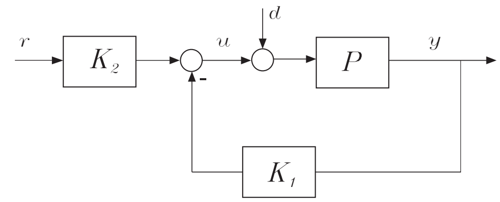

Fig. 3. 2-DOF control configuration.

- If the set-point response exhibits excessive overshoot, decrease b online (starting at one) until the best compromise between rise time and overshoot is achieved.

Remark 4.1: Although this work focuses on input disturbances, disturbances occurring at the output of the plant can be regarded as unmeasured set-point changes. In this sense, the servo/regulation tradeoff can alternatively be thought of as an input/output disturbance tradeoff. Thus, if output disturbances are equally important, (18) represents a reasonable tuning for γ even in the 2-DOF scenario.

B. Switched PI Control

Another strategy to combine both good set-point and disturbance processing is to switch between suitable controllers for each purpose. The potential advantages of switched linear control have been reported in [19], among others. Consider the following scheme:

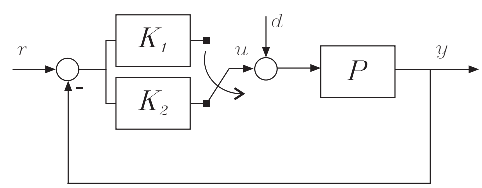

Fig. 4. Switched feedback scheme.

- By default, the system operates in *Regulator* mode. Thus, K_1 is tuned as before using Table III ($\gamma = \lambda$).
- When a set-point change is signaled, we commute to K_2, which is tuned using Table III with $\gamma = |\tau|$. This way, during the set-point tracking, a *Servo-type* tuning is used.
- After t_{sp} seconds, the system switches back to K_1.

The role of t_{sp} is analogous to b in the linear 2-DOF case. If $t_{sp} = 0$, there is no switching and the scheme reduces to the conventional PI controller given by K_1 (K_2 is never active). By increasing t_{sp}, K_2 will be active for a longer period of time, reducing the overshoot after a reference change. Obviously, t_{sp} should be chosen, at maximum, equal to the settling time of the set-point response.

Recently, PI control with reset action — or PI+CI, following the nomenclature introduced in [20]—, has been proposed in different articles [20], [21], [22]. As depicted in Fig. 5, the PI+CI controller can be regarded as a conventional PI compensator with two possible integral gains. Instead of using impulsive reset action based on the error signal as in [20], [21], [22], two alternative tunings based on switching

TABLE III

TUNING RULES FOR EXTREME AND INTERMEDIATE VALUES OF γ.

γ	K_c	T_i	Objective
$\lvert\tau\rvert$	$\frac{1}{K_g}\frac{T_i}{\lambda+\lvert\tau\rvert+h-T_i}$	$\frac{\tau(h+\lambda+\lvert\tau\rvert)-\lambda\lvert\tau\rvert}{\tau+h}$	Set-point tracking
λ	$\frac{1}{K_g}\frac{\tau}{\lambda+h}\left(\frac{h+2\lambda-\lambda^2/\tau}{h+\lambda}\right)$	$\frac{\tau(h+2\lambda)-\lambda^2}{\tau+h}$	Disturbance attenuation
$(3\tau+4h)\frac{\lambda+h}{\tau-\lambda}$	$\frac{1}{K_g}\frac{4(\tau-\lambda)}{4h+3\lambda}$	$4(\lambda+h)$	*Servo/Regulator* tradeoff (ESIMC)

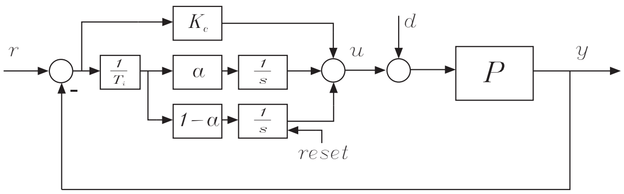

Fig. 5. Unity feedback scheme using the PI+CI controller.

and the proposed design are suggested in Table IV. By assuming positive logic and that set-point changes occur at $t=0$, the reset signal is:

$$\text{reset}(t)=\begin{cases}1 & \text{if} \quad 0\le t\le t_{sp}\\ 0 & \text{if} \quad t>t_{sp}\end{cases} \tag{22}$$

The rationale behind Table IV is explained next. Note that the PI+CI controller offers only a restricted implementation of the general switched scheme, since the reset mechanism just acts over the integral term. Therefore, T_i is chosen as in Table III ($\gamma=\lambda$) when the reset is inactive, and α is such that $\frac{\alpha}{T_i}$ is equal to the other extreme value of T_i given in Table III for $\gamma=|\tau|$. The idea is to recover the largest possible value of T_i when the reset is active. This way, during the set-point transient the integral gain is reduced to improve the tracking (diminishing the overshoot). As K_c is not altered by the reset mechanism, the two options in Table IV correspond to selecting K_c using $\gamma=\lambda$ or $\gamma=|\tau|$. The first option chooses a *Servo*-type tuning rule (Table III, case $\gamma=|\tau|$) when the reset is active, and increases the integral gain for regulation purposes when the reset is inactive. On the other hand, option 2 uses a *Regulator*-type tuning rule (Table III, case $\gamma=\lambda$) for normal operation, and reduces the integral gain to improve the set-point response.

V. SIMULATION EXAMPLES

In this section, we go through two simulation examples. In the first one, the SIMC and ESIMC rules are compared. The second one applies the proposed method to the 2-DOF PI and the PI+CI controllers. The main purpose of these examples is to briefly sketch possible applications of the proposed design, rather than illustrating completely established results.

Example I. (*Revisiting SIMC*) For FOPTD systems, the SIMC rule [9], [3] is

$$K_c=\frac{\tau}{K_g(\lambda+h)} \qquad T_i=\min\{\tau,4(\lambda+h)\} \tag{23}$$

and, as already commented, was devised in a 1-DOF setting for balanced servo/regulation operation of stable plants. From Section IV.A, the proposed ESIMC tuning rule in the third row of Table III represents an extended version of the SIMC tuning. To illustrate this, let us consider $P=\frac{e^{-s}}{20s+1}$ ($K_g=1,h=1,\tau=20$) and $\lambda=h=1$. The corresponding time responses for SIMC and ESIMC are depicted in Fig. 6 (top). As it can be seen, the two methods yield very similar results (indeed, the same happens in general for any ratio between the time delay and the time constant). More precisely, the PI parameters are $K_c=10,T_i=8$ (SIMC) and $K_c=10.8571,T_i=8$ (ESIMC). If we now consider

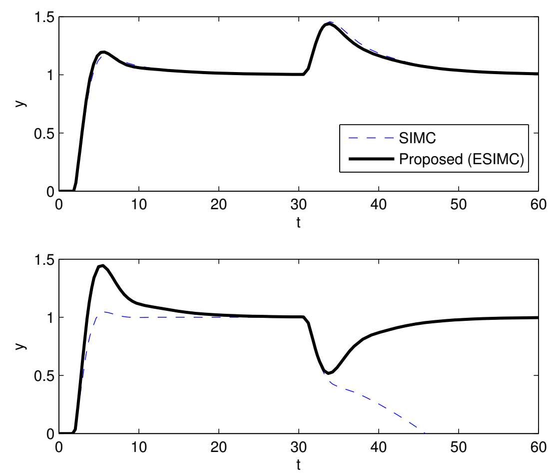

Fig. 6. Time responses for $P=\frac{e^{-s}}{20s+1}$ (top) and $P=\frac{e^{-s}}{-20s+1}$ (bottom).

$P=\frac{e^{-s}}{-20s+1}$ ($K_g=1,h=1,\tau=-20,\lambda=1$) , Fig. 6 (bottom) shows that the ESIMC tuning rule leads to an acceptable response with $K_c=-12,T_i=8$. However, the original SIMC rule ($K_c=-10,T_i=-20$) produces an unstable pole/zero cancellation between the plant and the controller and renders the closed-loop internally unstable. Although in Fig. 6 (bottom) the closed-loop goes unstable after the load disturbance (which excites the unstable mode), in practice instability would occur earlier.

Example II. (*2-DOF vs switched PI+CI*) Following Section V, here we compare the common 2-DOF control strategy (19) with the recently proposed PI+CI controller. For the plant $P=\frac{e^{-s}}{10s+1}$, Fig. 7 (bottom) displays the results obtained using conventional 1-DOF and 2-DOF PI controllers, and the PI+CI controller using Option 1 in Table IV. In all the cases, the *feedback* tuning is the same: $K_c=\frac{\tau}{K_g(\lambda+h)},T_i=\frac{\tau(h+2\lambda)-\lambda^2}{\tau+h}$ ($\gamma=\lambda=1.2$). Substituting the concrete values into the tuning expressions, we get $K_c=4.545,T_i=0.6512$. Regarding the 2-DOF PI

TABLE IV

TUNING RULES FOR THE PI+CI CONTROLLER.

Option	K_c	T_i	α
1	$\frac{1}{K_g}\frac{T_i}{\lambda+\lvert\tau\rvert+h-T_i}$	$\frac{\tau(h+2\lambda)-\lambda^2}{\tau+h}$	$\frac{\tau(h+2\lambda)-\lambda^2}{\tau(h+\lambda+\lvert\tau\rvert)-\lambda\lvert\tau\rvert}$
2	$\frac{1}{K_g}\frac{\tau}{\lambda+h}\left(\frac{h+2\lambda-\lambda^2/\tau}{h+\lambda}\right)$	$\frac{\tau(h+2\lambda)-\lambda^2}{\tau+h}$	$\frac{\tau(h+2\lambda)-\lambda^2}{\tau(h+\lambda+\lvert\tau\rvert)-\lambda\lvert\tau\rvert}$

controller, $b = 0.25$ (decreasing b further does not reduce the overshoot significantly). As for the PI+CI controller, $\alpha = \frac{\tau(h+2\lambda)-\lambda^2}{\tau(h+\lambda+|\tau|)-\lambda|\tau|} = 0.1087$ according to Table IV, and $t_{sp} = 3$. In this particular example, it seems that the PI+CI

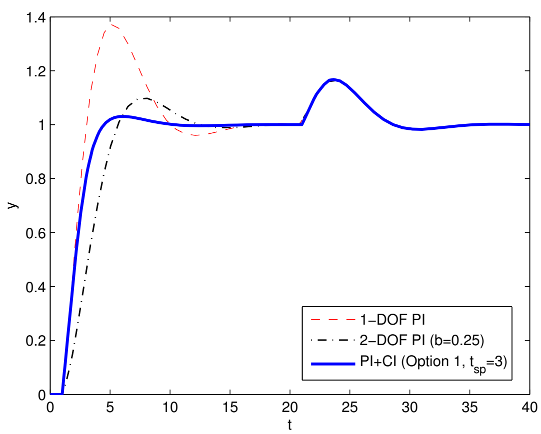

Fig. 7. Time responses for 1-DOF PI, 2-DOF PI and PI+CI schemes.

controller may be advantageous with respect to the 2-DOF PI controller. In particular, a set-point response with shorter rise time and lower overshoot is attained. However, to establish this thesis completely, a more thorough study is necessary.

VI. SUMMARY AND CONCLUSIONS

Tuning rules for PI control have been obtained from an analytical $\mathcal{H}_\infty$ methodology. The resulting tuning expressions take into account set-point tracking and disturbance attenuation and are valid for both stable and unstable plants. As reported in [23], even in the stable case, there is some room to improve the SIMC rule. In this line, the presented design constitutes a good framework to improve the SIMC settings, including unstable plants in the discussion. The presented tuning rules have also been applied to systematic tuning of the PI+CI controller, which has been compared with the more common 2-DOF PI control law.

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