Control Structure Selection for Reactor, Separator, and Recycle Processes

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We consider control structure selection, with emphasis on “what to control”, for a simple plant with a liquid-phase reactor, a distillation column, and recycle of unreacted reactants. Plants of this kind have been studied extensively in the plantwide control literature. Our starting point is a clear definition of the operational objectives, constraints, and degrees of freedom. Active constraints should be controlled to optimize the economic performance. This implies for this case study that the reactor level should be kept at its maximum, which rules out many of the control structures proposed in the literature from being economically attractive. Maximizing the reactor holdup also minimizes the “snowball effect”. The main focus is on the selection of a suitable controlled variable for the remaining unconstrained degree of freedom, where we use the concept of self-optimizing control, which is to search for a constant setpoint strategy with an acceptable economic loss. Both for the case with a given feed rate where the energy costs should be minimized and for the case where the production rate should be maximized, we find that a good controlled variable is the reflux ratio $L/F$. This applies to single-loop control as well as multivariable model predictive control.

Introduction

A common feature of many chemical processes is the presence of recycle. Variations of a plant with reaction, separation, and mass recycle (see Figure 1) have been extensively studied in the literature (with different parameters and with and without a distillation column).

Gilliland et al. used this plant to study how the dynamics and steady-state behavior are changed by the positive feedback introduced by the recycle. The changes in steady-state relative gain array for the distillation column caused by introduction of the recycle have been studied, and others found that control of internal compositions, either distillate or reactor composition, helps the control of the bottom composition. Luyben followed up Gilliland’s points and focused on the high sensitivity that the recycle flow rate in some cases has to the feed flow rate. He called this the “snowball effect” and, as a remedy, proposed to let the reactor holdup vary and, as a generic rule, that “one flowrate somewhere in the recycle loop should be flow controlled”. Wu and Yu proposed that a better way of avoiding snowballing is with a constant reactor composition.

The recycle plant in Figure 1 has 4 degrees of freedom at steady state: 1 for the throughput (feed rate $F_0$), 1 for the reactor (holdup $M_r$), and 2 for the distillation column (e.g., reflux and boilup); see also Table 2. In the literature several alternative sets of controlled variables have been proposed for the case with a given feed rate $F_0$ and given (and controlled) product composition $x_B$:

(a) “Conventional” (denoted $x_0$ in the following): control of $M_r$ and $x_0$ (fixed reactor holdup and “two-point” distillation column control).

(b) “Luyben’s structure” (LS) with varying reactor holdup: control of $F$ and $x_0$.

(c) “Balanced structure I” (with varying reactor holdup): control of $x_r$ (reactor composition) and $x_D$.

(d) “Balanced structure II” (with varying reactor holdup): control of $F/F_0$ and $x_0$.

(e) “Luyben’s rule” ($D$ or $F$) applied to cases with constant reactor holdup: control of $M_r$ and $D$ or $F$ (structures CD or CF in work by Wu and Yu).

(f) “Reflux ratio” ($L/F$): control of $M_r$ and $L/F$ (this paper).

Here Luyben’s structure (LS) and the balanced structures are “unconventional” in the sense that the reactor level is left floating, which may at first seem impossible. However, the reactor level has a steady-state effect through its effect on the conversion and will therefore be indirectly given by specifying some other variable,
for example, \( x_r \) or \( F \). (If desired, for example, for safety reasons, one may install a reactor level controller as an inner cascade, with the level set point replacing the flow used for level control as a degree of freedom. This will not affect the steady-state behavior.)

The above works raise some issues that need to be studied further. First, in most of the above works, the overall operational objectives for the plant were not clearly defined. Second, a liquid-phase reactor should normally be operated at maximum holdup (liquid level) in order to optimize steady-state economics, whereas the reactor at levels in the “unconventional” structures of Wu and Yu\(^8\) and Luyben. This has an impact on the steady-state economics, an issue that has been overlooked by most researchers so far. Third, “Luyben’s rule” of controlling a flow in the recycle loop \((D \text{ or } F)\) has not been properly substantiated. To the contrary, Wu and Yu\(^8\) found that “Luyben’s structure” (LS) resulted in snowballing in the reactor holdup and that “Luyben’s rule with constant reactor holdup” \((D \text{ or } F)\) could handle only very small throughput changes.

The objective of this paper is to study in a systematic manner the selection of controlled variables for the reactor with the recycle process. To this end, we will use the general procedure of Skogestad\(^9\), where we first define the economic and operational control objectives and identify the available degrees of freedom. The goal is to find a self-optimizing control structure where acceptable operation under all conditions is achieved with constant set points for the controlled variables. However, before describing this procedure and applying it to the case study, we discuss in some more detail the so-called snowball effect.

**Plant Data.** The plant and design data are taken from work by Wu and Yu\(^1\). The model is simple and assumes a binary feed \((x_0 = 0.9 \text{ mol-A/mol} \text{ and } F_0 = 460 \text{ kmol/h})\), an isothermal reactor with maximum holdup 2800 kmol, and a first-order reaction \( A \rightarrow B \) with \( k = 0.341 \text{ h}^{-1} \). The distillation column has 22 stages including reboiler and condenser, liquid feed at stage 13, constant relative volatility \( \alpha_{AB} = 2 \), and constant molar flows. The purity requirement for the product is \( x_B = 0.0105 \text{ mol-A/mol} \). From the total mass balance of component A, the nominal reactor concentration is

\[
x_r = \frac{F_0(x_0 - x_B)}{kM} = \frac{460(0.9 - 0.0105)}{0.341 \times 2800} = 0.43 \quad (1)
\]

**2. Snowball Effect**

Luyben\(^4\) introduced the term “snowball effect” to describe what can happen, for the recycle process in Figure 1, in response to an increase in the fresh feed rate \( F_0 \).

For our process, where all of the feed is converted to the product, the increase in \( F_0 \) must be accompanied by a corresponding increase in the conversion in the reactor. Assume that we in Figure 1 have a liquid-phase continuous stirred tank reactor with a first-order reaction. The amount of A converted in the reactor is then \( kM x_r [\text{mol} \cdot \text{A/mol}] \). We see that there are three options for increasing the conversion: \(^3\)

1. Increase the reaction constant \( k \left[ \text{s}^{-1} \right] \) (e.g., by increasing the reactor temperature).
2. Increase the reactor holdup \( M_r \) [mol].
3. Increase the reactor mole fraction \( x_r [\text{mol-A/mol}] \) of reactant A.

We assume here that option 1 (increasing \( k \)) is not available.

Option 2 (increasing the reactor holdup) is probably the “default” way of dealing with a feed-rate increase when seen from a design person’s point of view. More specifically, a design person would increase all extensive variables (including flows) in the process proportionally to \( F_0 \), such that the intensive variables (compositions) in the process were kept constant. This is also the idea behind the “balanced” control structures of Wu and Yu\(^8\).

However, changing the reactor holdup (volume) during operation may not be possible, or at least not desirable, because for most reactions it is economically optimal to use a fixed maximum reactor volume in order to maximize per pass conversion.

Assuming \( k \) and \( M_r \) to be constant, the only remaining way to increase conversion is to follow option 3 and increase \( x_r \), which can be done by recycling more unreacted A. However, the effect of this is limited, and the snowball effect occurs because even with infinite recycle D, the reactor concentration cannot exceed that of pure A \((x_r = 1)\). More precisely, for the process in Figure 1 the material balance equations for component A and total mass are\(^b\)

**Overall process:**

\[
F_0 x_0 = B x_0 - kM x_r; \quad F_0 = B
\]

**Column:**

\[
F x_0 = B x_0 + D x_0; \quad F = B + D
\]

Here \( x_0, x_r, \) and \( x_B \) denote the molar fractions of component A in streams \( F_0, D, \) and \( B, \) respectively. By eliminating \( x_r \), we find

\[
F = F_0 \frac{kM(x_D - x_B)}{kM x_0 - F_0(x_0 - x_B)} \quad (2)
\]

If the reactor holdup is large relative to the feed rate, then we have almost complete conversion in one reactor pass and no recycle, so \( D \approx 0 \) and \( F \approx F_0 \); that is, the column feed rate \( F \) increases linearly with the fresh feed rate \( F_0 \). For larger values of \( F_0 \), the denominator in eq 2 will approach zero (and \( x_r \) will approach \( x_B \)), and we will experience “snowballing” with very large increases in \( D \) and \( F \) in response to only moderate increases in \( F_0 \). If the reactor holdup is too small compared to \( F_0 \) that is, if

\[
M_r \leq F_0(x_0 - x_B) \quad (3)
\]

then the desired steady state is infeasible (even with infinite flow rates for \( D \) and \( F \)). In practice, because of constraints, the flow rates will not go to infinity. Most likely, the liquid or vapor rate in the column will reach its maximum value, and the observed result of snowballing will be a breakthrough of component A in the bottom product; that is, we will find that we are not able to maintain the product purity specification \( x_B \).

To avoid this snowballing, Luyben and co-workers and Wu and Yu\(^8\) propose to use a varying reactor holdup (option 2), rather than the “conventional” control structure with constant holdup (option 3). Their simulations confirm that a variable holdup results in less snowballing in \( D \) and \( F \), but these simulations are misleading because they do not consider the reactor holdup. In fact, the Luyben structure (LS), with fixed \( D \) or \( F \), may result in snowballing in the reactor holdup.\(^8\) This is confirmed
by Figure 2, where we see that an increase in the feed rate may result in the following:

(a) Conventional structure (constant \( x_0 \)) with constant reactor holdup: snowballing in recycle flow (this is the snowballing considered by Luyben).

(b) Luyben structure (LS) with varying reactor holdup: snowballing in reactor holdup.

(c) Luyben rule (constant D or F) with constant reactor holdup: snowballing inside the column.

Actually, the snowballing in the recycle flow with the conventional structure is not even as poor as that shown in Figure 2. This is because we here used an intermediate value for the constant reactor holdup \( (M_r = 2800 \text{ kmol}) \), whereas from eq 2, we find that the lowest value of \( F \) for a given value of \( F_0 \) is when the reactor holdup \( M_r \) is at its maximum; so with a fixed maximum holdup, the conventional structure \( (x_0) \) actually gives smaller flows \( (D \text{ and } F) \) than the Luyben structure (LS) in all cases.

In summary, the "snowball effect" is a real operational problem if the reactor (or some other unit in the recycle loop) is "too small", such that we may get close to or even encounter cases where the feed rate is larger than the reactor (or rather the system) can handle. The "snowball effect" makes control more difficult and critical, but it is not a control problem in the sense that it can be avoided by use of control. Rather, it is a design problem which could have been avoided by installing a sufficiently large reactor to begin with. In conclusion, for an existing plant the best remedy against snowballing is to use the maximum reactor holdup.

3. General Procedure for Selecting Controlled Variables

A plant generally has several operational degrees of freedom (manipulated variables; there are six for our recycle plant; see Table 2). The objective of the control system is to adjust these manipulated variables to assist in achieving acceptable operation of the plant. Thus, to design a control system in a systematic manner, we first need to define the operational requirements (constraints) and the goal of the operation. In general, we have upper and lower constraints on all extensive variables in a process and on many intensive variables. The goal of the operation is quantified by defining a scalar cost function \( J \) to be minimized. The optimum (minimum value of \( J \)) usually lies at some constraints, and usually most of the degrees of freedom are consumed to satisfy these "active" constraints. However, in many cases there are unconstrained degrees of freedom, and the difficult issue is to decide what to control (that is, what to keep at a constant setpoint) in order to satisfy these. If we used optimal setpoints and there were no uncertainty or disturbances, then this choice would not matter. However, there will always be uncertainty and disturbances, and the optimal setpoints for the controlled variables should be insensitive to such changes. In addition, the shape of the objective function should be "flat", so that an implementation error will give a small loss.9 To address this in a systematic manner, we will consider the economic loss imposed by keeping a given set of variables constant.

We assume that \( J \) is the economic cost, determined mainly by the plant's steady-state behavior, and from ref 9, we adopt the following procedure for selecting the controlled variables.

**Step 1: Degree of Freedom Analysis.** Determine the degrees of freedom available for steady-state optimization. The easiest way is to count the number of manipulated variables and subtract the number of variables with no steady state that need to be controlled (e.g., reboiler and condenser levels in distillation).

**Step 2: Cost Function and Constraints.** Define the optimal operation problem by formulating a scalar cost function \( J \) to be minimized, and specify the constraints.

**Step 3: Identification of the Important Disturbances.** Here "disturbances" include process disturbances, implementation errors in the controlled variables (sum of the steady-state control error and measurement noise), and the effect of changes and errors (uncertainty) in the model.

**Step 4: Optimization.** The steady-state optimization problem is solved both for the nominal case and for the identified range of disturbances.

**Step 5: Identification of Candidate-Controlled Variables \( c \).** Active constraints should normally be controlled because this optimizes steady-state cost. To select between the remaining unconstrained candidates, we proceed to step 6.

**Step 6: Evaluation of the Loss.** Evaluate the loss for alternative sets of controlled variables \( c \). Here the loss is the difference between the cost with constant setpoints \( c_0 \) and the theoretical optimal cost (with setpoints reoptimized for each disturbance \( d \))

\[
L = J(c_0,d) - J_{\text{opt}}(d)
\]  

"Self-optimizing" controlled variables \( c \) with a small loss \( L \) are preferred.

**Step 7: Further Analysis.** Normally several candidates give an acceptable loss, and further analysis may be based on a controllability analysis.

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**Figure 2.** Snowballing: steady-state values of the recycle flow (D), reactor holdup (M_r), and column boilup (V) as a function of feed rate \( F_0 \) for alternative control structures.
between the value of the product to be maximized. We here select to be minimized, or equivalently, a profit functionables for the recycle plant. To quantify the goals of control, introduced above, to select the controlled vari-
ables with no steady-state effect 2
unconstrained degrees of freedom 1
boiler level 4
case I: min operation cost (energy) case II: max production rate

<table>
<thead>
<tr>
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<th>Step 1: Degree of Freedom Analysis (See Table 2)</th>
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<tr>
<td>objective function minimize V</td>
<td>objective function maximize F₀</td>
<td></td>
</tr>
<tr>
<td>constraints</td>
<td>constraints</td>
<td></td>
</tr>
<tr>
<td>reactor level Mₗ ≤ 2800</td>
<td>reactor level Mₗ ≤ 2800</td>
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<tr>
<td>product quality x₀ ≤ 0.0105</td>
<td>product quality x₀ ≤ 0.0105</td>
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</tr>
<tr>
<td>feed rate F₀ = F₀,max = 460</td>
<td>vapor boilup V ≤ V_max = 1500</td>
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<tr>
<td>disturbances</td>
<td>disturbances</td>
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<tr>
<td>feed rate F₀,max ±20%</td>
<td>maximum vapor boilup V,max ±20%</td>
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</tr>
<tr>
<td>implementation error ±20%</td>
<td>implementation error ±20%</td>
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<td>− active constraints at the optimum</td>
<td>active constraints at the optimum</td>
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</tr>
<tr>
<td>Mᵣ, xᵣ, F₀</td>
<td>Mᵣ, xᵣ, V</td>
<td></td>
</tr>
<tr>
<td>= unconstrained degrees of freedom</td>
<td>= unconstrained degrees of freedom</td>
<td></td>
</tr>
<tr>
<td>nominal optimum: V = 1276</td>
<td>nominal optimum: F₀ = 497.8</td>
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<table>
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<th>Step 5: Identify Candidate-Controlled Variables for Unconstrained DOF (c)</th>
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<td>F, L, D, L/D, L/F, L/V, x₀, FF₀, DF₀</td>
<td>F, L, D, L/D, L/F, L/V, x₀, FF₀, DF₀</td>
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<th>Step 7: Further Analysis</th>
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<tr>
<td>ratio control (L/F or L/D) is easier than composition control (x₀)</td>
<td>control L/F or L/D (+ active constraint Mᵣ, xᵣ, F₀)</td>
</tr>
</tbody>
</table>

4. Selection of Controlled Variables for the Recycle Plant

In this section, we use the concept of self-optimizing control, introduced above, to select the controlled variables for the recycle plant. To quantify the goals of operation, we define a scalar economic cost function J to be minimized, or equivalently, a profit function P [$/s] to be maximized. We here select P [$/s] as the difference between the value of the product B and the feed F₀ and subtract the operational costs for distillation and recycling:

\[ P = -J = p_B F_0 - p_V V - p_D D \]  \( (5) \)

Here \( p_B \) [$/mol] is the difference between the product and feedstock prices, and we have used \( B = F_0 \). \( p_V \) is the energy cost related to distillation (because the column has a liquid feed and total condenser, the vapor flows to be evaporated and condensed are approximately the same, and \( p_V \) [$/mol] is the sum of the price for reboiling and condensing). The recycle cost \( p_D \) may include costs for pumping and preprocessing (e.g., heating) the stream D. This cost is neglected here, but for gas-phase systems with compression, the term is usually important.

In general, the optimal way of operating the plant depends on the relative prices. However, for our problem the following two constraints are always active:

(a) Because the product (mostly B) is more valuable than the feedstock (mostly A), it is optimal to put as much unreacted A into the product as possible; that is, it is always economically optimal to operate with the bottom purity at its constraint (i.e., \( x_B = 0.0105 \)).

(b) Because there is no economic penalty involved in increasing the reactor holdup, it is optimal for this reaction to keep \( M_r \) at its upper bound (i.e., \( M_r = 2800 \)). This maximizes conversion "per pass", which reduces recycle and thereby the load on the distillation column.

We will in the following consider two different cases.

**Case I: Given Feed Rate.** With \( F_0 \) given and negligible recycling costs \( (D_0 = 0) \), the profit \( P \) is only influenced by the energy costs \( p_V \) for heating and cooling in the distillation column. Thus, with a given feed rate \( F_0 \), optimal operation is obtained by minimizing the boilup \( V \).

**Case II: Variable Feed Rate.** With \( F_0 \) as an unconstrained degree of freedom, we find that it is optimal to increase the feed rate \( F_0 \) as much as possible (because the profit \( P \) increases linearly with \( F_0 \)) when we increase all other flows in proportion to \( F_0 \). However, there are always capacity constraints, and we assume here that the first one to become active is the vapor flow constraint \( V \leq V_{max} \) in the distillation column. With \( V = V_{max} \) and negligible recycling costs, the profit \( P \) is only influenced by the feed rate \( F_0 \). Thus, with variable feed rate, optimal operation is obtained by maximizing the feed rate (and the production rate) \( F_0 \).

All of the results in this section are based on a steady-state analysis. Table 1 summarizes the results which are further discussed below.

**4.1. Case I: Given Feed Rate (Minimum Energy).**

**Step 1: Degree of Freedom Analysis.** From Table 2 we see that there are 4 degrees of freedom at steady state, including \( F_0 \).

**Step 2: Cost Function and Constraints.** As noted above, the objective is to minimize the vapor boilup
(energy), i.e., $J = V$. There are constraints on the reactor holdup ($M_r$), product quality ($x_D$), and column capacity (boilup $V$). In addition, the feed rate $F_0$ is given.

**Step 3: Disturbances.** The main disturbance is in the feed rate $F_0$, and we consider ±20% changes. We also consider disturbances in the feed composition, $x_D = 0.9 \pm 0.1$, but these turn out to be of much less importance. The implementation error is assumed to be ±20% in each of the candidate variables, and we also consider $\pm 0.002$ implementation error (possibly caused by poor dynamic control) in the product composition $x_D$. (Other possible disturbances, not considered here, include a change in the reaction rate constant $k$ and a change in the reactor holdup $M_r$.)

**Step 4: Optimization.** Table 3 shows the results of the nominal optimization.

As expected, the two constraints on $x_D$ and $M_r$ are active. Because the feed rate $F_0$ is given, we are then left with one unconstrained degree of freedom.

**Step 5: Candidates for Control.** We choose to control the active constraints in order to optimize operation; i.e., $M_r$ and $x_D$ are controlled. This rules out the "unconventional" structures with variable reactor holdup, including the "Luyben structure" and the "balanced structures".

Some of the candidates for the remaining degree of freedom are listed in Table 3. Two candidate variables have already been eliminated:

- The reactor composition $x_r$ cannot be specified independently and is thus not a candidate for control. This follows from eq 1 because the feed is given ($F_0$ and $x_0$) and $x_D$ and $M_r$ are controlled at their constraints.
- The boilup $V$ in the column is not a candidate for control because specifying it below its minimum (optimum) value results in infeasible operation.

**Step 6: Evaluation of the Loss.** Figure 3 shows the loss in energy (i.e., increase in boilup $V$) imposed by keeping alternative controlled variables fixed at their nominal setpoints. The losses due to implementation errors (third row) and disturbances in $F_0$ (first row) are quite large for some variables. For example, we see from the first row that with $L$ constant (left plot) a decrease in the feed rate by 10% results in using about 5% more than the optimal; this is reduced to less than 0.1% if we keep $x_D$ constant and less than 0.01% if we keep $L/F$ constant (right plot). The disturbances in feed composition $x_0$ (second row plots) result in very small losses in all cases. The loss due to a back-off in the bottom composition from 0.0105 to 0.0085 is about 3% for all structures (last row).

In summary, Figure 3 shows that control of $x_D$, $L/F$, or $L/D$ gives small losses (right plots), while control of $F$, $D$, $L$, $V$, $F/F_0$, or $D/F_0$ gives large losses (left plots). In particular, we find that Luyben's rule of fixing one flow in every recycle loop, corresponding to fixing $D$ or $F$, results in very large losses and even infeasibility, because $V$ goes to infinity when $F_0$ increases (left plot in the second row).

**Step 7: Other Considerations.** The conventional control configuration with maximum reactor holdup ($M_r$), constant product composition $x_D$, and constant setpoint for the distillate composition ($x_B$) has very good self-optimizing properties with small economic losses. However, it may be costly to obtain online measurements of $x_D$, and two-point distillation composition control (of $x_D$ and $x_B$) is known to be a difficult control problem because of interactions. In any case, the analysis shows that control of the internal distillate composition $x_D$ is not really needed because, in terms of economic loss, control of $L/F$ (or $L/D$) performs almost equally well. The latter results in a simple control problem and is therefore preferred. Figure 4 shows a possible control structure involving the following single loops: $M_r \rightarrow F$, $L/F \rightarrow L$, $x_D \rightarrow V$, $M_0 \rightarrow D$, and $M_8 \rightarrow B$.

**4.2. Case II: Maximize the Feed Rate.** We now consider the case where the feed rate is a degree of freedom and should be maximized. This case is of more practical importance because small losses in the production rate usually have a large impact on the overall plant economics.

**Step 1: Degree of Freedom Analysis.** As before, there are 4 degrees of freedom at steady state; see Table 2.

**Step 2: Cost Function and Constraints.** The goal here is to maximize the production rate, i.e., to minimize $J = -F_0$, and there are constraints on vapor boilup, reactor holdup, and product composition.

**Step 3: Disturbances.** The main disturbance is in the actual value of the (maximum) boilup $V_{\text{max}}$, which may vary, for example, because of variations in the column pressure or available heat to the column.

**Step 4: Optimization.** Table 3 shows the results from the nominal optimization. We find as expected that all three constraints are active, including the maximum constraint on the vapor boilup. This leaves 1 unconstrained degree of freedom.

To understand why the production rate is limited, consider Figure 5. At low production rates ($F_0$), there is almost a linear relation between $F_0$ and $x_D$. As $F_0$ is increased, the load to the distillation column increases ($F = F_0 + D$ increases), and because $V = V_{\text{max}}$ is constant, we eventually experience "snowballing" with breakthrough of product B in the top of the column. This results in a decrease (rather than the desired increase) in the fraction $x_D$ of A in the reactor, and the production rate drops. This happens at $F_0 = 497.8 \text{ kmol/h}$ (the optimal point).

**Step 5: Candidates for Control.** We consider the same candidate variables as those in case I.

**Step 6: Evaluation of the Loss.** Figure 6 shows the loss in the production rate due to a disturbance in $V$ and due to implementation error. Although the details are different, the results are similar to those of case I, with small losses for control of $x_D$, $L/F$, and $L/D$.

**Step 7: Other Considerations.** Again, because controlling $L/F$ or $L/D$ gives a much easier control problem for the distillation column, it will be preferred over control of $x_D$. A possible control structure is shown in Figure 7. To be able to handle also case I, we have included a cascade flow control loop where we obtain $F_0 = F_0^\ast$ by adjusting $V$, but this control ($FC$) loop is not used in case II, where we have maximum vapor boilup ($V = V_{\text{max}}$).
Comment: The same variable, $L/F$ or $L/D$, turned out to be a good unconstrained controlled variable for both cases I and II. This is generally attractive because it may reduce the effort in reconfiguring the loops when, for example, the economic conditions change from case I (given production) to case II (maximum production).

Figure 3. Case I: losses in energy ($V$) with alternative controlled variables ($F_0 = 460, M_r = 2800, x_0 = 0.0105$).
The Luyben rule with constant $F$ for the case with constant reactor holdup yields instability. It is not able to maintain the desired bottom composition even for small increases in the feed rate. This confirms the steady-state results in Figure 3 and the findings of Wu and Yu.8 This is easily explained: As the feed rate $F_0$ is increased, we must with constant $F = F_0 + D$ reduce the recycle $D$ to the reactor (which is the opposite of what we would like to do). This results in snowballing inside the distillation column with accumulation of unreacted component A, and operation eventually becomes infeasible.

The Luyben structure (LS; with varying reactor holdup) clearly yields the best dynamic response in $x_B$; this is because the varying reactor holdup serves as a surge tank, which helps to smooth (average out) the feed-rate disturbance. However, the response in $x_B$ for the Luyben structure is unrealistic because we have allowed the reactor level $M_r$ to exceed its maximum value, and we see from the right plot in Figure 8 that there is actually snowballing in the reactor level.8 To guarantee feasibility ($M_r \leq M_{r,max}$) for feed-rate changes, we would for the Luyben structure need to “back away” from the reactor level constraint (using a nominal holdup significantly smaller than $M_{r,max}$), which would give nonoptimal economic operation with about 50% higher energy usage ($V$) in the distillation column or, even worse, the inability to handle the desired feed rate because of capacity limitations in the distillation column.

On the other hand, if we for the other structures (with constant holdup) introduce a back-off in bottom composition $x_B$ from 0.0105 to 0.0085 (in order to handle the control variations in Figure 8), then the increase in energy usage ($V$) is only by about 3% (see the lower plot in Figure 3). Alternatively, we may avoid the need for back-off in $x_B$ (and the resulting 3% energy increase) by using a product tank with mixing to average out the dynamic variations in $x_B$.

5. Closed-Loop Simulations

In Figure 8 we show for case I (given feed rate $F_0$, and no capacity limit on $V$) the closed-loop dynamic responses in the bottom composition to a 20% increase in feed rate $F_0$ for the following structures:

**Conventional ($x_0$):**

- $M_r \leftrightarrow F$, $x_0 \leftrightarrow L$, $x_B \leftrightarrow V$, $M_D \leftrightarrow D$, and $M_B \leftrightarrow B$
- Reflux ratio ($L/F$):
  - $M_r \leftrightarrow F$, $L/F \leftrightarrow L$, $x_0 \leftrightarrow V$, $M_D \leftrightarrow D$, and $M_B \leftrightarrow B$
- Luyben rule ($F$):
  - $M_r \leftrightarrow D$, $F$ constant, $x_0 \leftrightarrow V$, $M_D \leftrightarrow L$, and $M_B \leftrightarrow B$
- Luyben structure (LS; varying reactor holdup):
  - $F$ constant, $x_0 \leftrightarrow L$, $x_B \leftrightarrow V$, $M_D \leftrightarrow D$, and $M_B \leftrightarrow B$

Note that we have used single-loop controllers and \( \leftrightarrow \) means “is paired with” or more precisely “is controlled by.” The pairings are based on a relative gain array analysis, and PI settings are found using the IMC tuning approach. We selected 0.25 min as the desired closed-loop time constant for the level loops and 2.5 min for the other loops. For the three first structures, we have constant maximum reactor holdup, $M_r = 2800$ kmol.

The conventional structure and reflux ratio structure yield very similar and acceptable dynamic responses.

6. Discussion

### 6.1. Alternative Sets of Active Constraints

We have in this paper considered case I with a given feed rate and case II with an unconstrained feed rate, and these resulted in two different control structures. What other cases are there? We here define a “case” in terms of the set of active constraints. For our recycle plant, the following four upper constraints are of interest:

- $x_B \leq x_{B,max}$
- $M_r \leq M_{r,max}$
- $F_0 \leq F_{0,max}$
- $V \leq V_{max}$

Here, as explained earlier, the economic conditions are such that the two first constraints are always active and at least one of the two latter constraints are active. We are then left with only three cases:

**Case I:** Constraint on $F_{0,max}$ is active, and $V$ is unconstrained. This happens for low $V$ or for one of the available feed rate $F_{0,max}$ (or large values of $V_{max}$), where it is optimal to process all of the available feed while minimizing the value of $V$.

**Case II:** Constraint on $V_{max}$ is active, and $F_0$ is unconstrained. This happens for high values of $F_{0,max}$.
Constraints on \( F_{\text{0,max}} \) and \( V_{\text{max}} \) are both active. This happens for intermediate values of the available feed rate \( F_{\text{0,max}} \), provided there is some penalty on recycle, i.e., \( p_D > 0 \).

**Figure 6.** Case II: losses in the production rate \( F_0 \) for alternative controlled variables (\( V = 1500, M_r = 2800, x_B = 0.0105 \)).
The details depend on the cost function \(-J = p_r F_0 - p_v V - p_D D\). The feed-rate range where case III is economically optimal is often quite small, especially if recycle costs are small compared to distillation costs. In this paper we have assumed no recycle costs \((p_D = 0)\), and we go directly from case I to case II. For example, with \(p_v = 0\) and \(V_{max} = 1500\), we have case I for \(F_{0,max} \leq 468.6\) and case II for \(F_{0,max} \geq 468.6\). With recycle costs included \((p_D > 0)\), it is optimal to use more energy in the distillation column, and we get a region where both constraints are active (case III). For example, with the cost function \(-J = F_0 - 0.01V - 0.1D\) \((i.e., p_F = 1, p_V = 0.01, p_D = 0.1)\) and \(V_{max} = 1500\), we have case I for \(F_{0,max} \leq 468.6\), case II for \(F_{0,max} \geq 468.6\). Note here that the economic maximum capacity of 493.2 is somewhat less than the achievable maximum capacity of 497.8 \([kmol/h]\).

In the above discussion we have considered the "available feed rate" \(F_{0,max}\) (inequality constraint \(F_0 \leq F_{0,max}\)). For the closely related case with a "given feed rate" \(F_0 = F_{0,max}\), we have the following: At low feed rates \(F_0\), we have case I with \(V\) unconstrained. As the feed rate increases, the boilup \(V\) also increases (the change in \(V\) for a small change in \(F_0\) may be large if we experience snowballing), and eventually the column reaches its capacity limit \(V_{max}\). With constant boilup \((V = V_{max})\), it may be possible to increase the feed rate further by reducing the distillate purity \(x_B\) and increasing the recycle (case III), but eventually the column becomes a bottleneck (case II), where it is not feasible to process any more feed while maintaining the given product composition.

Comment: The fact that the distillation column is a bottleneck in case II does not necessarily mean that the production rate can be much increased by increasing its capacity \(V_{max}\), because if the system is close to snowballing, then increasing \(F_{0,max}\) is the only effective way of increasing the plant capacity.

6.2. Decentralized Control and Reconfiguration of Loops. The focus in this paper is to decide on which variables to control, and we have recommended to use the "reflux ratio" structure with control of \(L/F\). The analysis has been based on steady-state economics and is independent of the actual implementation. However, in the closed-loop simulations, we assumed a decentralized control, where each controlled variable was paired with a manipulated input. A main problem with decentralized control is that a reconfiguration of loops is generally required when the active constraints change. Let us consider this in more detail for our proposed reflux ratio structure.

We have already proposed pairings for cases I and II. In the intermediate case III, there are no unconstrained degrees of freedom; that is, the economic optimal control structure is to use all four steady-state degrees of freedom to control the active constraints. A possible control structure for case III is then \(M_r \leftrightarrow F, x_B \leftrightarrow L, F_0 = F_{0,max}, V = V_{max}, M_D \leftrightarrow D,\) and \(M_V \leftrightarrow B\). Note that the reflux \(L\) is used here to control the bottom composition. In summary, we then have for the "reflux ratio" structure (in all cases we use \(M_D \leftrightarrow D\) and \(M_B \leftrightarrow B\)) the following:

Case I: \(M_r \leftrightarrow F, L/F \leftrightarrow L, F_0 = F_{0,max}, x_B \leftrightarrow V\)
Case II: \(M_r \leftrightarrow F_0, L/F \leftrightarrow L, V = V_{max} x_B, x_B \leftrightarrow F\)
Case III: \(M_r \leftrightarrow F, F_0 = F_{0,max}, V = V_{max}, x_B \leftrightarrow L\)

We note that two loops (control of \(M_r\) and \(x_B\)) need to be reconfigured as we go from case I to case II. To minimize the need for reconfiguration, we may use the inflow \(F_0\) to control the reactor level in all cases (this corresponds to setting the production rate at the column bottleneck (\(V\)) in all cases). We then get the control structure in Figure 7:

Case I: \(M_r \leftrightarrow F_0, L/F \leftrightarrow L, F_0 \leftrightarrow V, x_B \leftrightarrow F\)
In this case no reconfiguration is required as we go between cases I and II. The disadvantage is that control of feed rate \( F_0 \) is indirect so \( F_0 \) will deviate from \( F_{0,\text{max}} \) when the process is disturbed. However, if we have a storage tank for the feed, then this does not matter as the variations will average out over time.

6.3. Multivariable Constraint Control (MPC). To avoid the logic in reconfiguring loops when switching between cases I–III, one may use a multivariable controller with explicit handling of constraints (e.g., model predictive control, MPC) that “automatically” reconfigures the control tasks when the active constraints change. However, also here one needs to decide on what variables to setpoint control to satisfy the unconstrained degrees of freedom (cases I and II). Thus, our recommendation of controlling the reflux ratio \( L/F \) applies also to MPC.

The objective of the model predictive controller would then be to control \( x_B \) (first priority) and \( L/F \) (second priority) at their setpoints (and possibly also the reactor level, condenser level, and reboiler level, but we assume these are controlled by a lower-layer-level control system), using the degrees of freedom \( F_0, V, \) and \( L \) (assuming here that \( F, D, \) and \( B \) are used for level control in the lower layer), subject to given constraints on \( F_0 \) and \( V \). The setpoints, which may vary with time, are supplied by the layer above MPC. This may be a steady-state optimizer or an operator.

6.4. Economics Not Important. We have in this study excluded the “unconventional” control structures with variable reactor holdup from being economically optimal. However, with a given feed rate \( F_0 \), low energy and recycle costs \( (p_0 \) and \( D_0 \) are small), and no capacity constraints \( \left(V_{\text{max}} \right) \), the economic penalty of using \( M_r < M_{r,\text{max}} \) may be small, and it may be more important to operate the plant as smoothly as possible, for example, to reduce the effect of disturbances on other parts of the plant. In such cases, a variable reactor holdup structure, such as one of the balanced structures, may be better because the reactor is effectively used as a surge tank to “average out” disturbances in the column feed rate. Nevertheless, we do not recommend the Luyben structure (LS) with a fixed flow in the recycle loop because it results in snowballing in the reactor holdup, see also Figure 8. This is also explained since in response to an increase in the feed rate clearly should increase the recycle (and not keep it constant).

7. Conclusion

We have presented a systematic approach for selecting controlled variables for the liquid-phase reactor with a recycle plant. To optimize economics, we need to control active constraints. For the cases of both minimizing operating costs (case I) and maximizing production rate (case II), it is optimal to keep the reactor holdup at its maximum. This makes the Luyben structure (LS) and the two balanced structures economically unattractive. For the unconstrained variables, we look for self-optimizing variables where constant setpoints give acceptable economic loss. In both cases I and II, the reflux ratio \( L/F \) or \( L/D \) appears to be such a variable. To avoid the so-called “snowball” effect, it has been proposed in the literature to “fix a flow in a liquid recycle loop”. However, the rule seems to have a limited basis because it leads to control structures that can handle only small feed-rate changes (constant reactor holdup) or that result in large variations in the reactor holdup (variable reactor holdup).

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