CONTROL STRUCTURE DESIGN

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This is a copy of the transparencies used for the presentation. For more detailed information see Chapter 10 in S. Skogestad and I. Postlethwaite, Multivariable feedback control - Analysis and design, Wiley, 1996.
THEORY: General formulation

- Find a controller $K$ which based on the information in $v$, generates a control signal $u$ which counteracts the influence of $w$ on $z$, thereby minimizing the closed-loop norm from $w$ to $z$. 
PRACTICE: Typical base level control structure
PRACTICE: Typical control hierarchy
Foss (1973):

The central issue to be resolved ... is the determination of control system structure.

Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?

There is more than a suspicion that the work of a genius is needed here, for without it the control configuration problem will likely remain in a primitive, hazily stated and wholly unmanageable form.

The gap is present indeed, but contrary to the views of many, it is the theoretician who must close it.
CONTROL STRUCTURE DESIGN

Tasks:

1. Selection of controlled outputs (a set of variables which are to be controlled to achieve a set of specific objectives)

2. Selection of manipulations and measurements (sets of variables which can be manipulated and measured for control purposes)

3. Selection of control configuration (a structure interconnecting measurements/commands and manipulated variables)

4. Selection of controller type (control law specification, e.g., PID, decoupler, LQG, etc.).

Note distinction between control structure and control configuration.

Tasks 1 and 2 combined: input/output selection
Task 3 (configuration): input/output pairing

Shinskey (1967, 1988)
Morari (1982)
Stephanopoulos (1984)
Balchen and Mumme (1988)
Nett (1989)

Skogestad and Postlethwaite (1996)
TASK 1: Selection of controlled outputs

Controlled output $y$: Measured output with reference ($r$)

Two distinct questions:
1. What should the controlled variables $y$? (includes open-loop by selecting $y = u$)
2. What is their optimal values ($y_{opt}$)?

Second problem: Extensively studied.
**Example 1.** *Room heating.*

\[ y = \text{room temperature} \]

Other cases: Less obvious.

**Example 2.** *Cake baking.*

Goal (purpose): well baked inside and nice outside

Manipulated input: \( u = Q \) (assume 15 minutes).

(a) Open-loop implementation: Heat input \( Q \)

(b) Closed-loop implementation:

\[ y = \text{oven temperature} \]

“Optimizer”: Cook book (look-up table)
SELECTION OF CONTROLLED OUTPUTS

The input $u$ (generated by feedback to achieve $y \approx r$) should be close to the optimal input $u_{opt}(d)$.

$$u - u_{opt} = G^{-1}(0)(y - y_{opt})$$

where

$$y - y_{opt} = \underbrace{y - r}_{e} + \underbrace{r - y_{opt}(d)}_{e_{opt}}$$

$\Rightarrow$ Select controlled outputs $y$ such that:

1. $G^{-1}(0)$ is small; inputs have a large effect on $y$.
2. $e_{opt} = r - y_{opt}(d)$ is small; $y_{opt}(d)$ depends only weakly on disturbances.
3. $e = y - r$ is small; good measurement and control of $y$.

Scale outputs such that $\|y - y_{opt}(d)\| \approx 1$ (due to measurement errors and disturbances)

Note: $\bar{\sigma}(G^{-1}(0)) = 1/\underline{\sigma}(G(0))$.

Conclusion. Simple tool:

- Prefer a set of controlled outputs with large $\underline{\sigma}(G(0))$. 

Distillation column example from Kjetil
Distillation column example from Kjetil
Distillation column example from Kjetil
TASK 2: Selection of manipulations and measurements

- $y$ – all candidate outputs (measurements)
- $u$ – all candidate inputs (manipulations)

Combinatorial growth: Detailed analysis time consuming.

Possibilities with 1 to $M$ inputs and 1 to $L$ outputs (Nett, 1989):

$$\sum_{m=1}^{M} \sum_{l=1}^{L} \binom{L}{l} \binom{M}{m}$$

$M = L = 2$: 4+2+2+1=9 candidates

$M = L = 4$: 225 candidates, etc.

TOOLS THAT AVOID COMBINATORIAL GROWTH DESIRED.

RGA is one such tool.
RGA for non-square plant

\[ \Lambda(G) = G \times G^\dagger \]

**HDA plant** (Cao, 1995).

- 5 outputs and 13 candidate inputs.
- \( \binom{13}{5} = 1287 \) combinations with 5 inputs / 5 outputs.
- \( \binom{13}{6} = 1716 \) combinations with 6 inputs / 5 outputs.

\[
G^T(0) =
\begin{bmatrix}
0.7878 & 1.1480 & 2.6640 & -3.0928 & -0.0703 \\
0.6055 & 0.8814 & -0.1079 & -2.3769 & -0.0540 \\
1.4722 & -5.0025 & -1.3279 & 8.8609 & 0.1824 \\
-1.5477 & -0.1083 & -0.0872 & 0.7539 & -0.0551 \\
2.5653 & 6.9433 & 2.2032 & -1.5170 & 8.7714 \\
1.4459 & 7.6959 & -0.9927 & -8.1707 & -0.2565 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.1097 & -0.7272 & -0.1901 & 1.2574 & 0.0217 \\
0.3485 & -2.9009 & -0.8223 & 5.2178 & 0.0853 \\
-1.5899 & -0.9647 & -0.3648 & 1.1514 & -8.5365 \\
0.0000 & 0.0002 & -0.5307 & -0.0001 & 0.0000 \\
-0.0323 & -0.1351 & 0.0164 & 0.1451 & 0.0041 \\
-0.0443 & -0.1859 & 0.0212 & 0.1951 & 0.0054
\end{bmatrix}
\]
RGA may be useful in providing an initial screening:

\[
\Lambda^T = \begin{bmatrix}
0.1275 & -0.0755 & 0.5907 & 0.1215 & 0.0034 \\
0.0656 & -0.0523 & 0.0030 & 0.1294 & 0.0002 \\
0.2780 & 0.0044 & 0.0463 & 0.4055 & -0.0006 \\
0.3684 & -0.0081 & 0.0009 & 0.0383 & -0.0018 \\
-0.0599 & 0.9017 & 0.2079 & -0.1439 & 0.0443 \\
0.1683 & 0.4042 & 0.1359 & 0.1376 & 0.0089 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0014 & -0.0017 & 0.0013 & 0.0099 & 0.0000 \\
0.0129 & -0.0451 & 0.0230 & 0.1873 & -0.0005 \\
0.0374 & -0.1277 & -0.0320 & 0.1163 & 0.0516 \\
0.0000 & 0.0000 & 0.0268 & 0.0000 & 0.0000 \\
0.0001 & 0.0001 & 0.0000 & 0.0001 & 0.0000 \\
0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0000 \\
\end{bmatrix}, \quad \Lambda^T_{\Sigma} = \begin{bmatrix}
0.77 \\
0.15 \\
0.73 \\
0.40 \\
0.95 \\
0.85 \\
0.00 \\
0.01 \\
0.18 \\
0.94 \\
0.03 \\
0.00 \\
0.00 \\
\end{bmatrix}
\]

- Five inputs with the largest inputs projection: 5, 10, 6, 1, and 3 (in that order).

- For this selection $\sigma(G_s) = 1.73$ whereas $\sigma(G) = 2.45$ for the overall system.
TASK 3: Selection of control configuration

Controller $K$ connects available measurements/commands ($v$) and manipulations ($u$):

$$u = Kv$$

Control configuration: The restrictions imposed on the structure of the overall controller $K$ by decomposing it into a set of local controllers (subcontrollers, units, elements, blocks) with predetermined links and with a possibly predetermined design sequence.

Typical restriction: one degree-of-freedom controller where input is $r - y$.

Some elements used to build up configuration:

- Decentralized controllers ($K$ diagonal)
- Cascade controllers (predetermined order for tuning)
- Feedforward elements
- Decoupling elements
- Selectors
Decentralized control. The control system consists of independent feedback controllers which interconnect a subset of the output measurements/commands and a subset of the manipulated inputs. These subsets should not be used by any other controller.

Usually: Rearrange order of inputs and outputs such that $K$ is block-diagonal.

Cascade control is when the output from one controller is the input to another. This is broader than the conventional definition of cascade control which is that the output from one controller is the reference command (setpoint) to another.

Feedforward elements link measured disturbances and manipulated inputs.

Decoupling elements link one set of manipulated inputs ("measurements") with another set of manipulated inputs. Often viewed as "feedforward" elements.

In addition to restriction on structure on $K$: Impose restrictions on sequence the subcontrollers are designed.
Cascaded controllers

(a) Extra measurements $y_2$ (conventional cascade control)

(b) Extra inputs $u_2$ (input resetting)
Why use control configurations?

- Decomposed configurations often quite complex.

So why use control configurations?

- Cost associated with obtaining good plant models (needed for centralized control).
- Cascade, decentralized, etc.: Controller is usually tuned one at a time with little modelling effort. Often ON-LINE tuning.
Other advantages decentralized/cascade/hierarchical configurations:

- Often easier to understand for operators
- Reduce the need for control links
- Allow for decentralized implementation
- Tuning parameters have direct and “localized” effect
- Tend to be insensitive to uncertainty
- Simpler implementation
- Reduced computation load
- Simple or even on-line tuning
- Longer sampling intervals for the higher layers
- Allow simple models when designing higher layers
- “Stabilize”\(^1\) the plant such that it is can be controlled by operators.

\(^1\)The terms “stabilize” and “unstable” as used by process operators may not refer to a plant that is unstable in a mathematical sense, but rather to a plant that is sensitive to disturbances and which is difficult to control manually.
THEORY FOR CONTROL CONFIGURATIONS

Partial control

<table>
<thead>
<tr>
<th>Control configuration</th>
<th>Meas./Control of $y_1$</th>
<th>Control objective for $y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential decentralized control</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sequential cascade control</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>“True” partial control</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Indirect control</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Set $y_2 = r_2$

$$y_1 = (G_{11} - G_{12}G_{22}^{-1}G_{21}) u_1 + (G_{d1} - G_{12}G_{22}^{-1}G_{d2}) d + G_{12}G_{22}^{-1}(r_2 - n_2)$$

$\triangleq P_u$  $\triangleq P_d$  $\triangleq P_r$

$P_d$ – partial disturbance gain
Some criteria for selecting \( u_2 \) and \( y_2 \) in lower-layer:

1. Lower layer must quickly implement the set-points from higher layers, i.e., controllability of subsystem \( u_2 \) / \( y_2 \) should be good. \((G_{22})\)

2. Provide for local disturbance rejection. \((P_d)\)

3. Impose no unnecessary control limitations on problem involving \( u_1 \) and/or \( r_2 \) to control \( y_1 \). \((P_u \text{ or } P_r)\)

“Unnecessary”: Limitations (RHP-zeros, ill-conditioning, etc.) not in original problem involving \( u \) and \( y \)
Example: Control of $5 \times 5$ distillation process

$$u = \begin{bmatrix} L & V & D & B & V_T \end{bmatrix}^T$$

$$y = \begin{bmatrix} y_D x_B & M_D & M_B & p \end{bmatrix}^T$$

Stabilize: Close three SISO loops for level and pressure:

$$y_1 = \begin{bmatrix} y_D & x_B \end{bmatrix}^T, \quad y_2 = \begin{bmatrix} M_D & M_B & p \end{bmatrix}^T$$

Many possible choices for $u_1$ and $u_2$. LV-configuration:

$$u_1 = \begin{bmatrix} L & V \end{bmatrix}^T$$

$DV$-configuration:

$$u_1 = \begin{bmatrix} D & V \end{bmatrix}^T$$
Figure 6: Typical distillation column with LV-control configuration

- $5 \times 5$ model.

Important issues:

- Disturbance sensitivity ($P_d$ should be small)
- Interactions (RGA-elements of $P_u$)

Because of interactions and cost of measurements: Often only one product composition controlled ("true" partial control).
**Decentralized feedback control**

![Diagram of decentralized feedback control]

**Figure 8:** Decentralized diagonal control of a $2 \times 2$ plant

Design of decentralized control systems involves two steps:

1. Choice of pairings (control configuration selection)
2. Design (tuning) of each controller, $k_i(s)$.

Magnitude of off-diagonal elements in $G'$ relative to its diagonal elements given by:

$$E \triangleq (G' - \tilde{G}_{diag})\tilde{G}_{diag}^{-1}$$

Important relationship:

$$\begin{align*}
(I + G K) & = (I + ET_{diag}) (I + \tilde{G}_{diag} K) \\
& \text{(overall)} \quad \text{(interactions)} \quad \text{(individual loops)}
\end{align*}$$

ETC...........

(quite a lot of theory available, RGA etc.)
Conclusions / Future work

1. Control structure design: Issues must be defined
2. Tools and theory are developing; Controllability analysis
3. Use feedback effectively - feedback hierarchies
4. Balance between centralized and cascade/decentralized implementations
5. Balance between complexity and performance
6. Balance between modelling effort and performance
7. Mathematical problem formulation is difficult (non-convex):
   - Optimal design of $K$ with given no. of non-zero elements
   - Penalize links and controller complexity
   - Controllability analysis approaches; branch and bound
Figure 1: Block diagram of a partial control system