Near-optimal control of batch processes
- by tracking of approximated sufficient conditions of optimality

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What are we trying to do?

- Seek linear combination of measurements: $c = Sy$
- Select the best matrix $S$ using systematic procedure
- Allow separation of problem into:
  - Economic optimization
  - Control structure design via selecting $S$
  - Implementation using for example PI controllers
Implementation strategies

• Online methods
  – Measurements used to update model (or state estimate)
  – Requires online optimization
  – High computational load

• Offline methods
  – Measurements used directly in output feedback structure
  – Requires offline analysis
  – Low computational load
Outline

• Goal: systematic control structure design
• Offline school of thought
• Remaining talk has three main parts:
  
a) Method development: $c(t) = S(t)y(t)$
b) Linear example with stable plant
c) Nonlinear batch distillation example
Mayer problem

\[
\min_{u(t)} J(x(t_f))
\]

\[
\dot{x} = f(x,u,d), \quad x(0) = x_0.
\]

\[
y = h(x)
\]
Necessary and sufficient conditions for optimal control

\[ H(t) = \lambda^T f(x,u,d), \quad \dot{\lambda} = -\frac{\partial H}{\partial x} \]

\[ H_u = 0 \]

\[ \lambda(t_f) = \frac{\partial J}{\partial x(t_f)} \]

\[ \delta^2 J = \int_0^{t_f} \begin{bmatrix} \delta x^T & \delta u^T \end{bmatrix} \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta u \end{bmatrix} dt. \]
Matching the Hamiltonian Matrix

$$\delta^2 J = \int_0^{t_f} \left[ \begin{array}{cc} \delta x^T & \delta u^T \end{array} \right] \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \left[ \begin{array}{c} \delta x \\ \delta u \end{array} \right] dt.$$

- Form linear combinations: $c = Sy = SCx$
- New approximation:

$$\delta^2 J \approx \frac{1}{2} \int_0^{t_f} \left( S(t)C(t)\delta x(t) \right)^T Q_c \left( S(t)C(t)\delta x(t) \right) + \delta u^T H_{uu} \delta u dt$$

- Obviously, we must select $R = H_{uu}$
- Also

$$\left( SC(t) \right)^T Q_c(t) \left( SC(t) \right) = H_{xx}(t).$$
Scaling matrix $Q_c$

- Variables “c” not known a priori
- Assume c’s independent
- Assume worst-case change in c to occur for all c’s at same d
- Scaling suggested:

$$Q_c(t) = \left(\text{diag}\left(SQ_y^{-1}(t)e\right)\right)^{-1}.$$

$$\Rightarrow (SC(t))^T \left(\text{diag}(SQ_y^{-1}e)\right)^{-1} (SC(t)) = H_{xx}(t).$$
A linear-quadratic problem

\[ J = \frac{1}{2} \int_0^{t_f} \| x \|_2^2 + \| u \|_2^2 \, dt \]

\[ \dot{x} = Ax + Bu, \]

\[ y =Cx \]

\[
A = \begin{bmatrix}
-3 & 1 & 1 \\
0 & -2 & 1 \\
0 & 0 & -3 \\
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
1 & 1 \\
\end{bmatrix}, \quad C_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]
Block diagram: output feedback

\[ K \xrightarrow{u} \text{PLANT} \xrightarrow{y} S \]

\[ c \]
Control strategy

- $S$ computed for times $\{0,1,2,3,4\}$
- Linear interpolation used between known points
- Static feedback with pure gain
- Tuned for $x_0=[1,1,1]$
- Look at responses for $[1,1,1]$
State trajectories

- $x_1$ vs. time
- $x_2$ vs. time
- $x_3$ vs. time

Text:

Open loop

$x_1 - x_2$

Optimal
Input trajectories

- $u_1$
- $u_2$

TIME

- Open loop
- $x_1 - x_2$
- Optimal
Average losses (%)

Can improve loss by better controller tuning
Batch distillation

Maximum recovery of methanol
Minimum purity: 90%
Measurements: tray compositions

MeOH/Allyl alchol
Computations

- Optimize for 49 vol%, 50 vol% and 51 vol% methanol to find $Q_y$ (www.gpops.org)
- Compute $H_{xx}$
- Solve

$$C^T S^T \text{diag}^{-1}(SQ_y^{-1}e)SC = H_{xx}$$

using direct search optimization for every hour
- Verify by simulations in closed loop
• Algorithm for control structure design

• Applicable to both linear and nonlinear systems

• A step on the way to automatic control structure design

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