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Robust Reliable Decentralized Control

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Abstract

Several researchers have considered the *analysis* of control system reliability for systems without plant/model mismatch [1, 2, 3, 4, 5, 6, 7, 8, 9]. In this manuscript we review several of the strongest and/or most recent of these proposed analysis tools, and show that the results are either conservative, computationally expensive, or incorrect. We then develop necessary and sufficient conditions for many forms of reliability defined in the literature, and since the model is always an imperfect representation for the true process, we extend the resulting analysis tools to uncertain systems.

1. Mathematical Background

Norm-bounded real or complex perturbations are collected in the block-diagonal matrix $\Delta_U = \mathrm{diag}\{\Delta_i\}$, as in Fig. 1. Frequency domain performance specifications can be treated as complex uncertainty (the block Δ_P). The *generalized plant* G in Fig. 1 is determined by the nominal model P, the size and nature of the uncertainty, and the performance specifications. The generalized plant G and the controller K can be combined to get the overall system matrix M. If we partition G to be compatible with K, then M is given by the *linear fractional transformation* $M = F_l(G,K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$. The LFT $F_l(G,K)$ is well-defined if and only if the inverse of $I - G_{22}K$ exists.

The structured singular value μ [10, 11, 12] provides the necessary and sufficient test for robustness to linear time invariant perturbations

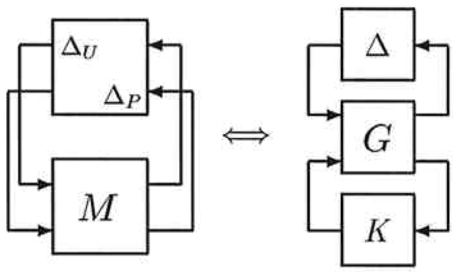

Figure 1: Robust performance and the $G-K-\Delta$ and $M-\Delta$ block structures.

$$\text{system robustness} \iff \mu < 1, \forall\omega \qquad (1)$$

where μ is a function of M (which depends on G and K) and the structure of the uncertainty Δ. For example, the test for *robust stability* is $\mu_{\Delta_U}(M_{11}) < 1$ and the test for *robust performance* is $\mu_\Delta(M) < 1$. Upper and lower bounds for μ can be calculated in polynomial time and are usually close [13, 14]. The pitfalls in attempting to calculate μ exactly in the presence of real Δs are discussed by Braatz et al. [15]. Similar necessary and sufficient tests exist for systems with arbitrary nonlinear (NL) [16], *arbitrarily fast* linear time varying (FLTV) [16], or *arbitrarily slow* linear time varying perturbations (SLTV) [17]. The necessary and sufficient tests for systems with NL, FLTV, or SLTV perturbations can be calculated in polynomial time.

2. Definitions of Reliability

Several strong forms of reliability to failure of actuators or sensors are defined in the open literature for systems without plant/model mismatch.

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Below we review these forms of reliability and extend these definitions to uncertain systems. We will primarily discuss reliability to *actuator* faults or failures, since very similar definitions/results hold for other process equipment.

Integrity is defined as follows [18, 1, 2, 6, 7, 19].

Definition 2.1 *The closed loop system demonstrates* integrity *if $\tilde{K}(s) = EK(s)$ stabilizes $P(s)$ for all $E \in \mathcal{E}_{1/0}$ where*

$$\mathcal{E}_{1/0} \equiv \{\text{diag}(\epsilon_i) \mid \epsilon_i \in \{0,1\}, i = 1,\ldots,n\}. \quad (2)$$

A closed loop system which demonstrates integrity remains stable as subsystem controllers are arbitrarily brought in and out of service. For a system to demonstrate integrity, the plant $P(s)$ must be stable. To have sensor or actuator failure tolerance when the *controller* is unstable, the failure must be recognized and the affected control loop taken out of service. It is clear that whether a system demonstrates integrity can be tested through 2^n stability (eigenvalue) determinations. The following definition extends integrity to uncertain systems.

Definition 2.2 *The closed loop system demonstrates* robust integrity *if the system is stable with $\tilde{K}(s) = EK(s)$ for all $E \in \mathcal{E}_{1/0}$ and all $\|\Delta\|_\infty \leq 1$.*

An uncertain system demonstrates robust integrity if it remains stabilized for any plant given by the uncertainty description, as subsystem controllers are arbitrarily brought in and out of service. For a system to demonstrate robust integrity, the plant must be stable under all allowed perturbations. To have failure tolerance when the controller is unstable, the failure must be recognized and the affected control loop taken out of service. Note that robust integrity implies integrity. It is clear that whether a system demonstrates robust integrity can be tested through 2^n nominal stability (eigenvalue) and 2^n robust stability (μ) calculations.

A very strong notion of reliability was defined by Campo and Morari [1] for decentralized controllers. The requirement is that the nominal closed loop system remains *stable* under arbitrary *independent* detuning of the controller gains. For decentralized control systems, this is equivalent to arbitrary detuning of the actuator/sensor gains to zero. Having stability with detuning is useful because it allows the operators to safely vary the closed loop speed of response depending on process operating conditions.

Definition 2.3 *The closed loop system is* decentralized unconditionally stable *(DUS) if $\tilde{K}(s) = EK(s)$ stabilizes $P(s)$ for all $E \in \mathcal{E}_D$ where*

$$\mathcal{E}_D \equiv \{\text{diag}(\epsilon_i) \mid \epsilon_i \in (0,1), i = 1,\ldots,n\}. \quad (3)$$

The closed loop system will not be DUS if either the plant $P(s)$ or controller $K(s)$ have poles in the open right half plane. To see this, let us consider the multivariable root locus [20] with equal detuning, $\epsilon_i = \epsilon$. For small ϵ, the closed loop poles approach the open loop poles. Since the closed loop poles are a continuous function of the controller gains [20], if any of the open loop poles are in the LHP then some of the closed loop poles will be unstable for sufficiently small ϵ. The following is the generalization to uncertain systems.

Definition 2.4 *The closed loop system is* robust decentralized unconditionally stable *(RDUS) if the system is stable with $\tilde{K}(s) = EK(s)$ for all $E \in \mathcal{E}_D$ and all $\|\Delta\|_\infty \leq 1$.*

By a similar argument as was used for DUS, the closed loop system will not by RDUS if any poles of the controller $K(s)$ or any plant given by the uncertainty description are in the open right half plane. For open loop unstable controllers or plants, some minimum amount of feedback is required for closed loop stability.

Actually, the definition of DUS given by Campo and Morari [1] requires that the closed loop system be stable for all $\epsilon_i \in [0,1]$—we will refer to this version as *closed decentralized unconditional stability* (CDUS). *Closed decentralized robust unconditional stability* (CRDUS) is defined similarly.

3. Review of Previous Research

Here we show that much of the existing tools for analyzing controller reliability are conservative, computationally expensive, or incorrect.

3.1. Integrity

Most research on the analyzing reliability considers only system integrity [18, 2, 3, 5, 6, 7, 19, 9]. Fujita and Shimemura [3] state that a necessary and sufficient condition for integrity with *stable* controllers is that all the principal minors of $I+PK$ are minimum phase. This condition is theoretically interesting, because this test does not require the calculation of matrix inverses. However, since there are 2^n principal minors, the calculation required by this test grows exponentially as a function of the plant dimension.

Fujita and Shimemura [3] also provide a *sufficient* condition for integrity when the controller is stable, in terms of the generalized diagonal dominance of $I + P(j\omega)K(j\omega)$. Applying the Perron-Frobenius Thm. [21] gives the following lemma (for details, see Delich [2]).

Lemma 3.1 *Assume $P(s)$ and $K(s)$ are stable, the diagonal elements of $I + P(s)K(s)$ are minimum phase, and $P(s)$ is irreducible. Then the closed loop system demonstrates integrity if*

$$\rho\left(\left|A(j\omega)\left(\overline{A(j\omega)}\right)^{-1}\right|\right) < 2, \forall\omega, \qquad (4)$$

where $A = I + PK$, $\overline{A}$ refers to the matrix with all offdiagonal elements of A replaced with zero, $|B|$ refers to the matrix with each element of B replaced with its magnitude, and ρ is the spectral radius.

The above assumption that P is irreducible can be removed with some added complexity in the theorem statement [5]. The spectral radius is readily computable with polynomial growth ($\sim n^3$) as a function of the plant dimension. However, the lemma is conservative as shown by the closed loop system with the following plant and controller:

$$K(s) = \frac{75s+1}{\lambda s + 0.01}\begin{bmatrix} -\frac{1}{0.878} & 0 \\ 0 & -\frac{1}{0.014}\end{bmatrix}; \qquad (5)$$

$$P = \frac{1}{75s+1}\begin{bmatrix} -0.878 & 0.014 \\ -1.082 & -0.014\end{bmatrix}. \qquad (6)$$

The inequality (4) is not satisfied for this system ($\rho \approx 2.1 > 2$), illustrating that the sufficient test can be conservative, even for 2×2 systems.

3.2. Robust Integrity

Robust integrity seems to have only been considered by Laughlin et al. [22], which provide computationally simple tests which are useful for cross-directional processes (e.g. paper machines, adhesive coaters [23], polymer extruders [24]). Their results do not extend to general plants; hence we will not discuss these tests further here.

3.3. Decentralized Unconditional Stability

Morari [8] considers stability with detuning of all loops *simultaneously*. This leads to a number of computationally simple necessary conditions for DUS, which are surveyed in the monograph by Morari and Zafiriou [25]. However, all of these conditions can be conservative for testing DUS, as illustrated by numerous examples in the monograph.

3.4. CDUS

Nwokah et al. have considered conditions under which a system with controller $K(s) = (1/s)I$ is CDUS. They claim (Thm. 3 of [26, 27], Thm. 1 of [28], Thm. 5.1 of [29], and Thm. 7 of [30]) that a *necessary* condition for $K(s) = (1/s)I$ to provide CDUS is that $P(0)$ is *all gain positive stable.* A matrix P is *all gain positive stable* if P, P^{-1}, and all their corresponding principal submatrices are *D-stable.* A matrix P is *D-stable* if $\mathrm{Re}\{\lambda_i(PD)\} > 0$, $\forall i$, $\forall D > 0$, where D is real and diagonal.

The following plant (from [1]) illustrates that the condition by Nwokah et al. is *not necessary*:

$$P(s) = \begin{bmatrix} 1 & 0 & 2 \\ \frac{1}{s+1} & 1 & \frac{-4s}{s+1} \\ 0 & 4 & 1 \end{bmatrix}. \qquad (7)$$

It can be shown via the Routh-Hurwitz stability criteria that the closed loop system for the above plant is stable for $K(s) = (1/s)I$ and remains stable with arbitrary detuning of the SISO loop gains. The eigenvalues of $P(0)$ are $\{\pm i\sqrt{3}, 3\}$, so $P(0)$ is not D-stable, and $P(0)$ is not all gain positive stable. We note here without details that the above plant also shows that all theorems in the above papers by Nwokah et al. regarding *decentralized integral controllability* are also *not necessary.*

3.5. RDUS and RCDUS

It seems that these forms of integrity have not been considered in the open literature.

4. Modeling Faults using μ

Braatz [31] describes in some detail the modeling of faults with either uncertainty and/or performance descriptions. These can be included with requirements on the stability or performance during faulty operation to give a μ condition. This μ condition provides a test for system reliability. In what follows we will illustrate how to model actuator gain variation for two cases: 1) without additional uncertainty, and 2) with additional uncertainty.

The nominal controller is defined to be $K(s)$. Then the controller with gain variation can be described by $\hat{K}(s) = EK(s)$, where $E = \text{diag}\{\epsilon_i\}$, and $\epsilon_{i,low} \leq \epsilon_i \leq \epsilon_{i,high}$. We can write the set of E described by the gain variation as $E = \bar{E} + W_r\Delta^r$, where $\bar{E} = \text{diag}\{\bar{\epsilon}_i\}$, $W_r = \text{diag}\{w_i\}$,

$$\bar{\epsilon}_i = \frac{\epsilon_{i,high} + \epsilon_{i,low}}{2}, w_i = \frac{\epsilon_{i,high} - \epsilon_{i,low}}{2}, \quad (8)$$

and Δ^r is a diagonal Δ-block with real independent uncertainties. Standard block diagram manipulations are used to arrive at the $G - K - \Delta$ block structure in Fig. 1, where $\Delta = \Delta^r$ and

$$G(s) = \begin{bmatrix} 0 & I \\ -P(s)W_r & -P(s)\bar{E} \end{bmatrix}. \quad (9)$$

The corresponding M matrix is

$$M(s) = -(I + K(s)P(s)\bar{E})^{-1}K(s)P(s)W_r. \quad (10)$$

Stability is obtained for all variations in gain if and only if $\mu_{\Delta^r}(M(j\omega)) < 1$, $\forall\omega$.

If we are interested in maintaining stability or performance with respect to both actuator gain variation and other perturbations, then the expressions for M and G are somewhat more complicated. Let the system without gain variation be described by $\hat{G}(s)$ with uncertainty $\hat{\Delta}$. Then the G and Δ which includes gain variation are

$$G = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12}W_r & \hat{G}_{12}\bar{E} \\ 0 & 0 & I \\ \hat{G}_{21} & \hat{G}_{22}W_r & \hat{G}_{22}\bar{E} \end{bmatrix}, \Delta = \text{diag}\{\hat{\Delta}, \Delta^r\}. \quad (11)$$

The corresponding M matrix is

$$M = \begin{bmatrix} \hat{G}_{11} + \hat{G}_{12}\bar{E}K(I - \hat{G}_{22}\bar{E}K)^{-1}\hat{G}_{21} & \hat{G}_{12}(I + \bar{E}K(I - \hat{G}_{22}\bar{E}K)^{-1}\hat{G}_{22})W_r \\ K(I - \hat{G}_{22}\bar{E}K)^{-1}\hat{G}_{21} & K(I - \hat{G}_{22}\bar{E}K)^{-1}\hat{G}_{22}W_r \end{bmatrix}. \quad (12)$$

5. Conditions for Reliability

The following are necessary and sufficient conditions for DUS and RDUS, which can be tested approximately in polynomial time as a function of the plant dimension.

Theorem 5.1 (DUS) *Assume $K(s)$ is decentralized. Define Δ^r to be a diagonal Δ-block with independent real uncertainties. Then the closed loop system is DUS if and only if $M(s)$ is internally stable and*

$$\mu_{\Delta^r}(M(j\omega)) \leq 1, \quad \forall\omega, \quad (13)$$

where $M(s) = -\frac{1}{2}\left(I + \frac{1}{2}K(s)P(s)\right)^{-1}K(s)P(s)$.

Proof: Let $\bar{E} = W_r = (1/2)I$ in (10). QED.

Theorem 5.2 (RDUS) *Assume $K(s)$ is decentralized, and that the uncertain system is described by $\hat{G}(s)$ and $\hat{\Delta}$. Define Δ^r to be a diagonal Δ-block with independent real uncertainties. Then the closed loop system is RDUS if and only if $M(s)$ is internally stable and*

$$\mu_{\Delta}(M(j\omega)) \leq 1, \quad \forall\omega, \quad (14)$$

where $\Delta = \text{diag}\{\hat{\Delta}, \Delta^r\}$, *and*

$$M(s) = \begin{bmatrix} \hat{G}_{11} + \frac{1}{2}\hat{G}_{12}K(I - \frac{1}{2}\hat{G}_{22}K)^{-1}\hat{G}_{21} & \frac{1}{2}\hat{G}_{12}(I + \frac{1}{2}K(I - \frac{1}{2}\hat{G}_{22}K)^{-1}\hat{G}_{22}) \\ K(I - \frac{1}{2}\hat{G}_{22}K)^{-1}\hat{G}_{21} & \frac{1}{2}K(I - \frac{1}{2}\hat{G}_{22}K)^{-1}\hat{G}_{22} \end{bmatrix}. \quad (15)$$

Proof: Let $\bar{E} = W_r = (1/2)I$ in (12). QED.

CDUS When $K(s)$ is stable, a necessary and sufficient test for CDUS is given by Thm. 5.1 except with the condition $\mu < 1$ replacing $\mu \leq 1$ in (13). When $K(s)$ is integral, μ in (13) will equal 1 at $\omega = 0$, because setting the proportional gain to zero in a controller with integral action will remove the feedback around the integrator, which will then be a limit of instability. Thus $\mu \leq 1$ in (13) will be a tight necessary condition for CDUS, but not sufficient. The following simple example shows that $\mu \leq 1$ is not sufficient for CDUS:

$$P(s) = \frac{1}{s+1}\begin{pmatrix} s & -1 \\ 1 & 1 \end{pmatrix}, K(s) = \frac{1}{s}I. \quad (16)$$

It can be shown by using the Routh criterion that the above system is DUS and $\mu \leq 1$. Loop #1 is not stable (for *any* ϵ_1) when Loop #2 is open (due to a pole-zero cancellation at $s = 0$), and so the system does not possess integrity and is not CDUS. The following more involved example illustrates that a system can possess integrity and be DUS without being CDUS.

$$P(s) = \frac{1}{s+4}\begin{pmatrix} \gamma\frac{s^2+s+10}{s+\alpha} & 1 \\ 1 & 1 \end{pmatrix}, K(s) = \frac{1}{s}I. \tag{17}$$

where $\gamma = (40\sqrt{55} - 256)/9$ and $\alpha = (62 - 8\sqrt{55})/9$. It can be shown via the Routh criterion that the above system is DUS and has integrity, and $\mu \leq 1$. It can also be shown that the first loop is not stable for $\epsilon_1 = 1/2$ and $\epsilon_2 = 0$ though it is stable for all other $\epsilon_i \in [0, 1]$.

CDUS *can* be checked through a finite number of stability and μ tests, by using Thm. 5.1 to check the interior of the ϵ-hypercube, and testing the boundary (the points, edges, faces, etc.) through additional μ tests. The number of μ tests required grows rapidly with the size of the system. Though the above examples show that CDUS is not equivalent to DUS, the set of plant which are DUS but not CDUS is nongeneric, i.e. any perturbation in such a plant will likely cause the plant to either become DUS or not be DUS. Since Thm. 5.1 provides an exact condition for DUS, finding computable exact conditions for CDUS is of diminished importance.

CRDUS can be defined similarly, and a similar discussion applies as for CDUS.

6. Conclusions

We review several of the strongest and/or most recent of proposed tools for analysis of system reliability, and show that these tools are either conservative, computationally expensive, or incorrect. In particular, it was observed that existing tools for testing integrity require an exponential growth in computation as a function of plant dimension. It was shown that the most well-known sufficient condition for integrity when the controller is stable (Lemma 3.1) can be conservative even for 2×2 systems. It was shown that conditions given by Nwokah and co-workers for closed decentralized unconditional stability are incorrect.

We then develop necessary and sufficient conditions for decentralized unconditional stability and its generalization to uncertain systems. Stability with arbitrary detuning (but not including zero) can tested in polynomial time for arbitrary nonlinear operator, arbitrary linear time varying, and arbitrarily-slow linear time varying uncertainty descriptions, and be approximated in polynomial time for either complex or real linear time invariant uncertainty descriptions. Braatz has used these conditions for the design of robust reliable decentralized controllers [31].

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