Combination of measurements as controlled variables for self-optimizing control

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Abstract

A new method for selecting controlled variables (c) as linear combination of measurements (y) is proposed based on the idea of self-optimizing control. The objective is to find controlled variables, such that a constant setpoint policy leads to near optimal operation in the presence of low frequency disturbances (d). We propose to combine as many measurements as there are unconstrained degrees of freedom (inputs, u) and major disturbances such that $\Delta c_{opt}(d) = 0$. To illustrate the ideas a gas-lift allocation example is included. The example show that the method proposed here give controlled variables with good self-optimizing properties.

1. Introduction

Although not widely acknowledged, controlling the right variables is a key element in overcoming uncertainty in operation. Control systems often consist of several layers in a hierarchical structure, each operating on a different time scale. Typically, layers include scheduling (weeks), site-wide optimization (day), local optimization (hours), supervisory/predictive control (minutes) and regulatory control (seconds). The layers are interconnected through the controlled variables c. Optimal operation for a given disturbance d can be found by solving the following problem.

$$\begin{aligned}
&\min_u J(x,u,d) \qquad (1)\\
&f(x,u,d) = 0\\
&g(x,u,d) \leq 0\\
&x \in X, u \in U, d \in D
\end{aligned}$$

where f is the process model, g the inequality constraints, u the independent variables (inputs), d the disturbances in which we cannot affect, and x the states. J is the scalar economic performance metric and since the economics are primarily decided by steady-state operation, only steady-state models are used in this analysis. Solution of (1) give the optimal inputs and states, $u_{opt}(d)$ and $x_{opt}(d)$ respectively, and also the optimal value of the measurements, $y_{opt}(d)$, as a function of d. As shown in Skogestad (2000*a*) we assume that all optimally active constraints are implemented (active constraint control). Self-optimizing control follows the idea of Morari et al. (1980) and may be summarized as:

Self-optimizing control (Skogestad 2000*a*) *is when an acceptable loss can be achieved using constant setpoints for the controlled variables (without the need to re-optimize when disturbances occur).*

The central issue when searching for the self-optimizing control structure is to decide how to best *implement* the optimal policy in the presence of uncertainty. This is accomplished by selecting the right set of controlled variables c to be kept at constant setpoints c_s, in spite of disturbances d and implementation error n. The goal is to minimize the loss, $L = J(c,d) - J_{opt}(d)$, with a constant setpoint strategy, where the loss is the difference between the value of the objective using a constant setpoint policy and the value of the true optimal objective. For a review of self-optimizing control see (Skogestad 2000*b*). Skogestad et al. (1998) propose two methods for selecting controlled variables with good self-optimizing properties based on a Taylor series expansion of the loss function. Candidate controlled variables are not limited to single measurements, as shown by Morud (1995) who, by seeking all possible directions of the output space, was able to find a linear combination of the measurements with good self-optimizing properties. Here a much simpler method is proposed.

2. Proposed method for selecting controlled variables as linear combinations of measurements

We show in this section, that if we neglect the implementation error in controlling c (e.g caused by poor control or measurement error), then it is possible from a linear point of view to find a linear combination of the available measurements with zero loss ("perfect self-optimizing control"). By eliminating the states x, we may write the measured variables y as a function of the independent variables (degrees of freedom) u and disturbances d.

$$y = f_y(u,d) \tag{2}$$

In general the set y also includes the independent variables u. The controlled variables c ("primary outputs") are to be selected as combinations of the measured variables ("secondary outputs"),

$$c = h(y) \tag{3}$$

where the generally non-linear function h is free to choose, except that we assume that the controlled variables are independent and that the number of controlled variables (c's) equals the number of degrees of freedom (u's). We will here consider the case where the function h(y) is linear. We may then write $c = h(y)$ as

$$\Delta c = H\Delta y \tag{4}$$

where the matrix H is free to choose. We assume that the operation is nominally optimal, that is, we have $c_s = c_{opt}(d^*)$ where d^* is the nominal disturbance. We assume that there is no implementation error ($n = 0$), which implies that we will have $c = c_s$ (constant) for all disturbances d. This constant setpoint policy will be optimal (with zero loss) provided the optimal value of $c(d)$ remains constant, that is, is $c_{opt}(d)$ is independent of d. This simple insight may be used to find the optimal linear combination (i.e. find the optimal choice for the matrix H). We consider small changes (disturbances) from the nominal disturbance. Then the change in the optimal value of the measurements is given by

$$\Delta y_{opt} = y_{opt,d} - y_{opt,d^*} = F(d - d^*) = F\Delta d \tag{5}$$

where the sensitivity matrix $F = \frac{dy_{opt}}{dd}$ may be obtained numerically by solving the optimization problem (1) for small changes in the disturbance variables d, and from this obtain $u_{opt}(d)$ as well as $y_{opt}(d)$. We assume that u_{opt} and x_{opt} are continuous in d in a neighborhood of the nominal point. From (4) the corresponding change in the optimal value of c is $\Delta c_{opt} = H\Delta y_{opt}$. Now require that $\Delta c_{opt} = 0$ which gives $\Delta c_{opt} = HF\Delta d = 0$. This need to be satisfied for any Δd so we must have that

$$HF = 0 \tag{6}$$

In other words, we should select H to be in the left null space of F ($H \in \mathcal{N}(F^T)$). We assume that we have n unconstrained degrees of freedom (the length of vectors u and c are n), use m independent measurements when forming c, and have k independent disturbances. We than have that F is a $m \times k$ matrix and H is a $n \times m$ matrix. By assuming $m \geq k$ and $m \geq n$ and by assuming independent inputs and disturbances it follows that $rank(F) = k$. The fundamental theorem of linear algebra (Strang 1988) tells that the left null space of F, $\mathcal{N}(F^T)$, has rank $m - r$, where $r = K = Rank(F)$. Since $H \in \mathcal{N}(F^T)$ we have that $dim(H) = m - k$ and by assuming that the number of controlled variables must be equal to the number of inputs we get $rank(H) = n$

$$m - k = n \Leftrightarrow m = n + k \tag{7}$$

so that $\#y = \#d + \#\, u$, e.g. the minimum number of measurements needed is equal to the number of inputs plus the number of disturbances. We then have

Theorem 2.1 *Assume we have n unconstrained independent variables u, k independent disturbances d, and m measurements y, of which at least $n+k$ are independent. It is the possible to select measurement combinations*

$$\Delta c = H\Delta y \tag{8}$$

such that $HF = 0$ where $F = \frac{dy_{opt}}{dd}$. Keeping c constant at its nominal optimal value then gives zero loss when there are small disturbances d. The matrix H is generally not unique.

In summary, the main idea is to select the selection matrix H such that $\Delta c_{opt} = H\Delta y_{m,opt} = 0$ by using $m = n+k$ independent measurements. If the number of available measurements exceeds the number of inputs and major disturbances, there is some freedom to choose these as to reduce the implementation error and to maximize the observability of the disturbances in the measurements, see Alstad & Skogestad (2002) for further details.

3. Example: Gas-lift allocation optimization

In many oil/gas fields the production of oil, gas and water are constrained by the processing capacity and other process constraints such as available flow-line transportation capacity. Wang et al. (2002) point out that the available literature does not provide robust procedures on how to formulate and solve typical optimization problems for such systems. Often, the "optimization" consider the constraints sequentially, or only sub-problems are considered (e.g by not including the transportation system to the processing facility). Dutta-Roy & Kattapuram (1997) considered the effect of including process constraints for a two-well case that share a common transportation line to the process. They found that failing to include the process constraints (in this case the transportation line) gave a sub-optimal solution of the problem.

Here, we focus on how to *implement* the optimal operation in the presence of low frequency disturbances. In typical oil/gas producing systems there are large uncertainties (e.g. reservoir properties, models) and few measurements, so methods that can help operate the process optimally when disturbances occur are of great value.

In this paper we consider the gas-lift structure in Figure 1 with the data given in Table 2. The model used is a distributed pseudo one-phase flow model (Taitel 2001) assuming black oil compositional PVT behavior (Golan & Whitson 1996). The valves are modeled as one-phase with a linear characteristic. The flow model represent a two-point boundary value problem and the partial differential equations are discretized using orthogonal collocation. The two wells (W_1 and W_2) are connected to a common transport line (T). We assume that the system is dynamically stable. Gas is injected through valves (CV_6 and CV_7) to increase the production from the reservoir by making the static pressure (head) less. The operating objective is to maximize the profit, $J = \sum_{i=o,g,gi} p_i m_i$ where indices o, g, gi are oil, gas and injected gas respectively, p_i is the price for phase and m_i is the mass rate for phase i. We have neglected water in this analysis. The inputs in this case are $u = [V_1\ V_2\ V_3\ V_4\ V_5\ V_6\ V_7]^T$ where V_i is the valve position for valve i. We assume that the level and pressure of the separator are controlled at the setpoints using CV_4 and CV_5 respectively. These setpoints can not be manipulated, thus removing 2 DOF. In typical offshore systems, the ratio of oil and gas (GOR, the ratio of stock-tank gas mass to stock-tank oil mass) from each well is not exactly known, so we assume that the low frequency disturbance is the ratio of gas and oil ($d = [GOR_1\ GOR_2]^T [\frac{kg\ gas}{kg\ oil}]$) in the reservoir, where the GOR is given at reservoir properties. The available measurements are the pressure upstream the valves for the wells (P_{V1} and P_{V2}) and the injection gas mass rates ($m_{gi,1}$ and $m_{gi,2}$). It is assumed that there is a upper limit in the gas processing capacity in the process, due to compressor limitations in the process. The optimally active constraints (for all disturbances) are $[m_{g,tot}\ V_1\ V_2\ V_3]$ so we have $DOF = 7-2-4 = 1$ unconstrained DOF. Since it is optimal to control the total gas mass flow at the constraint, we can reformulate the objective to only consider the cost of injecting the gas into the well ($J = \sum_{i=o,gi} p_i m_i$). In this case we have assumed that $p_o = 0.17[\$/kg]$ and $p_{gi} = -0.05[\$/kg]$ corresponding to a oil price of $ 20 per barrel. The cost for recycling gas in the system has been assumed to be half the sale price of natural gas which was assumed to be $0.1\$/Sm^3$. Following the procedure in Section 2 we have that $m = n+k = 1+2 = 3$, so we need three measurements. We select the measurements P_{V1}, P_{V2} and $m_{gi,1}$as measurements. The optimal sensitivity function (F) is calculated by imposing the above constraints and upon requiring $HF = 0$, this result in the controlled variable $c_{LC} = Hy = [0.76\ -0.65\ 0.09][P_{V1}\ P_{V2}\ m_{gi,1}]^T$. The loss is calculated for several structures and is given in Table 1. We see that controlling $c = c_{LC}$ have good self-optimizing properties with the lowest average and worst case loss. A constant setpoint policy for the other controlled variables give a higher loss.

Table 1. Loss for the alternative control variables for the gas optimization case

		Loss (in million $/year)			
Rank	c	GOR_1 $0.03 \to 0.06$	GOR_2 $0.10 \to 0.13$	$GOR_1 : 0.03 \to 0.06$ $GOR_2 : 0.10 \to 0.13$	Average
1	c_{LC}	0.0	0.0	0.16	0.05
2	P_{V1}	1.5	1.0	1.9	1.5
3	$m_{gi,2}$	2.0	0.5	4.5	2.3
4	$m_{gi,1}$	0.8	1.4	5.3	2.5
5	P_{V2}	3.2	2.7	4.1	3.3

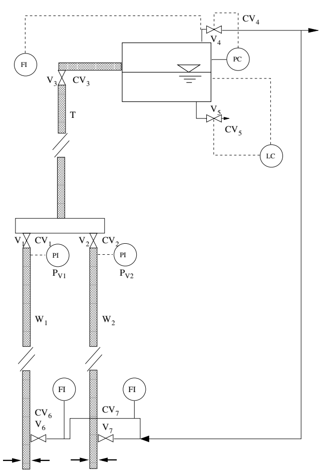

Figure 1. Figure of the well network

Table 2. Data for the gas lift allocation example

Parameter	Value	Unit	Comment
$L_{W1,W2}$	1500	m	Length well 1 and 2
$D_{W1,W2}$	0.12	m	Diameter well 1 and 2
L_T	300	m	Length transportation line
D_T	0.25	m	Diameter transportation line
$P_{res,1}$	150	bara	Pressure reservoir well 1
$P_{res,2}$	155	bara	Pressure reservoir well 2
$PI_{res,1}$	$1E-7$	$\frac{m^3}{s\ Pa}$	Production index well 1
$PI_{res,2}$	$0.98E-7$	$\frac{m^3}{s\ Pa}$	Production index well 2
P_sep	50	bara	Pressure separator
ρ_1	750	$\frac{kg}{m^3}$	Black oil density reservoir 1
ρ_2	800	$\frac{kg}{m^3}$	Black oil density reservoir 2
M_g	20	$\frac{kg}{kmole}$	Molecular weight gas
GOR_1^0	0.03	$\frac{kg}{kg}$	Nominal gas oil ratio
GOR_2^0	0.10	$\frac{kg}{kg}$	Nominal gas oil ratio
$m_{g,tot}$	15	$\frac{kg}{s}$	Maximum gas capacity

4. Conclusion

We have derived a new method for selecting controlled variables as linear combination of the available measurements, that from a linear point of view have perfect self-optimizing properties if we neglect implementation error. The idea is to calculate the optimal sensitivity function ($\Delta y_{opt} = F\Delta d$) and select controlled variables as linear combination of the measurements $c = Hy$, such that $HF = 0$. The method has been illustrated on a gas-lift allocation example. The example illustrate that in a constant set-point control structure, selecting the right controlled variables are of major importance.

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