Multiple Steady States and Instability in Distillation. Implications for Operation and Control†

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The fact that distillation columns, even in the ideal binary case, may display multiple steady states and unstable operating points has only recently been recognized. This article addresses some implications of these phenomena for the operation and control of distillation columns. Under manual operation, the multiplicity and instability will result in inability to reach separations corresponding to unstable operating points and may furthermore cause abrupt changes and hysteresis in operating conditions. It is shown that an unstable operating point may be stabilized by feedback control of a single product composition or tray temperature (one-point control). The steady-state multiplicity does, in this case, not represent any severe limitation in operation, but if the control is not sufficiently tight, the column may settle in sustained oscillations (stable limit cycle). Finally, the impact of open-loop instability on the achievable closed-loop performance with both product compositions under feedback control is discussed.

1. Introduction

Distillation is undoubtedly the most studied unit operation in the process control literature. Apart from the industrial importance of distillation, this is probably due to the fact that distillation columns possess several inherent properties, e.g., strong interactions (ill-conditioning), sluggish responses, and strong nonlinearities, that make tight control of the product compositions a challenging task. However, with level and pressure loops closed, all published work so far have assumed the columns to be open-loop stable. This is mainly a result of the fact that most authors employ dynamic models with the common assumptions of inputs, e.g., reflux and boilup, yields several solutions in terms of the outputs, e.g., product compositions. A different type of multiplicity, input multiplicity, that also may occur in distillation is discussed in Jacobsen and Skogestad (1993) and is not treated here. When we talk about multiplicity and multiple steady states in the following, we always refer to output multiplicity unless otherwise stated.

In Jacobsen and Skogestad (1994), the stability of distillation columns is studied and it is shown that columns which display multiple steady states will have at least one solution that corresponds to an unstable operating point. It is also shown that, for a given column, instability is most likely with large internal flows, i.e., large reflux and boilup.

Jacobsen and Skogestad (1991, 1994) show that the existence of multiplicity and instability in general will depend on the specific choice of independent inputs. A two-product distillation column has many inputs (flows) that may be manipulated. Assuming given feed conditions, these are typically the product flow rates \( D_w \) and \( B_w \), the reflux \( L_w \) (the index \( w \) denotes mass basis), the boilup \( V \) (indirectly adjusted with reboiler heat input \( Q_R \)), and the condensation rate \( V_T \) (indirectly adjusted with condenser heat removal \( Q_D \)) (see Figure 1). However, all these flows may not be specified independently. A typical two-product distillation column has 2 degrees of freedom at steady state (still assuming given feed conditions), and thus, only two of the five flows may be specified independently at steady state. From a control point of view, this may be understood from the fact that the column pressure and the liquid levels of the reboiler

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and condenser need to be controlled, and three inputs must be used for this purpose. Typically, the condensation rate \( V_T \) is used to control the pressure, and the product flows \( D_c \) and \( B_w \) are used to control the condenser and reboiler levels, respectively. This leaves reflux \( L_w \) and boilup \( V \) as the independent inputs. In control terms, this is denoted by the \( L_wV \) configuration and is probably the most widespread configuration in industry. Jacobsen and Skogestad (1991, 1994) also find that this is the configuration for which steady-state multiplicity and instability are most likely to occur. However, many other configurations are possible, and the choice of a proper configuration for composition control has been studied extensively in the literature (see e.g., Skogestad et al., 1990). The configurations considered in this paper are the \( L_wV \) configuration, where \( L_w \) [kg/min] and \( V \) [kmol/min] are used for quality control; the \( LV \) configuration, where \( L \) [kmol/min] and \( V \) [kmol/min] are used for quality control; and the \( DV \) configuration, where \( D_o \) [kg/min] and \( V \) [kmol/min] are used for quality control. As noted above, the \( LV \) configuration is primarily of theoretical interest because liquid flows only in rare cases may be manipulated on a molar rate basis. The \( L_wV \) and \( LV \) configurations differ if the chemical components in the distillate have different molecular weights.

Our two previous articles (Jacobsen and Skogestad, 1991, 1994) present fundamental results regarding the existence of multiple steady states and unstable operating points. In this article, we discuss some of the practical implications for operation and control. While multiplicity and instability most certainly have been experienced during industrial operation, it has not been properly understood. The aim of the present article is to explain which effects may be caused by steady-state multiplicity and instability in distillation.

We start the article by presenting an example column that will be used for illustration throughout the article. The column operates with the \( L_wV \) configuration and is shown to display multiple steady-state solutions and unstable operating points. We first discuss how the multiplicity and instability may affect the column behavior when the independent inputs are manipulated by an operator, i.e., manual operation. We then show that an unstable operating point usually may be stabilized by feedback control of a single product composition or tray temperature. Proper distillation control involves feedback control of both product compositions, and we

![Figure 1. Two-product distillation column.](image-url)
Boilup $V$ and Table 2. Steady-State Solutions for the Methanol–Propanol Column with $V$ in the Range 48–63 kg/min, and Constant Molar Flows for the methanol-propanol column with $V$ = 2.0 kmol/min.

Table 2. Steady-State Solutions for the Methanol–Propanol Column with $V$ = 2.0 kmol/min, $L_w$ in the Range 48–53 kg/min, and Constant Molar Flows (No Energy Balance)

<table>
<thead>
<tr>
<th>$L_w$ kmol/min</th>
<th>$D_1$ kmol/min</th>
<th>$L_{w_1}$ kg/min</th>
<th>$y_D$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.064</td>
<td>0.936</td>
<td>48.00</td>
<td>0.5339</td>
</tr>
<tr>
<td>II</td>
<td>1.143</td>
<td>0.857</td>
<td>50.00</td>
<td>0.5828</td>
</tr>
<tr>
<td>III</td>
<td>1.462</td>
<td>0.538</td>
<td>50.00</td>
<td>0.9924</td>
</tr>
<tr>
<td>IV</td>
<td>1.556</td>
<td>0.444</td>
<td>50.00</td>
<td>0.9969</td>
</tr>
<tr>
<td>V</td>
<td>1.650</td>
<td>0.350</td>
<td>53.00</td>
<td>0.9984</td>
</tr>
</tbody>
</table>

3. Steady-State Multiplicity and Instability

Figure 2 and Table 2 show steady-state solutions for the methanol–propanol column for different values of mass reflux $L_w$ with boilup $V$ fixed at 2.0 kmol/min. From Figure 2, we see that, with boilup fixed at this value, there are three steady-state solutions in terms of the product compositions for mass reflux in the range 48.8–52.2 kg/min. For example, for $L_w = 50.0$ kg/min and $V = 2.0$ kmol/min, we obtain the three solutions II, III, and IV in Table 2. Note from Table 2 that the solutions II, III, and IV have different values of the molar reflux $L$ such that specifying the molar flows $L$ and $V$ would yield unique solutions for the product compositions.

Indeed, as shown in Jacobsen and Skogestad (1991), the multiplicity is in this case caused by a multiplicity between the mass reflux $L_w$, which is the manipulated input, and the molar reflux $L$, which determines separation. For a binary mixture, we have

$$L = L_w/M \quad M = y_D M_1 + (1 - y_D) M_2$$

(1)

where $M_1$ and $M_2$ denote the molecular weight of the light and heavy component, respectively. Differentiating (1) with respect to $L$ yields

$$\left(\frac{\partial L_w}{\partial L}\right)_V = M + L(M_1 - M_2)\left(\frac{\partial y_D}{\partial L}\right)_V$$

(2)

The transformation from $L_w$ to $L$ is singular when $\left(\frac{\partial L_w}{\partial L}\right)_V = 0$, or equivalently $\left(\frac{\partial L}{\partial L_w}\right)_V = \infty$, and corresponds to a limit point around which there locally exist two steady-state solutions. For further details and a discussion on operating conditions that favor multiple steady states, we refer the reader to Jacobsen and Skogestad (1991).

The maximum eigenvalue (pole), obtained from local linearizations of the full nonlinear dynamic model, are also shown at selected operating points in Figure 2. From the figure, we see that solutions with a negative slope between reflux $L_w$ and top composition $y_D$ corresponds to unstable operating points with poles in the right half plane (RHP). The eigenvalues at the singular points are zero, as expected. Note that the stability with the $L_w V$ configuration is independent of the tuning of the pressure and level control loops (Jacobsen and Skogestad, 1994). For proof of the instability, we refer to Jacobsen and Skogestad (1994).

The rest of the article is devoted to a discussion on how the observed steady-state multiplicity and instability may affect the operation and control of distillation columns.

4. Manual Operation

Many industrial columns are operated manually in the sense that only the reboiler and condenser levels and column pressure are under feedback control while an operator adjusts the remaining two independent inputs to keep the product compositions close to some specified values. The multiplicity and instability presented above will have several implications for this case. First, it is very difficult to obtain product compositions corresponding to unstable operating points by manual manipulation of reflux and boilup. The reason is that stabilization requires consistent and reasonably fast feedback which the operator is unlikely to provide. Second, one should expect the operating conditions to change drastically as a manipulated variable is taken past a singular (turning) point. Finally, the fact that there are two stable operating points for certain values of the inputs implies that the product compositions will depend not only the current values of the manipulated inputs but also on the past history of operating conditions. This is, hysteresis may be experienced in operation.

We illustrate the potential difficulties involved in the manual operation of a column with multiple steady states and instability through nonlinear simulations of the methanol–propanol column. The desired operating point is operating point III in Table 2, i.e., $y_D = 0.9234$ and $x_B = 0.0078$, which is unstable with the $L_w V$ configuration. Assume that the required reflux and boilup have been computed by means of some steady-state simulator (note that the inputs are uniquely determined when $y_D$ and $x_B$ are specified). Thus, the column is started up with $L_w = 50.0$ kg/min and $V = 2.0$ kmol/min.

However, according to Table 2, there are three possible steady-state solutions for these values of the inputs. Only two of these are stable, namely, operating points II and IV. Assume that the column initially settles at operating point II with $y_D = 0.5828$ and $x_B = 0.0035$. Because the top product is too unpure and the bottom

![Figure 2. Steady-state solutions as a function of mass reflux $L_w$ for the methanol–propanol column with $L_w V$ configuration. On the upper plot, the corresponding maximum eigenvalue is shown at some of the steady-state solutions. Boilup $V = 2.0$ kmol/min.](image-url)
product too pure, the operator decides to increase the reflux \( L_w \) in a stepwise fashion. This is illustrated in Figure 3 together with the response in top composition \( y_D \). In the beginning, the top composition increases slightly with increasing reflux, as expected. However, as the operator increases the reflux from 52.0 to 53.0 kg/min (at \( t = 50 \) min), the top composition starts to increase drastically. The reason is that the reflux has been increased past the lower singular point in Figure 2 and the column goes through what is known as a catastrophic jump (see e.g., Poston and Stewart, 1978). The operator observes that the top product has become too pure and the bottom product too unpure and decides to reduce reflux first to 52.5 kg/min (at \( t = 80 \) min). However, as seen from Figure 3, this does not have the desired effect, and the operator finally decides to reduce the reflux all the way to the initial value of 50.0 kg/min. However, due to the steady-state multiplicity, the column now settles to operating point IV with \( y_D = 0.9969 \) and \( x_B = 0.1038 \). Thus, the operator is unable to reach the desired operating point.

Indeed, for the methanol–propanol column, it is almost impossible to obtain purities in the top product \( y_D \) in the range 0.733–0.987 by manual manipulation of the reflux \( L_w \) while keeping \( V \) constant at 2.0 kmol/min (see Figure 2). Similarly, it is almost impossible to obtain purities in the bottom product \( 1 - x_B \) in the range 0.9765–0.9948 by manipulation of boilup \( V \) while keeping \( L_w \) fixed at 50.0 kg/min.

In conclusion, the simulations in Figure 3 illustrate three different effects that may be observed under manual operation of columns with multiple steady states: (1) inability to reach separations corresponding to unstable operating points, (2) catastrophic jumps as the column goes through a singular point, and (3) hysteresis in operation.

Note that, in some columns, either one or both of the stable solution branches do not completely overlap the unstable branch (see, e.g., Figure 8 in Jacobsen and Skogestad (1991)). In this case, the column is likely to go globally unstable at some point in the sense that the reboiler (missing lower branch) or condenser (missing upper branch) runs dry. The reason is that there exists no stable solution as the column is taken past one of the two singular points.

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**Figure 3.** Nonlinear open-loop response of the methanol–propanol column to step changes in mass reflux \( L_w \). Boilup \( V = 2.0 \) kmol/min.

**Figure 4.** Steady-state solutions as a function of feed composition \( z_F \) for the methanol–propanol column. The dashed line corresponds to unstable operating points. Reflux \( L_w = 50 \) kg/min, boilup \( V = 2.0 \) kmol/min. Roman numbers II–IV refer to Table 2 with \( z_F = 0.5 \) and VI to \( z_F = 0.45 \).

**Figure 5.** Nonlinear open-loop response of the methanol–propanol column to disturbances in feed composition \( z_F \). Reflux \( L_w = 50 \) kg/min and boilup \( V = 2.0 \) kmol/min. Roman numbers II–IV refer to Table 2 with \( z_F = 0.5 \). Operating point VI corresponds to \( z_F = 0.45 \).

**Effect of Disturbances.** Jacobsen and Skogestad (1991, 1994) discuss multiple steady states and instability with respect to the manipulated inputs only. However, when there are multiple steady states with respect to the manipulated inputs, then there will also be multiple steady states with respect to other independent parameters like feed flow rate, feed composition, feed liquid fraction, tray efficiency, etc. (often denoted bifurcation parameters in the nonlinear dynamics literature). All these parameters will vary to some extent during operation and may, similarly to the inputs, cause the column operation to go from open-loop stable to open-loop unstable.

To illustrate this, consider Figure 4, which shows steady-state solutions for the methanol–propanol column with \( L_w = 50.0 \) kg/min, \( V = 2.0 \) kmol/min, and feed composition \( z_F \) in the range 0.40–0.60. From the figure, we see that there are multiple steady-state solutions in terms of the product compositions for \( z_F \) in the range 0.46–0.54. This implies that disturbances in the feed composition may take the column through a singular point and thus cause a sudden "jump" to another solution branch. This is illustrated in Figure 5, which shows the response in the top composition \( y_D \) to a change in the feed composition \( z_F \) from 0.50 to 0.45 (assuming that the column initially is at operating point IV in Table 2). The simulation illustrates how the disturbance causes the top composition to "jump" to the lower solution branch and settle at operating point VI (see Figure 4). When the feed concentration returns to \( z_F \)
In this section, we consider feedback control of a single product composition or tray loop unstable operating point I11 of the methanol-propanol column. We have seen that manual operation of distillation columns with multiple steady states is a very difficult task. In this section, we consider feedback control of unstable distillation columns and show that an open-loop unstable operating point may be stabilized using feedback control of a single product composition or tray temperature. We also discuss to what extent the existence of an open-loop unstable pole influences the achievable closed-loop performance. The latter is important, as the existing literature on distillation control assumes open-loop stable models. However, before we go on to discuss the application of feedback control, we need to understand how the use of mass reflux $L_w$ rather than molar reflux $L$ affects the overall dynamics, apart from the stability, of a column.

5. Operation with Feedback Control

We have seen that manual operation of distillation columns with multiple steady states is a very difficult task. In this section, we consider feedback control of unstable distillation columns and show that an open-loop unstable operating point may be stabilized using feedback control of a single product composition or tray temperature. We also discuss to what extent the existence of an open-loop unstable pole influences the achievable closed-loop performance. The latter is important, as the existing literature on distillation control assumes open-loop stable models. However, before we go on to discuss the application of feedback control, we need to understand how the use of mass reflux $L_w$ rather than molar reflux $L$ affects the overall dynamics, apart from the stability, of a column.

5.1. Overall Dynamics. The analysis presented in Jacobsen and Skogestad (1994) shows that the dominant pole (corresponding to the largest time constant), and thus the low-frequency dynamics, is strongly influenced by the transformation between mass and molar reflux. However, Jacobsen and Skogestad (1994) do not discuss the effect of the transformation on the higher frequency dynamics which are important for feedback control.

Figure 6 shows the magnitude and phase of the transfer functions for both molar and mass reflux at operating point III of the methanol-propanol column. Note that, in order to get comparable units, the magnitudes of the two transfer functions differ by 180° at low frequencies. This is as expected since the transfer function for $L_w$ is unstable. However, the phases approach each other at intermediate frequencies and become identical at high frequencies. The magnitudes of the two transfer functions are also almost identical at intermediate and high frequencies. The fact that they are almost identical also at low frequencies is a coincidence for this operating point, because the dominant poles happen to be of similar magnitude (≈0.0986 and 0.0855 min$^{-1}$). However, the dynamics of the two systems differ since the phases differ. In general, one will find that the magnitudes, similar to the phases, differ at low frequencies and become almost identical at high frequencies.

The main conclusion to draw from Figure 6 is that it is mainly the low-frequency dynamics corresponding to the dominant pole that is influenced by the transformation between mass and molar reflux. The initial response (high-frequency dynamics) is almost unaffected. To explain this, consider the relation between the magnitude of two transfer functions

$$\left|\frac{\partial y_D}{\partial L_w}(j \omega)\right| = \left|\frac{\partial y_D}{\partial L_v}(j \omega)\right|\left|\frac{\partial L_v}{\partial L_w}(j \omega)\right|$$

Differentiating the transformation $L_w = LM$ with respect to $L$ yields

$$\left|\frac{\partial L_w}{\partial L_v}(j \omega)\right| = |M^*| + \left|L^*(M_1 - M_2)\right|$$

Thus, at high frequencies, where the magnitude of $\left|\frac{\partial y_D}{\partial L_v}(j \omega)\right|$ is small, the main difference between the magnitude of the two transfer functions is simply the factor $|M^*|$. The magnitudes of the two transfer functions are significantly different only at low frequencies, where the magnitude of $\left|\frac{\partial y_D}{\partial L_v}(j \omega)\right|$ is large. Similar arguments apply also to the phases of the two transfer functions.

Note that also the transfer function zeros will be affected by the transformation between mass and molar flows. This is not discussed here, but Jacobsen (1993) finds that the transformation even may cause nonminimum-phase (RHP) zeros in the transfer function $\frac{\partial y_D}{\partial V}(s)$, i.e., from boilup $V$ to bottoms composition $x_b$. However, for most columns, including the methanol-propanol column discussed in this article, the effect of the transformation on the transfer function zeros is relatively small.

5.2. Performance Limitations Imposed by RHP Poles. The analysis presented above shows that the main difference between a model with molar reflux and a model with mass reflux is the location of the dominant pole, which may even be unstable in the latter case. When a column is open-loop unstable, feedback control (in addition to level and pressure control) is required for stabilization. From control theory, it is well-known that RHP poles alone do not impose any upper limitation on the bandwidth of the closed-loop system; on the contrary, they impose a lower limit on the allowable bandwidth. Problems with respect to stabilization will therefore only arise if there simultaneously are upper bandwidth limitations (e.g., RHP zeros or delays) at frequencies comparable to the RHP pole ("the system goes unstable before we are able to do anything about it") or if there are input constraints ("we do not have sufficient power to counteract the instability").

Although unstable systems usually may be stabilized by feedback control, the existence of a RHP pole in the open-loop system does have some impact on the achieve-
able control performance. Here we consider scalar systems, but similar results, although more limited, are obtained for multivariable systems if one considers the maximum singular value of the sensitivity function $\sigma(S)$ instead of $|S|$.

Consider the sensitivity function $S = (1 + gc)^{-1}$ of a closed-loop system. Here $S = (r - y)/r = y/d$, where $y$ is the controlled output, $r$ is the reference signal, and $d$ is a disturbance acting on the output $y$. Note that the feedback control improves the system response for frequencies where $|S(j\omega)| < 1$ but deteriorates the system response where $|S(j\omega)| > 1$. Ideally, we want $S = 0$ at all frequencies, that is, perfect following of reference signal and complete disturbance rejection. However, for all real systems, $|S(j\omega)| = 1$ at high frequencies. In addition, for an open-loop system with a pole excess of at least two (satisfied for any real system) and a single real RHP pole $p$, the following constraint applies to $|S(j\omega)|$ (e.g., Freudenberg and Looze, 1985)

$$\int_0^\infty \log|S(j\omega)| \, d\omega = \pi p$$  

(5)

(With no RHP pole, $p = 0$, eq 5 reduces to the well-known Bode Integral.) From (5), we see that we need a frequency range with $|S| > 1$ and that the presence of a RHP pole increases the area where $|S| > 1$. However, (5) does not impose any practical design limitation, as the area for $|S| > 1$ may be smoothed out over an arbitrarily large frequency range, and the peak of $S$ may accordingly be made arbitrarily small. Thus, the RHP pole will not represent a control limitation if there are no other bandwidth constraints present in the system.

For an open-loop system with a real RHP pole $p$ and a real RHP zero $z$, the following constraint applies (Freudenberg and Looze, 1985, 1988)

$$\int_0^\infty \log|S(j\omega)| W(z,\omega) \, d\omega = \pi \log\frac{p + z}{p - z}$$  

(6)

(With no RHP pole, $p = 0$, the integral equals zero.) The weight $W$ is given by

$$W(z,\omega) = \frac{2z}{z^2 + \omega^2}$$  

(7)

The form of $W$ (asymptotically it equals $2\omega$ for frequencies up to $\omega = z$, where is cuts off with a $-2$ slope) implies in most practical cases that essentially all the area for $|S| > 1$ has to be at frequencies lower than $z$, and the sensitivity function must have a peak $|S| > 1$ at $\omega = z$. The peak will have to become increasingly large as the bandwidth frequency (where $|S|$ first reaches 1) approaches $z$. From (6), we also see that, as the RHP zero approaches the RHP pole, the peak goes to infinity.

Note that the right-hand side of (6) is symmetric in the sense that the same area is obtained regardless of whether $p$ is to the left or to the right of $z$ in the RHP. Thus, it may seem that the control problem is similar for the two cases. However, this is somewhat misleading. If a system has a single RHP pole $p$, a single RHP zero $z$, and $p > z$, then stabilization is impossible using a linear stable controller (Youla et al., 1974). Thus, in practice, we must require $p < z$ in order to be able to stabilize an open-loop unstable plant. With a restricted structure of the controller, e.g., a PI controller, we must require the RHP pole to be some distance inside the RHP zero in order to be able to stabilize the column.

For the distillation column, $p = \lambda_{\text{max}}$ and “RHP zeros” are most likely caused by delays in measurements and actuators. Using a first-order Padé approximation for a delay $\theta_4$, results in a RHP zero at $z = 2/\theta_4$. We must then approximately require $p > 2/\theta_4$, or equivalently $\theta_4 < 2/p$, in order to be able to stabilize the system.

5.3. One-Point Composition Control. Tight control of distillation columns requires feedback control of both product compositions (two-point control). However, in order to simply stabilize an open-loop unstable column, one-point control will suffice. This is also the way most industrial columns with composition control are operated. An unstable column operating with the $L_0V$ configuration may be stabilized by controlling either top or bottom composition or any other variable related to composition, e.g., a temperature on any tray inside the column. The analysis presented above for SISO systems then applies.

For operating point III of the methanol–propanol column, the RHP pole is at $p = 0.0855 \text{ min}^{-1}$ and it will be impossible to stabilize the column if the delay exceeds $2/p = 23 \text{ min}$. With a simple controller, stabilization is even more difficult. Indeed, by optimizing the parameters of a PI controller such that the pole with the largest real value is minimized, we find that we are unable to stabilize the column if the deadtime $\theta_4$ exceeds 11 min. Composition measurements in industrial columns (GC analysis) may typically have deadtimes up to 30 min, and one should then use faster temperature measurements in order to stabilize the column.

Nonlinear Simulations. Figure 7 shows nonlinear simulations of the methanol–propanol column using a single-loop PI controller (the tuning $K_c = 3.0$ and $\tau_1 = 20.0$, $C(s) = K_c(1 + 1/\tau_1 s)$, was selected to yield reasonably fast response; note that Ziegler–Nichols tuning
rules resulted in a closed-loop unstable system) between top composition $Y_D$ and mass reflux $L_w$ and a measurement delay $\theta_d = 1.0$ min. The figure shows the responses to setpoint changes in $Y_D$ from operating point II (open-loop stable) to operating point III (open-loop unstable) and then further on to operating point IV (open-loop stable) with boilup $V$ fixed at 2.0 kmol/min (see Figure 2 and Table 2). A logarithmic measurement $Y_D = \log(1 - y_D)$ was used as input to the controller. From the figure, we see that the controller is able to stabilize the open-loop unstable operating point III with a RHP pole at 0.0855 min$^{-1}$. The simulations also demonstrate that the same controller yields reasonable performance at all of the three widely differing operating points. The reason is that the initial response (high-frequency dynamics) in terms of logarithmic composition $Y_D$ is similar at all operating points (Skogestad and Morari, 1988). From the plot of mass reflux $L_w$ (lower plot in Figure 7), we see that the input $L_w$ only changes dynamically. At steady state, there is no change in the input, showing that the three operating points are multiple solutions.

One should be careful about detuning a controller in an open-loop unstable process, as the bandwidth may become lower than the minimum allowable and the operating point closed-loop unstable. This is illustrated in Figure 8, where the controller gain has been reduced by a factor of 2 (to $K_c = 1.5$) compared to Figure 7 (note that reducing the integral action has no effect on stability since the gain $K_c = 3.0$ is sufficient for stabilization). Operating point III now becomes closed-loop unstable, and a small setpoint change causes the system to drift away. However, this does not imply that the column goes globally unstable in the sense that physical constraints are violated. Since there exists stable steady-state solutions above and below the unstable solution, the column goes into a stable limit cycle. If the controller gain is reduced further, the limit cycle will continue, but now with a larger amplitude and a longer period of each cycle. As the controller gain is reduced toward zero, the limit cycle will, in phase space, closely follow the stable steady-state solutions with abrupt jumps at the singular points. This is illustrated by the phase plot for the case $K_c = 0.1$ in Figure 8. Note that there are cases where no solution exists outside a singular point. In this case, the column is likely to go globally unstable, as either the condenser or reboiler runs dry.

5.4. One-Point Temperature Control. As mentioned above, composition measurements are often significantly delayed, and it may therefore prove difficult to stabilize an unstable operating point by feedback control of a composition. However, the RHP pole of an unstable operating point is shared by all outputs of the system, including the compositions and temperatures on all trays inside the column. An unstable operating point may therefore be stabilized by applying feedback control to any of these outputs. Most columns have temperature measurements on selected trays inside the column, and these may be utilized to stabilize the column when composition measurements are significantly delayed. Temperature measurements are relatively inexpensive and usually have negligible delays. Below we demonstrate through simulations of the methanol-propanol column how temperature control may be used to stabilize an unstable operating point.

We apply feedback control to the temperature on tray 3, $T_3$, using boilup $V$ (note that we may stabilize the system using either of the independent inputs $L_w$ or $V$; the one with the most direct effect is usually preferred) and a pure proportional controller with gain 0.2, i.e.,

$$dV(s) = \frac{0.2}{s + 1}[T_3(s) - T_{3'}(s)]$$

where we have assumed a first-order lag of 1 min as the only control limitation. Figure 9 show the nonlinear response in top composition $Y_D$ to disturbances in feed composition $Z_F$ from operating point III of the methanol-
propanol column. The disturbance sequence applied is the same as in Figure 5, and we see that the temperature control stabilizes the column around the open-loop unstable operating point III. From Figure 9, we also see that the required control action in the boilup V is relatively small.

5.5. Two-Point Composition Control. As pointed out above, one-point control is sufficient to stabilize an unstable operating point, but preferably both product compositions should be under feedback control. There exist a large amount of literature on two-point control of distillation columns, but everything is based on open-loop stable models. From the discussion above on the effect of RHP poles and RHP zeros on achievable control performance, we would expect some performance deterioration when an operating point is open-loop unstable, in particular when there are significant measurement delays. Below we design specific controllers for the methanol–propanol column to consider closer the impact of a RHP pole on the resulting control performance.

We will again consider operating point III of the methanol–propanol column. In order to compare the achievable performance for the stable model with molar reflux and the unstable process with mass reflux, we design controllers with optimized performance for both cases. We employ linear models and scale the outputs according to their nominal values so that $1 - y^g$ corresponds to magnitude 1 for the top composition and $x^g$ corresponds to magnitude 1 for the bottom composition. As a design objective, we use the structured singular value, $\mu$ (see e.g., Skogestad et al., 1988). This implies that we may include model uncertainty in the design. We use a relative uncertainty weight in each channel given by

$$w_i(s) = 0.20 \frac{5\delta_{L} s + 1}{0.5\delta_{L} s + 1}$$

(9)

This means that we approximately allow for a deadtime $\theta_d$ in addition to 20% gain uncertainty in each input. The performance weight used is given by

$$w_P(s) = \frac{1}{P} \frac{\tau_{CL} s + P}{\tau_{CL} s}$$

(10)

This implies that the worst case peak of $\sigma(S(j\omega))$ should be less than $P$ and the worst case closed-loop time constant should be less than $\tau_{CL}$.

We design controllers for different values of $\theta_d$, and for each design, the performance weight is adjusted until a $\mu$ value of 1 is achieved. A $\mu$ value of 1 implies that the specified performance is obtained for all plants within the uncertainty description. The performance weight is adjusted so that a reasonable trade-off between the maximum peak $P$ and the closed-loop time constant $\tau_{CL}$ is obtained. The optimal controllers are found through so-called DK iterations, i.e., using $H_\infty$ optimization (K step) and $\mu$ calculations (D step). For this purpose, we employ the Matlab $\mu$-Toolbox (Balas et al., 1993). The controllers thereby obtained contain around 50 states for our designs but may be reduced to around 10 states without significant loss in performance.

Table 3 gives the results for designs with $\theta_d$ between 0.1 and 15 min. For $\theta_d = 0.1$ min, we see from Table 3 that there is almost no difference between the achieved performance for the unstable process ($L_w V$ configuration) and the stable model ($LV$ configuration). As expected from (4), we find that the controllers obtained with this small measurement delay are very similar apart from the scaling factor $M^*$ between $L$ and $L_w$.

With larger measurement delays, we see from Table 3 that we must allow for a lower bandwidth as well as a higher peak in the sensitivity function for the open-loop unstable process compared to the open-loop stable model. This is as expected from the sensitivity integral in (6). With a delay of 3 min, we can only guarantee half the bandwidth and also get a significantly higher peak in the sensitivity function $\sigma(S)$ for the unstable system. With a delay of 5 min (RHP zero $z \approx 0.4 \text{ min}^{-1}$), the response for the $LV$ configuration is still reasonable ($\tau_{CL} = 100 \text{ min}$ and $P = 1.70$), while the response for the open-loop unstable $L_w V$ configuration is poor ($\tau_{CL} = 270 \text{ min}$ and $P = 2.50$). Furthermore, the controller obtained using the model assuming molar inputs would in this case yield a closed-loop unstable system when applied to the plant with the $L_w V$ configuration (after taking the scaling $M^*$ into account).

**Nonlinear Simulations.** Figure 10 shows responses to setpoint changes in top composition $y_D$ for the open-loop stable $LV$ configuration and the open-loop unstable $L_w V$ configuration using the optimized controllers for
The simulations include 5-min measurement delay (approximated using an eighth-order lag) and 20% gain uncertainty. The simulations demonstrate the fact that the $L_w,V$ configuration has a much larger overshoot as well as a longer settling time than the $LV$ configuration.

The results in Table 3 and Figure 10 confirm that an open-loop RHP pole indeed does influence the control performance and, in particular, when there simultaneously are control limitations such as large measurement delays. This implies that, when an operating point is open-loop unstable with the $L_w,V$ configuration and the system in addition has significant measurement delays, one should consider some modification to avoid the performance deterioration caused by the open-loop instability. One possibility is to reconfigure the level control system to obtain a different configuration, e.g., the $D,V$ configuration, which yields open-loop stable operating points provided the level control is sufficiently tight (see discussion below). Another possibility is to utilize a fast temperature measurement on a tray inside the column in a cascaded controller structure such that the fast inner temperature loop stabilizes the column. For instance, we could stabilize operating point III of the methanol–propanol column using a proportional controller between boilup $V$ and the temperature on tray 3 as in (8). We are then left with reflux $L_w$ and the setpoint $T_3^*$ as our independent inputs, and with a composition measurement delay $\theta_0 = 5$ min, we find that the achievable robust performance with the $L_w,V$ configuration is $P = 1.7$ and $T_{CL} = 95$. This is comparable to the values $P = 1.55$ and $T_{CL} = 90$ obtained with the $LV$ configuration using the same cascaded controller structure.

### 6. Operation with Other Configurations

So far, we have only discussed operation with reflux $L_w$ and boilup $V$ as independent variables. One may argue that this is the most fundamental configuration since these are the two flows that affect the separation of the column in a direct manner. This may be one reason why the $L_w,V$ configuration is the most widespread configuration in industry. However, Jacobsen and Skogestad (1991) also show that this is the configuration for which steady-state multiplicity and the related instability are most likely to occur. All other configurations are likely to yield unique operating points, but as shown by Jacobsen and Skogestad (1994), they may still yield unstable operating points. A prerequisite for instability in this case is that the operating point is unstable with the $L_w,V$ configuration and that the level control is relatively slow. While instability with the $L_w,V$ configuration is caused by a single pole crossing the imaginary axis, the instability with other configurations will be brought about by a pair of complex eigenvalues crossing the imaginary axis, i.e., a Hopf bifurcation (Jacobsen and Skogestad, 1994). Jacobsen and Skogestad (1994) found that, for most columns, the Hopf bifurcation will be supercritical, implying that a stable limit cycle (sustained oscillations) occurs as the steady state becomes unstable.

**Nonlinear Simulations.** We will again consider the methanol–propanol column in Table 1, but we now employ the $D,V$ configuration. This configuration is obtained from the $L_w,V$ configuration by changing the condenser level control from using distillate flow $D_w$ to using reflux $L_w$. In the simulations, we assume holdups of 2.0 kmol in the reboiler and condenser (increased to avoid hitting constraints during oscillatory behavior).

![Figure 11](image-url)

*Figure 11.* Nonlinear open-loop simulation of the methanol–propanol column with $D_w,V$ configuration and condenser level controller $dL_c = 0.05dM_{Dw}$. Responses to increases in distillate flow $D_w$ with boilup $V = 2.0$ kmol/min. Dashed line: Corresponding response assuming molar reflux; i.e., $dL = 0.05dM_D$.

The condenser level is controlled using a proportional controller with gain 0.05, i.e., $dL_w = 0.05dM_{Dw}$ corresponding to a closed-loop time constant of 20 min. We assume perfect control of the reboiler level, but this assumption is not important since the tuning of the reboiler level control does not influence stability with the $D_w,V$ configuration.

Figure 11 shows responses in top composition $y_D$ to step changes in distillate flow $D_w$ starting at operating point IV in Table 2. The boilup $V$ is kept fixed at 2.0 kmol/min. Initially, as $D_w$ is increased from 14.25 to 15.5 kg/min, the response in $y_D$ is close to first order, which is the assumed predominant behavior of distillation columns in the literature (e.g., Davidson, 1956; Moczek et al., 1965). However, as $D_w$ is increased further to 16.5 kg/min, the response becomes more oscillatory, although still stable. Upon increasing $D_w$ to 17.5 kg/min, however, the stability is lost and the column instead settles in a stable oscillatory behavior.

Figure 11 also shows the corresponding response in $y_D$ using the same level controller gain but assuming the reflux flow given on a molar rate basis, i.e., $dL = 0.05dM_D$ (dashed line). We see that, in this case, the response in $y_D$ is nonoscillatory, stable, and close to first order for all values of $D_w$.

The observed behavior in Figure 11 is explained by the fact that, initially, the column is at operating point IV, which is stable with the $L_w,V$ configuration (see Figure 2) and hence also stable with the $D_w,V$ configuration (Jacobsen and Skogestad, 1994). However, as $y_D$ decreases with increasing $D_w$, the column enters a region of operation where it will be unstable if operated with the $L_w,V$ configuration, and a certain gain in the condenser level controller is therefore required to obtain stability with the $D_w,V$ configuration. Jacobsen and Skogestad (1994) show that a column with the $D_w,V$ configuration goes unstable approximately when the gain of the condenser level controller is smaller than the RHP pole of the column operating with the $L_w,V$ configuration. With $D_w = 17.5$ kg/min and $V = 2.0$ kmol/min, the methanol–propanol column has a RHP pole $p = 0.053$ if operated with the $L_w,V$ configuration. Thus, the column is unstable also with the $D_w,V$ config-
uration since we have a gain of 0.05 in the condenser level controller.

7. Detecting Open-Loop Instability during Operation

As we have seen, steady-state multiplicity and instability may cause several undesirable phenomena under manual operation and will also limit the achievable performance under feedback control. We provide here some suggestions as to how (potential) open-loop instability may be detected, without experiencing undesirable phenomena, in a column under operation. We discuss two different modes of operation: manual operation with configurations other than the $L_w V$ configuration and operation with one-point composition or temperature control.

7.1. Manual Operation. Provided the level control is reasonably tight, open-loop instability is unlikely to be experienced with configurations other than the $L_w V$ configuration. Thus, if one suspects problems with instability using the $L_w V$ configuration, one should switch to a different configuration, e.g., to the $D_w V$ configuration. As shown here, it will then also be possible to detect whether the column actually is unstable with the $L_w V$ configuration.

First consider operation with the $D_w V$ configuration. The transformation between the $D_w V$ and $L_w V$ configurations is for the top composition $y_D$ given by

$$\left( \frac{\partial y_D}{\partial L_w} \right)_V(s) = \left( \frac{\partial y_D}{\partial L_w} \right)_V(s) \left( \frac{\partial L_w}{\partial L_w} \right)_V(s)$$  \hspace{1cm} (11)

We assume no input or output multiplicity [output multiplicity is unlikely with the $D_w V$ configuration (Jacobson and Skogestad, 1991), while input multiplicity occurs only in special cases (Jacobson, 1993)] with the $D_w V$ configuration. This implies that the steady-state gain $(\partial y_D / \partial L_w)_V(0)$ always is negative (Jacobson and Skogestad, 1991). Assuming no input multiplicity with the $L_w V$ configuration implies that the steady-state gain $(\partial y_D / \partial L_w)_V(0)$ is positive when the $L_w V$ configuration is stable and negative when the $L_w V$ configuration is unstable (Jacobson and Skogestad, 1992). By inspection of (11), we see that this implies that the steady-state gain $(\partial y_D / \partial L_w)_V(0)$ is positive when the $L_w V$ configuration is stable and negative when the $L_w V$ configuration is unstable. Thus, if an increase in $D_w$ with $V$ constant results in an increase in the reflux $L_w$ at steady state, the column will be unstable if operated with the $L_w V$ configuration.

Note that the increase in reflux with increasing distillate flow discussed above applies to steady state only. The reflux will always decrease initially when the distillate flow is increased because initially the top composition is unchanged and the condenser level controller thus causes $L_w$ to decrease. Mathematically, this is understood by considering (11); if the $D_w V$ configuration is stable, i.e., no RHP poles in $(\partial y_D / \partial L_w)_V(s)$, while the $L_w V$ configuration is unstable, i.e., $(\partial y_D / \partial L_w)_V(s)$ has a RHP pole $p$, then $(\partial L_w / \partial D_w)_V(s)$ must contain a RHP zero $z = p$ that cancels the unstable pole. A RHP zero in $(\partial L_w / \partial D_w)_V(s)$ implies that there will be an inverse response in $L_w$ to changes in $D_w$ with $V$ constant. This is illustrated by the simulation in Figure 12.

In conclusion, a RHP pole with the $L_w V$ configuration will imply an inverse response, corresponding to a RHP zero $z = p$, in reflux $L_w$ to changes in distillate flow $D_w$ with the $D_w V$ configuration. This fact may be utilized to detect a potential RHP pole with the $L_w V$ configuration through experiments with the stable $D_w V$ configuration.

We do not discuss operation with other configurations in detail here, but similar arguments to those used for the $D_w V$ configuration apply also to other configurations. For instance, if $(\partial y_D / \partial L_w)_V(0)$ is positive with the $L_w B_w$ configuration, then the column will be unstable if operated with the $L_w V$ configuration. Note that also in this case will there be an inverse response in boilup $V$ to changes in bottoms flow $B_w$.

7.2. Operation with One-Point Control. One of the surprising features of the steady-state multiplicity discussed in this article is that, at unstable operating points, the steady-state gains

$$\left( \frac{\partial x_i}{\partial L_w} \right)_V(0) < 0; \quad \left( \frac{\partial T_i}{\partial L_w} \right)_V(0) > 0$$  \hspace{1cm} (12)

Here $x_i$ denotes the fraction of light component and $T_i$ the temperature on tray $i$, including the reboiler and condenser. Thus, at an unstable operating point, separation in the top of the column becomes worse as the reflux is increased. This is definitely counterintuitive. Similarly, the gains

$$\left( \frac{\partial x_i}{\partial V} \right)_L(0) > 0; \quad \left( \frac{\partial T_i}{\partial V} \right)_L(0) < 0$$  \hspace{1cm} (13)

at an open-loop unstable operating point. This unexpected behavior may be used to detect potential open-loop instability in a column operating with one-point feedback control. For example, with $V$ fixed, increasing a tray temperature through feedback control will at steady state result in an increase in the reflux $L_w$. Note that, similar to what was discussed above, there will also in this case be an inverse response for $L_w$.

8. Conclusions

This article discusses the implications of steady-state multiplicity and open-loop instability for the operation and control of distillation columns.

In a column operated manually with reflux and boilup as the manipulated inputs, the following phenomena may be observed due to multiplicity and instability: inability to reach certain separations because they correspond to unstable operating points; large and abrupt changes in operating conditions for relatively small disturbances or changes in the manipulated inputs; hysteresis in operation, that is, the separation
depends not only on the present value of the manipulated inputs and feed conditions but also on the past history of these.

With configurations other than the $L_m V$ configuration, e.g., the $D_a V$ configuration, steady-state multiplicity is unlikely. However, if the level control is relatively slow, an operating point may become unstable and sustained oscillations corresponding to a limit cycle will then occur.

An unstable column may usually be stabilized by feedback control of a product composition or tray temperature. In this case, the following phenomena may be observed at open-loop unstable operating points: (1) increasing purity in the top, with boilup constant, results in decreased reflux; similarly, increasing purity in the bottom, with reflux constant, results in decreased boilup; this is opposite to what one intuitively would expect; (2) sustained oscillations if the composition or temperature control is not sufficiently tight.

With both product compositions under control, the existence of open-loop instability will limit the achievable control performance to some extent. This effect becomes increasingly marked with large measurement delays.

Acknowledgment

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Nomenclature (See also Figure 1)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>bottoms flow (kmol/min)</td>
</tr>
<tr>
<td>$c$</td>
<td>controller transfer function</td>
</tr>
<tr>
<td>$D$</td>
<td>distillate flow (kmol/min)</td>
</tr>
<tr>
<td>$d$</td>
<td>disturbance acting on output</td>
</tr>
<tr>
<td>$F$</td>
<td>feed rate (kmol/min)</td>
</tr>
<tr>
<td>$g$</td>
<td>process transfer function</td>
</tr>
<tr>
<td>$L$</td>
<td>reflux flow rate (kmol/min)</td>
</tr>
<tr>
<td>$M$</td>
<td>molecular weight, usually of top product (kg/kmol)</td>
</tr>
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<td>$M_1$</td>
<td>pure component molecular weight of most volatile component (kg/kmol)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>pure component molecular weight of least volatile component (kg/kmol)</td>
</tr>
<tr>
<td>$M_l$</td>
<td>tray liquid holdup (kmol)</td>
</tr>
<tr>
<td>$M_D$</td>
<td>condenser holdup (kmol)</td>
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<tr>
<td>$M_R$</td>
<td>reboiler holdup (kmol)</td>
</tr>
<tr>
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<tr>
<td>$N_f$</td>
<td>feed stage location (1 reboiler)</td>
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<td>$P$</td>
<td>maximum peak of sensitivity function</td>
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<td>right half plane pole (min⁻¹)</td>
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<td>$Q_h$</td>
<td>heat input to reboiler</td>
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<tr>
<td>$Q_r$</td>
<td>heat removal in condenser</td>
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<td>$q_f$</td>
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<td>$RHP$</td>
<td>right half plane</td>
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<td>$r$</td>
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<td>sensitivity function</td>
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<td>$T_i$</td>
<td>temperature on tray $i$</td>
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<td>$V$</td>
<td>boilup from reboiler (kmol/min) (determined indirectly by heating $Q$)</td>
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<td>vapor flow to condenser (kmol/min)</td>
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<td>mole fraction of most volatile component in bottom product</td>
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<td>$x_i$</td>
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<td>$y$</td>
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<td>$y_D$</td>
<td>mole fraction of most volatile component in distillate (top product)</td>
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<tr>
<td>$z$</td>
<td>right half plane zero (min⁻¹)</td>
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Greek Symbols

<table>
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<th>Description</th>
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<tr>
<td>$\alpha$</td>
<td>$[y/x_i]/[1-y_i]/(1-x_i)$ = relative volatility (binary mixture)</td>
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<tr>
<td>$\lambda_i(A)$</td>
<td>$i$th eigenvalue of $A$</td>
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<tr>
<td>$\lambda_{max}(A)$</td>
<td>maximum eigenvalue = dominant pole</td>
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<tr>
<td>$\mu$</td>
<td>structured singular value</td>
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<tr>
<td>$\sigma$</td>
<td>maximum singular value</td>
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<td>closed-loop time constant (min)</td>
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<tr>
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<td>deadline (min)</td>
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<tr>
<td>$\omega$</td>
<td>frequency (min⁻¹)</td>
</tr>
</tbody>
</table>

Superscripts

* = nominal steady-state value

Subscripts

$w$ = flow rate (kg/min)

Literature Cited


Rademaker, O.; Rijnsdorp, J. E.; Maarleveld, A. Dynamics and Control of Continuous Distillation Units; Elsevier: Amsterdam, 1975.


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