Selection of closed-loop time constant $\tau_c$

**Issues**

1. **Upper bound due to effective time delay (robustness)**
   
   $\text{SVC rule: } \tau_c \geq \tau_0 (= \tau_{min})$

2. **Lower bound due to disturbance rejection (performance)**
   
   $K_C \geq \frac{W_0}{|Y_{max}|}$  
   \( W_0 = \text{input magnitude required for disturbance rejection} \)
   \( |Y_{max}| = \max y \)

   Gives

   $\tau_c \leq \tau_{c,max}$

   by use of

   $K_C = \frac{1}{\kappa} \frac{\tau}{\tau + \theta}$

3. **Upper bound due to input saturation (avoiding too large $u$)**

   **Disturbances on input $u(y)$**

   \[ \frac{\Delta u}{\Delta u} = \frac{c_g}{1+c_g} = \frac{1}{2\tau + 1} \text{ (same as } \frac{1}{\tau}!) \]

   (so we do not overshoot here!)

   **BUT** could be that there is a requirement $\tau_c \geq \tau$

   because fast changes in $u$ are not desired.

   "Filtering" of $du$ required.

   **Disturbances on output $y(y)$**

   \[ \frac{\Delta y}{\Delta y} = \frac{c_g}{1+c_g} = \frac{1}{2\tau + 1} \quad (\text{This we must be able to handle}) \]

   \[ \frac{\Delta y}{\Delta y} = \frac{g(0)}{g} = \frac{1}{\kappa} \]

   \[ \text{Initial response:} \quad g(s) = \frac{K_C}{s} \Rightarrow g(0) = \frac{K_C}{s} \Rightarrow \Delta u = \frac{1}{\kappa s} \Rightarrow \Delta y = \frac{1}{\kappa \tau} \frac{1}{\kappa \tau} \]

   \[ \text{Overshoot initially is given by the "speed-up"} \]

   \[ \text{Max. allowed overshoot} \Rightarrow \Delta y = \Delta y_{max} \Rightarrow \frac{\Delta y}{\Delta y_{max}} = \frac{1}{\kappa} \Rightarrow \Delta y = \frac{1}{\kappa \tau} \]

   Useful magnitude for SS disturbance rejection method

   \[ \text{Maximum speed-up allowed is } \frac{\Delta y_{max}}{\Delta y} \]
(1) \[ T_c \geq \theta \quad \text{(robustness)} \]

(2) \[ T_c \leq \frac{\|y_{\text{max}}\|}{\|y_0\|} \cdot T \quad \text{(speedup required for disturbance rejection)} \]

\[ \|y_0\| = \text{output magnitude w/o control (due to disturbances)} \]

(3) \[ T_c \geq \frac{\|u_y\|}{\|u_{\text{max}}\|} \cdot T \quad \text{(maximum speedup because input may saturate when there are output disturbances)} \]

\[ \|u_y\| = \text{input change required to reject output disturbance (setpoint change)} = \frac{\|u_{\text{max}}\|}{K} \]

(4) \[ T_c \leq T_{c, \text{setpoint}} \quad \text{(Response time required for acceptable setpoint tracking)} \]

\[ \frac{\text{Note:}}{\text{Generally make as small as possible}} \]

(5) \[ T_c \geq T_{c, \text{input}} \quad \text{(Response time filtering of input disturbances)} \]