Problems with Specifying $\Delta T_{min}$ in the Design of Processes with Heat Exchangers

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We show in this article that the common method of specifying $\Delta T_{min}$ for individual heat exchangers may lead to wrong decisions and should be used with care when designing heat exchanger systems. In particular, design with constraints on $\Delta T_{min}$ may result in operation conditions that are not optimal when the resulting areas are installed. In addition, different $U$ values for the heat exchangers are not easily handled. We propose an alternative method (simplified TAC) to avoid these problems and compare it with the $\Delta T_{min}$ method on three vapor compression (refrigeration) cycle case studies.

1. Introduction

In process design, one seeks to optimize the future income of the plant. This might be realized by minimizing the total annualized cost (TAC), $J_{TAC} = J_{operation} + J_{capital} [\$$ year$^{-1}]$; see problem 1 below. However, finding $J_{capital}$ requires detailed equipment and cost data, which is not available at an early design stage.

An alternative simple and common approach for design of processes with heat exchangers, especially at an early design stage, is to specify the exchanger minimum approach temperature (EMAT = $\Delta T_{min}$) in each heat exchanger; see problem 2 below. The idea is that this specification should give a reasonable balance between minimizing operating costs $J_{operation}$ (favored by a small $\Delta T_{min}$) and minimizing capital costs $J_{capital}$ (favored by a large $\Delta T_{min}$). In practice, $\Delta T_{min}$ may require less area, but the outlet temperature $T_{min}$ will therefore tend to give designs with relatively large heat exchanger areas. In

\[
\min_{u_1} (J_{operation} + J_{capital})
\]

subject to model equations and operational constraints (this applies to all problems in this article). The degrees of freedom, $u_1$, include all the equipment data (sizes) and operating variables. The annualized capital cost is often obtained as,

\[
J_{capital} = \sum_{i \in \text{Units}} (C_{\text{fixed},i} + C_{\text{variable},i} S_i^u)/T
\]

Here, $S_i$ is the characteristic size for the unit (area in $m^2$ for heat exchangers), and the cost factors ($C_{\text{fixed},i}$ and $C_{\text{variable},i}$), and cost scaling factor $n_i$ are constants for each unit (e.g., heat exchangers). $T$ is the capital depreciation time, for example, $T = 10$ years. The operating costs $J_{operation}$ are given by the prices of feeds, products, and utilities (energy) plus other fixed and variable operating costs; e.g., see eq 8 below.

**Problem 2. Simplified Optimal Design with Specified $\Delta T_{min}$**

\[
\min_{u_2} (J_{operation})
\]

subject to $T_i - \Delta T_{min} \geq 0$  

Here, the degrees of freedom, $u_2$, include the heat transfer in the heat exchangers ($Q_i$) and the operating variables (flows, works, splits etc.). After solving this problem, one can calculate the heat exchanger areas $A_i$ from the resulting temperatures (using $Q_i = f(U_i \Delta T_{da})$). Note that problem 2 will favor designs where the temperature difference $\Delta T$ is close to $\Delta T_{min}$ throughout the heat exchangers because this improves energy efficiency but does not cost anything. Specifying $\Delta T_{min}$ will therefore tend to give designs with relatively large heat exchanger areas.

![Figure 1. Effect of different values for $\Delta T_{min}$](image)
addition, different $U$ values cannot be handled easily as they are not part of the optimization problem in eq 3. An indirect approach is to use different $\Delta T_{\text{min}}$ for each heat exchanger in eq 3.

Let us now consider steady-state operation, where the equipment data, including heat exchanger areas, are given and the degrees of freedom $u_3$ include only the operating variables. For each heat exchanger, there is at steady-state one operating variable that may be chosen as the effective area $A_i$ (in practice, $A_i$ may be changed using a bypass). However, we must require $A_i \leq A_{i,\text{max}}$, where $A_{i,\text{max}}$ is the “installed area”, for example, found from problem 1 or 2. We then have:

**Problem 3. Optimal Operation with Given Heat Exchanger Areas.**

$$\begin{align*}
\min_{u_3} & \quad J_{\text{operation}} \\
\text{subject to} & \quad A_i - A_{i,\text{max}} \leq 0
\end{align*}$$

(4)

Note that in many cases, including the examples in this article, it is optimal to have $A_i = A_{i,\text{max}}$.

The solution to problem 3 in terms of optimal stream data (temperatures) will be the same as to problem 1 but not generally the same as to problem 2; see the motivating example below. To understand this, note that, in problem 3, with the areas given, there is no particular incentive to make the temperature difference $\Delta T$ “even” (due to $\Delta T_{\text{min}}$) throughout the heat exchangers. Provided there are degrees of freedom, we will therefore find that $\Delta T$ from problem 3 varies more through the heat exchangers than $\Delta T$ from problem 2. In particular, the $\Delta T_{\text{min}}$ obtained from problem 3 is often smaller than that specified in design (problem 2); see the introductory example below. Thus, the optimal nominal operating point (solution to problem 3) is not the same as the nominal simplified design point (solution to problem 2). From this, it is clear that specifying $\Delta T_{\text{min}}$ in design is not a good approach.

The objective of this article is to study the $\Delta T_{\text{min}}$ method (problem 2) in more detail and suggest an alternative simple design method (called the simplified TAC method) for heat exchanger systems. The optimization problems are solved using the gPROMS software.

### 2. Motivating Example: Ammonia Refrigeration Cycle

The ammonia refrigeration cycle for cold storage presented in is shown in Figure 2. We use the following conditions:

- $Q_{\text{loss}} = 20 \text{ kW}$
- Ambient temperature $T_a = 25 \degree \text{C}$
- Cold storage (indoor) temperature set point $T_c = -12 \degree \text{C}$

![Figure 2. Ammonia refrigeration system.](image)

**Table 1. Structure of Model Equations for the Ammonia Case Study**

<table>
<thead>
<tr>
<th>$\Delta T_{\text{min}}$ Method</th>
<th>Simplified TAC (eq 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_{\text{vap}}$ [°C]</td>
<td>10.0</td>
</tr>
<tr>
<td>$\Delta T_{\text{min}}$ [°C]</td>
<td>10.0</td>
</tr>
<tr>
<td>$A_{\text{con}}$ [m$^2$]</td>
<td>4.50</td>
</tr>
<tr>
<td>$A_{\text{vap}}$ [m$^2$]</td>
<td>4.00</td>
</tr>
<tr>
<td>$H_{\text{vap}}$ [kJ/kg]</td>
<td>8.50</td>
</tr>
<tr>
<td>$P_{\text{h}}$ [bar]</td>
<td>1.00</td>
</tr>
<tr>
<td>$P_{\text{b}}$ [bar]</td>
<td>1.74</td>
</tr>
<tr>
<td>$\Delta T_{\text{sub}}$ [°C]</td>
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</tr>
<tr>
<td>$\Delta T_{\text{con}}$ [°C]</td>
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</tr>
<tr>
<td>$m$ [mol s$^{-1}$]</td>
<td>1.07</td>
</tr>
<tr>
<td>$W_{\text{c}}$ [W]</td>
<td>6019</td>
</tr>
<tr>
<td>COP [-]</td>
<td>3.32</td>
</tr>
</tbody>
</table>

* Given data is shown in boldface. $^b C_0$ adjusted to get same total heat exchanger area as the $\Delta T_{\text{min}}$ method. $^c C_0$ adjusted to get same $\Delta T_{\text{min}}$ as used in the $\Delta T_{\text{min}}$ method.

- Heat transfer coefficient for the evaporator and condenser, $U = 500 \text{ W m}^{-2} \text{ °C}^{-1}$
- The ammonia leaving the evaporator is saturated vapor (which is always optimal for this cycle$^b$)

The temperature controller is assumed to adjust the compressor power to maintain $T_c = T_c^0$, which indirectly sets the load $Q_c = Q_{\text{loss}}$. The main model equations are given in Table 1.

**2.1. $\Delta T_{\text{min}}$ Design Method.** The operational cost is given by the compressor power ($J_{\text{operation}} = W_c$), so with the $\Delta T_{\text{min}}$ method, the optimal design problem, as in problem 2, becomes:

$$\begin{align*}
\min_{u_2} & \quad (W_c) \\
\text{subject to} & \quad \Delta T_{\text{vap}} - \Delta T_{\text{min},\text{vap}} \geq 0 \\
& \quad \Delta T_{\text{con}} - \Delta T_{\text{min},\text{con}} \geq 0
\end{align*}$$

(5)

where the degrees of freedom $u_2$ include the heat transferred in the two-heat exchanger ($Q_e$) but note that $Q_c = 20 \text{ kW}$ plus three other operating variables$^1$ (e.g., two pressures and refrigerant flow). We choose $\Delta T_{\text{min}} = 10 \degree \text{C}$ in both the evaporator and the condenser.$^3$ The resulting heat exchanger areas $A$ are then obtained from $Q = J(U\Delta T)da$.

As noted in the introduction, the solution to eq 5 does not generally give optimal operation with the resulting areas. The reoptimized operation problem with given areas (problem 3) becomes:

$$\begin{align*}
\min_{u_3} & \quad (W_c) \\
\text{subject to} & \quad A_{\text{vap}} - A_{\text{vap}}^{\text{max}} \leq 0 \\
& \quad A_{\text{con}} - A_{\text{con}}^{\text{max}} \leq 0
\end{align*}$$

(6)

where $A_{\text{vap}}$ and $A_{\text{con}}$ are the results of the $\Delta T_{\text{min}}$ method design problem (eq 5). The degrees of freedom $u_3$ include $A_{\text{vap}}$ and
3. Proposed Simplified TAC Method

The original cost function for problem 1 requires quite detailed cost data plus a lot of other information, which is not available at an early design stage. The simplified problem 2 on the other hand is easy to formulate and solve, but here we cannot easily handle different U values for heat exchangers and the optimal design point is not generally the same as the optimal operating point (even nominally). Therefore, the objective is to find a better simplified formulation. The starting point is to replace the equipment cost (eq 2) in problem 1 with a simplified expression. First, we assume that the structure of the design is given such that we need not consider the fixed cost terms (i.e., we set $C_{\text{fixed},i} = 0$). Second, we only consider heat exchanger costs. For a vapor compression cycle, this is justified if the capital cost for the compressor is proportional to the compressor power $W_c$, which corresponds to assuming $n_l = 1$ for the cost exponent for the compressor. We can then include the capital cost for the compressor in the operating cost of the compressor. Third, we assume that all heat exchangers have the same cost factors ($C_{\text{variable},i} = C_0$ and $n_l = n_l$).

The resulting “simplified TAC” optimal design problem becomes:

$$\min_{u_i} (J_{\text{operation}} + C_0 \sum_i A_i^n)$$

subject to model equations and operating constraints, and where $u_i$ includes the heat exchanger areas $A_i$ plus the operating variables. In the general case,

$$J_{\text{operation}} = \sum p_{F_i} F_i - \sum p_{F_j} F_j + \sum p_{Q_k} Q_k + \sum p_{W_{ij}} W_{ij} [\text{S-year}^{-1}]$$

where $F_i$ are feeds, $P_j$ are products, $Q_k$ are utilities (energy), $W_{ij}$ are the mechanical work, and the $p$’s are respective prices. For a heat exchanger network problem, this can be reduced to $J_{\text{operation}} = \sum p_{Q_i} Q_i [\text{S-year}^{-1}]$, which in many cases simplifies to $J_{\text{operation}} = Q_H [\text{S-year}^{-1}]$, where $Q_H$ is the supplied heat.\(^5\) For the refrigeration cycles considered in this article, $J_{\text{operation}} = W_c [\text{S-year}^{-1}]$.

In the examples, we choose $n = 0.65$ for the heat exchangers and use $C_0$ as the single adjustable parameter (to replace $\Delta T_{\text{min}}$). There are several benefits compared with the $\Delta T_{\text{min}}$ method:

- The heat exchanger temperatures depend on $UA_i$ for each heat exchanger. Thus, different U values for each exchanger are easily included.
- The optimal design (eq 7) and the optimal operation (problem 3) have the same solution in terms of optimal stream data. This follows because the term $C_0 \sum_i A_i^n$ is constant in operation.
- The assumption of using the same $C_0$ for all heat exchangers is generally much better than assuming the same $\Delta T_{\text{min}}$.

On the other hand, compared to the $\Delta T_{\text{min}}$ design method, the simplified TAC design method requires calculation of $\Delta T$ inside all exchangers during the optimization, and the optimization problem is also a bit more difficult to solve. However, the proposed method does not require any additional data compared with the $\Delta T_{\text{min}}$ method.
3.1. Revisit of Ammonia Case Study. The optimization problem (eq 7) for the proposed simplified TAC method becomes\textsuperscript{13}:

$$\min (W_s + C_0(A_{\text{con}}^n + A_{\text{vap}}^n)) \quad (9)$$

The right two columns of Table 2 show the optimal design with $n = 0.65$ and two different values of $C_0$. First, $C_0 = 818$ gives the same total heat exchanger area and almost the same capital cost as the $\Delta T_{\text{min}}$ method, but the area is better distributed between the evaporator and condenser. This results in a 3.60% reduction in operating cost ($W_s$) compared with the $\Delta T_{\text{min}}$ method (0.36% after reoptimizing the operating point for the $\Delta T_{\text{min}}$ method). Second, $C_0 = 8250$ gives $\Delta T_{\text{min}} = 10.0 \, ^\circ\text{C}$ as specified in the $\Delta T_{\text{min}}$ method. The compressor work is increased with 107% (114%) but the heat exchanger area is reduced by 60%, and this is the only design that truly satisfies the $\Delta T_{\text{min}}$ we selected initially.

The simplified TAC method confirms that subcooling is optimal, and we see that the degree of subcooling increases with decreasing heat transfer area (increased $C_0$).

Note that the heat transfer coefficients $U_i$ were assumed to be equal, but the simplified TAC method will automatically distribute the heat transfer area optimally, also if the heat exchangers have different heat transfer coefficients. For example, with $U_{\text{vap}} = 2U_{\text{con}}$ the energy savings (for the same heat exchangers cost) are even larger (6%) using the simplified TAC method compared with the $\Delta T_{\text{min}}$ method.

4. Other Case Studies

4.1. CO\textsubscript{2} Air-Conditioner. CO\textsubscript{2} as a working fluid in air-conditioners and heat pumps is gaining increased popularity because of its low environmental impact.\textsuperscript{7,8} We consider a transcritical CO\textsubscript{2} air-conditioning unit with the following data:

- Heat transfer coefficient: $U = 500 \, \text{W m}^{-2} \text{K}^{-1}$ for the evaporator, condenser, and internal heat exchanger
- Ambient temperature: $T_H = 30 \, ^\circ\text{C}$
- Set point for room temperature: $T_C = 20 \, ^\circ\text{C}$
- Heat loss into the room: $Q_{\text{loss}} = 4.0 \, \text{KW}$

The details about the model are found in ref 3. In the optimization, we have included an internal heat exchanger (with area $A_{\text{inh}}$) that transfers heat from before the compressor to before the valve. Otherwise the flowsheet is as for the ammonia cycle shown in Figure 2.

For solving problem 2, we use a design $\Delta T_{\text{min}} = 5.0 \, ^\circ\text{C}$ in all heat exchangers. Again, we find that reoptimizing for operation (problem 3) gives a better operating point with 3.70% less compressor power. The results given in Table 3 are similar to the ammonia cooling cycle, although there is no subcooling because $P_0$ is above the critical pressure.

Interestingly, with the simplified TAC method we obtain $A_{\text{inh}} = 0.0 \, \text{m}^2$, which means that it is not optimal from an economical point of view to pay for the area for the internal heat exchanger (although the internal heat exchanger would of course be used if it were available free of charge). This is a bit surprising because we have not included the fixed cost of installing a heat exchanger, which would make it even less desirable to invest in an internal heat exchanger. On the other hand, if we require a lot of superheating before the compressor then it might be better to achieve this superheating in an internal heat exchanger, but this is not discussed here.

With $C_0 = 253$, we get the same total heat transfer area as for the $\Delta T_{\text{min}}$ method, but the shaft work is reduced by 4.26% (0.58% compared to reoptimized). $C_0 = 185$ gives the same cost of heat exchanger area (without even considering the savings of completely removing a heat exchanger), and $W_s$ is reduced by 12.22% (8.85%). With $C_0 = 877$, we get the only design with $\Delta T_{\text{min}} = 5.0 \, ^\circ\text{C}$. The heat exchanger cost is reduced by 41% and the compressor power is increased by 49% (55%) compared with the $\Delta T_{\text{min}}$ method.

4.2. PRICO LNG Process. The PRICO LNG process\textsuperscript{9} is a simple configuration utilizing mixed refrigerants. Details about the model are presented elsewhere.\textsuperscript{10} Note that we are not considering constraints on compressor suction volume and pressure ratio for the compressor. This will be important in an actual design, but we have tried to keep the case study simple to illustrate the effect of specifying $\Delta T_{\text{min}}$.

A design $\Delta T_{\text{min}}$ of 2.0 $^\circ\text{C}$ is used for the $\Delta T_{\text{min}}$ method. From Table 4, we see that reoptimizing reduces the energy usage ($W_s$) by 4.8%. This is achieved by increasing the pressure ratio (by 25.5%) and reducing the refrigerant flowrate (by 16.7%). The composition of the refrigerant is also slightly changed, but this is not shown in Table 4. We were quite surprised by the rather large improvement obtained by reoptimizing with fixed heat transfer areas considering the relatively low value for the initial $\Delta T_{\text{min}}$.

With the simplified TAC method, we get a 4.1% reduction (0.2% increase compared to reoptimized) in $W_s$ for the same total heat transfer area ($C_0 = 2135$). The small increase in $W_s$ compared with the reoptimized $\Delta T_{\text{min}}$ design is because the simplified TAC method minimizes the heat exchanger cost and not the total area. With the same cost ($C_0 = 2090$), the TAC method confirms that subcooling is optimal, and we see that the degree of subcooling increases with decreasing heat transfer area (increased $C_0$).
method gives a reduction in compressor power of 4.3% (0.1%). The saving compared with the reoptimized case is small because of the small $\Delta T_{\text{min}}$ resulting in very large heat exchangers. A more reasonable design is achieved with $C_0 = 7350$, which gives a design with a true $\Delta T_{\text{min}}$ of 2.0 °C. The heat exchanger capital cost is reduced by 40%, but the compressor power is increased by 17.0% (22.2%).

5. Discussion

5.1. $\Delta T_{\text{min}}$ Method. There are some main points that are important to note from this analysis of the $\Delta T_{\text{min}}$ method.

1. $\Delta T_{\text{min}}$ is treated as an important parameter in heat exchanger design, but the theoretical basis seems weak as “violating” $\Delta T_{\text{min}}$ in operation may give lower operating costs.

2. The $\Delta T_{\text{min}}$ method will not always give the optimal operating point, so suboptimal setpoints might be implemented.

3. The size distribution between the heat exchanger will not be optimal, although this may be partly corrected for by individually adjusting the value of $\Delta T_{\text{min}}$ for each heat exchanger.

4. More seriously, the results might lead to wrong structural decisions, and this cannot be changed by iterating on the $\Delta T_{\text{min}}$ values. In the ammonia case study, one would incorrectly conclude that subcooling is not optimal and thus implement a liquid receiver after the condenser. This would during operation achieve no subcooling. From the true optimum, however, we see that some subcooling is optimal.

5. One potential advantage with the $\Delta T_{\text{min}}$ method is that it only requires an overall energy balance for the heat exchangers. However, for more complex cases a more detailed model of the heat exchangers is needed so in the general cases this advantage is lost.

In summary, the $\Delta T_{\text{min}}$ method is not satisfactory for realistic design problems.

5.2. Other Approaches. The question is whether there are other ways of specifying temperature differences. In terms of area, rather than specifying $\Delta T_{\text{min}}$, it would be better to specify the mean temperature difference $\Delta T = 1/A \int \Delta T \text{d}A$ because $\Delta T$ is directly linked to the heat transfer area by (assuming constant heat transfer coefficient):

$$Q = UA\Delta T$$  \hspace{1cm} (10)

However, this method has several drawbacks that make it unattractive to use. First, it might be cumbersome to calculate the integral of the temperature and second, and more importantly, there are no general rules in selecting values for $\Delta T$. We therefore propose to use the simplified TAC method.

5.3. Heat Exchanger Network Design. The results in this article show that one should not use a constraint on $\Delta T_{\text{min}}$ (EMAT) for the design of individual heat exchangers. What about the standard heat exchanger network (HEN) design problem, where a constraint on the heat recovery approach temperature $\Delta T_{\text{min}}$ (HRAT) for the network is used? Our results show that $\Delta T_{\text{min}}$ invalidates this approach. First, the stream data (inlet and outlet temperatures and flows) are fixed for the standard HEN problem. It then follows, that specifying $\Delta T_{\text{min}}$ (HRAT) is equivalent to specifying the heat recovery or, equivalently, the required hot utility ($Q_h$). The solution to this particular design problem will therefore result in optimal operating data if we install the resulting areas (and remove the $\Delta T_{\text{min}}$ specification). However, note that the simplified TAC formulation in eq 7 with $J_{\text{operation}} = \sum p_0 Q_i$ is much more general than the standard heat exchanger network design problem, where the stream data are specified.

6. Conclusion

We have shown that the method of specifying $\Delta T_{\text{min}}$ for the design of heat exchangers, (min $J$ subject to $\Delta T \geq \Delta T_{\text{min}}$), may fail to give an optimal operating point. In the ammonia refrigeration case study, the $\Delta T_{\text{min}}$ method fails to find that subcooling in the condenser is optimal. As a simple alternative, we propose the simplified TAC method (min $(J + C_0 \sum A_i^r)$), where $C_0$ replaces $\Delta T_{\text{min}}$ as the adjustable parameter. A high value of $C_0$ corresponds to increasing the investment (capital) costs relative to the operating (energy) costs and favors small areas and a larger $\Delta T_{\text{min}}$. Thus, $C_0$ can be adjusted to get a desired value for $\Delta T_{\text{min}}$ for the total area or it can be obtained from cost data. With the alternative method, different heat transfer coefficients $U_i$ can also be accounted for.

Another important conclusion is related to the temperature difference profile in the heat exchanger. According to exergy or entropy minimization rules of thumb, it is optimal to have even driving forces, which suggests that $\Delta T$ should be constant in heat exchangers. The results presented here, however, suggest that this is not true. The $\Delta T_{\text{min}}$ approach (problem 2) favors a more constant $\Delta T$ profile (part a in Figure 3), but in optimal operation (problem 3) we find that the temperature difference is small in one end (part b of Figure 3).

Literature Cited


(12) The simple cycle in Figure 2 has five operating variables, but two of these have been specified (given load and saturated vapor).

(13) In a more realistic design, one may also consider additional constraints such as maximum compressor suction volumes and pressure ratio, but this is not discussed here.

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