Controllability of Integrated Plants

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1 Introduction

The control system for a complete plant must address many objectives, including, but not limited to safety, performance, disturbance rejection, economically optimal operation, operational discontinuities and tracking.

Several options for solving this problem have been proposed, including a completely centralized controller, vertically divided control subsections and horizontal decompositions (multilevel). A multitude of reasons can be launched for each solution, given a certain emphasis or preference.

Today’s most common practice of using a control structure with levels, will in most cases probably prevail compared to total centralized control due to aspects such as robustness, operator acceptance, cost of equipment and modeling etc. Another issue that favors a decentralized control scheme is that tuning is often simpler and requires less information. However complicated, a model is just a simplification and all controllers must be fit to match the desired specification on the real system.

The Tennessee Eastman problem has been published to motivate work in the area of dynamics and control. The system consists of a reactor, condenser, separator and stripper, and while being highly realistic, also is highly non-ideal. The system is open-loop unstable and shows signs of serious interactions.

Initial evaluations were performed on a linear model of the system. Eigenvalues of the system matrix revealed 9 poles in the right half plane (RHP) or close to the

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imaginary axis (pure integrators). Controlling the temperature of the reactor stabilized two unstable poles, while seven additional loops gave the system a maximum eigenvalue of \(-0.75 h^{-1}\).

The system employing this controller system was still somewhat oscillatory and displayed large overshoots on almost all measurements. The controller designed through this synthesis did not stabilize the non-linear system, i.e. the non-linear effects moved the system into instability.

The conclusion is that there is a need for better tools for the analysis and design of large decentralized control systems.

In this paper we will first discuss integrated control in general, with focus on main strategies rather than details before we look into some examples. Existing and novel analysis tools are applied to investigate the effect of interaction and disturbances and their implications on the total control. The measures are applied on a recycle system example.

2 Choice of main control strategy

2.1 Objectives of the control system.

A control system is required to fulfill a multitude of objectives. These include:

- Safe operation of the plant. The control system must ensure that injuries to humans and damage to process equipment are avoided.

- Operation in accordance with environmental regulations. The release of pollutants should be within legal limits.

- The control system must facilitate startup and shutdown of the plant and of individual units within the plant.

- Maintain consistent operating conditions and quality specifications.

- Economically optimal operation. Within the limits imposed by the objectives mentioned above, the control system should identify the economically optimal operating conditions, and enable the operators to keep the operating conditions close to these conditions. The economically optimal operating conditions will depend on a number of external factors, such as the price and quality of the available feedstock, the market prices for the range of product qualities the plant can produce, and possibly ambient weather conditions. In addition scheduled or unscheduled shutdown of individual processes within a plant can affect the optimal operating conditions. The control system must therefore be able to identify and track changes in the optimal operating conditions.

- The control system should be easy to understand for the operating personnel, and the effect of changes should be easy to anticipate.

During the history of process control, the emphasis placed on each of these objectives have varied. Of course, safety and the ability to start up, operate and shut down the plant have always been emphasized. The importance of environmental regulations have increased as the regulations have become stricter. The importance of operator understanding and acceptance has become clear as the development in hardware has
enabled the implementation of more complex control systems. In the early years of the chemical process industry, the computational power necessary to calculate the economically optimal operating conditions (accurately and with frequent updates) for a large industrial plant was simply not available.

2.2 Typical control system organization.

The organization of a typical control system for a large modern chemical plant reflects how present practice in process control has evolved from the practice established in earlier years of process control. The organization of a control system for a typical large chemical plant is illustrated in Fig. 1.

At the lowest level one finds the regulatory control system, which keeps a set of measurements at setpoints specified by the operators or the higher levels in the control system. The regulatory control system is usually highly decentralized, consisting mainly of single input single output (SISO) feedback loops. Feedforward may be used in some cases, and cascades are often used for flow control. True multivariable controllers are relatively rare. Thus, although the regulatory control system is implemented using electronics and computers, it has changed little conceptually from the days of SISO pneumatic controllers.

The middle level in the control system hierarchy may be termed supervisory control. This level in the control system usually coordinates several alternative manipulated variables in order to avoid saturation in critical loops in the regulatory control system, and maintain a set of measurements at setpoints or within maximum and minimum limits. The supervisory control system commonly uses a model of the process to predict the future values of the controlled variables. If no constraints are
active or are predicted to become active, some manipulated variables are commonly reset to predefined ideal resting values. Supervisory control first appeared in the seventies, and has become very popular during the eighties. The top level in the control system hierarchy is steady state optimization. At this level the optimal plant conditions are calculated simultaneously for part of the plant or for the entire plant. Plant wide steady state optimization became routine in the sixties and seventies, and is now performed regularly in many modern plants.

Usually, the lower level uses linear models, the middle level uses linear models but allows for constraints, while the top level uses a nonlinear model (or a linear model which is updated based on linearization).

Some work can be found in the literature about how to decompose the control system in a manner similar to what is described above (e.g., Morari and coworkers, 1980, Umeda and coworkers, 1978). However, there is little justification in the literature for why this decomposition is needed, the most comprehensive discussion about the need for decomposition appears to be given by Mesarovic (1970).

2.3 Centralized control system.

As explained above, the control system in most large plants can be decomposed into three different layers, and the middle layer can be further decomposed into separate units with limited amount of information interchange between each unit. It is clear that a control system which can be decomposed in this way cannot be truly optimal in the sense that it will achieve a lower rate of profit (within the constraints imposed by safety considerations and environmental regulations) over any prolonged period of time compared to a hypothetical control system that uses a dynamic model of the entire plant to optimize plant operation continuously. All control actions in such a hypothetical, ideal control system would be perfectly coordinated, and the control system would use dynamic optimization in real time instead of steady state optimization performed comparatively infrequently\(^1\). Most chemical processes contain nonlinearities, and there are also nonlinearities in the relationship between quality and price for feedstocks and products. A hypothetical, ideal control system would take full advantage of these nonlinearities (although there are still many unresolved issues in nonlinear control theory), in contrast to a traditional control system which is based mainly on linear control theory.

With the increasing availability of computing power it is therefore pertinent to consider whether the conventional decomposition of the control system should be discarded and attempts should be made to design and implement control systems which more closely approximates the hypothetically optimal. Such a control system will in the following be termed an "optimal centralized control system", because it will consist of only one "layer", in which all the tasks of the three different layers of a traditional control system are centralized.

2.4 Drawbacks of an optimal centralized control system.

The advantage of the optimal centralized control system has already been pointed out: When an optimal centralized control system is operating successfully, plant

\(^1\) Vendors of supervisory control systems may claim that their package performs online economic optimization, but the optimization performed in supervisory control systems is usually much too simple to be considered a realistic economic optimization.
profitability will be higher than for the same plant controlled by a traditional control system. The drawbacks of an optimal centralized control system are not so clear, and need to be considered more carefully.

- **Robustness.** The complexity of an optimal centralized control system will make it difficult to analyze whether the system is robust with respect to model uncertainty and numerical inaccuracies. Analyzing robustness need not be trivial even with a traditional control system. The ultimate test of robustness will be in operation of the plant. A traditional control system can be applied gradually, first the regulatory control, then section by section of the supervisory control level. When problems arise, it will therefore be easier to analyze the cause of the problem with a traditional control system than for an integrated control system.

- **Economics.** It is clear that the view of economics taken above is much too simplistic. The costs of designing and maintaining a control system must also be considered. These costs will be much higher for an optimal centralized control system than for a traditional control system, because one cannot reduce problem complexity by decomposing the problem when designing an optimal centralized control system. The need for the use of control systems with the minimum degree of complexity is emphasized by Reeves et al. (1991), for the reasons of system cost, maintainability and reliability. The top (optimization) layer in a traditional control system only uses a steady state model, which is much easier to build than the plant wide dynamic model needed for an optimal centralized control system. The middle, supervisory control level requires a dynamic model of its section of the plant, but can take advantage of the simplification in plant behavior caused by the control action of the lower (regulatory) control level.

It should also be understood that regularity is very important for plant economics, as it often takes a long time to start up a chemical plant. Thus, if plant shutdown occurs more frequently with an optimal centralized control system than with a traditional control system, it is unlikely that overall profit will increase when installing an optimal centralized control system even if the profit per hour of operation increases significantly.

- **Startup and shutdown.** Common operating practice during startup is that virtually all controls are initially put on manual. The loops of the regulatory control system are then put in service when the equipment that they control approaches normal operating conditions. When the regulatory control level is in service, the supervisory control can be turned on. Shutdown is performed in the reverse sequence. Thus, there may be significant scope for improvement in the startup and shutdown procedures of a plant, as quicker startup and shutdown can reduce plant downtime. However, a model of a plant which in addition to normal operating conditions also is able to describe startup and shutdown is necessarily much more complex than a model which only covers the range of conditions encountered in normal operation. Building such a model will be difficult and costly. Startup and shutdown of a plant with an optimal centralized control system that does not cover startup and shutdown, may be more difficult than with a traditional control system, because it may
not be possible to put an optimal centralized control system gradually into service.

- **Operator acceptance and understanding.** Control systems that are not accepted by the operators are likely to be taken out of service. An optimal centralized control system will often be complex and difficult to understand. Operator understanding obviously makes acceptance easier, and a traditional control system, being easier to understand, often has an advantage in this respect. Plant shutdowns may be caused by an operator with insufficient understanding of the control system. Such shutdowns should actually be blamed on the control system (or those who designed and installed the control system), since operators are an integral part of the plant operation, and their understanding of the control system must therefore be ensured.

On the other hand, it is clear that operator acceptance of a control system will be influenced by the available alternatives. Thus optimal centralized control systems have a better chance of being accepted for processes where traditional control systems have failed. One such example appears to be the Light Metal Electrolysis studied by Balchen and coworkers (e. g. Strand, 1991).

- **Hardware and software failure.** In a traditional control system, if a hardware or software failure occurs in the higher levels of the control system, the operators retain the help of the regulatory control system in keeping the plant in operation. A hardware backup system for the regulatory control level is much cheaper than for the higher levels in the control hierarchy, as the regulatory control system can be decomposed into simple control tasks (mainly single loops). In contrast, an optimal centralized control system requires powerful computers, and it is therefore more costly to provide a backup system. However, with continued increase in availability and decrease in price for computing power this argument may weaken.

- **Existing traditional control systems.** Where existing control systems perform reasonably well, it makes more sense to put the effort into improving the existing control system rather than to make the risky decision to design a new control system. This will apply also to many new plants, as many chemical processes are not well known. For such processes it will therefore be necessary to carry out model identification and validation on the actual process. During this period some minimum amount of control will be needed. The regulatory level in a traditional control system requires much less information about the process, and can operate during this period.

2.5 Conclusions.

Control systems for large chemical plants will probably continue to evolve from their traditional structure. Major breaks with tradition are most likely to occur where traditional methods for process control have been clearly unsuccessful.

Current research into improving the supervisory control algorithms (e. g. Lee et al., 1991) appears to be effort well spent. However, improved supervisory control does not imply that the other levels in the control hierarchy become less important. A well designed and structured regulatory control system is a prerequisite for
applying supervisory control, as a poorly designed regulatory control system may impose inherent control limitations which cannot be removed by the higher levels in the control system. Also, a well designed regulatory control system will simplify the design of a supervisory control system, and will be of help to the process operators during startup and shutdown, and when the higher levels of the control hierarchy are out of service.

3 General views on controllability

In this section we consider controllability aspects of the regulatory (mostly decentralized) control system.

The importance of the regulatory control system is not limited to the stability and flexibility requirements, but also the interaction with the supervisory level. The regulatory control determines the achievable performance of the supervisory control level, through the available “speed” of manipulation and the prospects of achieving the instructions imposed by the optimizing control level.

Some examples for which the importance of the regulatory control level is rather obvious are the following.

1) Distillation column control (fig. 2). Here the main issue for the regulatory control is: which inputs to use for inventory control loops. This gives rise to different control structures such as the conventional LV-configuration (where L and V are not used for regulatory control), as well as the DV-, DB-, DBV-configuration and so on. These configurations have very different control characteristics, and failing to find a good configuration may severely inhibit the performance of the remaining 2×2 problem for quality control (e.g., Shinskey, 1984, Skogestad et al., 1990).

2) Control of recycle systems. The issue is whether the recycle or the purge should be manipulated directly. This is discussed in later chapters.

3) Control of FCC (Fluid catalytic cracking) as shown in fig. 2. The issue for regulatory control is: Selection of secondary control objectives (outputs) to be used as manipulators for the supervisory control. It is clear (without going into detail) that the composition of the product, not the temperature in the reactors, is the
Figure 3: Heat exchanger network where bypasses should be placed to control temperatures $Y_{h1}$ and $Y_{c1}$

major objective. But to enable the supervisory control level to control the product composition, the regulatory control loops for temperature must be appropriately configured before applying supervisory control.

4) Control of outlet temperatures in heat exchanger network (fig. 3) The issue for regulatory control is: select appropriate bypass placements for control. The various choices of bypass placement and pairing of the loops may have different characteristics with respect to both control and flexibility.

The main difference between the alternative configurations (structures) in the above example is related to their disturbance rejection properties. That is, some structures have better "built-in" disturbance rejection, and this is related to where the disturbances are "directed". Later in this article we will also look into the conflict between designing regulatory controllers for a single unit versus taking other units into considerations.

4 Analysis tools

We will give a brief description of the analysis tools that will be applied later in this paper. Consider a plant model of the form

$$y = Gu + G_d d$$

where $y$ are the controlled variables, $u$ are the manipulated inputs and $d$ are the disturbances to the system. All variables are scaled to be within the interval -1 to 1, that is, their desired or expected magnitudes should be normalized to be less than 1.

Recommended scalings:

- Inputs ($u$): An $u_i$ of magnitude 1 should correspond to the allowable input signal (e.g., the input reaching its constraint).

- Outputs ($y$): An $y_j$ of magnitude 1 should correspond to the largest allowed controlled output.
• Disturbances ($z$): A $z_k$ of magnitude 1 should correspond to the largest expected disturbance.

_Singular value analysis_ The singular value decomposition of any matrix $G$ is $G = U \Sigma V^H$ with the matrix $\Sigma$ having the singular values $\sigma_i$ on the main diagonal. There will be $r = \text{rank}(G)$ singular values, and the ratio between the largest ($\sigma_1$) and smallest ($\sigma$) is often denoted the condition number, $\gamma(G) = \frac{\sigma_1}{\sigma}$. A plant with a large condition number is called ill-conditioned. Note that $\gamma(G)$ depends on the scaling of the inputs and outputs.

_Relative gain array._ The relative gain array (RGA) is calculated from the plant gain matrix ($G$) and gives valuable information about the interaction between the different control loops. Skogestad and Hovd (1990) give a thorough survey of the RGA and it’s properties. It is defined at each frequency as

$$\Lambda = G \times \begin{bmatrix} G^{-1} \end{bmatrix}^T$$

where $\times$ denotes element-by-element multiplication. It is worth noting that the RGA is independent of scaling, and must only be rearranged (not recomputed) when considering different control pairings. Plants with large RGA-values are both ill-conditioned ($\gamma(G)$ is large) and strongly interactive ($G$ has significant offdiagonal elements).

_Open-loop disturbance gain._ When no feedback is applied the effect of disturbances on the outputs is given as:

$$y = G_d d$$

If the magnitude of $G_d$ (i.e. $\| G_d \|$) is greater than one, feedback is required to counteract the disturbance. In the frequency domain this will give the minimum bandwidth requirements of the control system.

_Closed-loop disturbance gain for decentralized control._ Under closed-loop control we have:

$$e = y - r = Sr - SG_d d$$

At low frequencies this may be written in terms of the individual loops, $S_{\text{diag}}$, as follows:

$$e = y - r \approx -S_{\text{diag}} G_{\text{diag}} G^{-1} r + S_{\text{diag}} G_{\text{diag}} G^{-1} G_d d$$

where $G_{\text{diag}}$ consists of the diagonal elements ($g_{ii}$) of $G$ and $S_{\text{diag}}$ is defined as $(I + G_{\text{diag}} C)^{-1}$, i.e. has elements $1/(1 + g_{ii} c_i)$ (Skogestad and Hovd, 1991). We define the closed-loop disturbance gain (CLDG) as $\Delta = G_{\text{diag}} G^{-1} G_d$. The elements $\delta_{ik}$ represent the apparent disturbance gain from disturbance $k$ to output $i$ when the other loops are closed, and should be used instead of $G_d$ when we want to consider the influence of disturbances using decentralized control.

Since $G_d$ and $G$ are scaled the magnitude $|\delta_{ik}|$ at a given frequency directly gives the necessary loop gain $|g_{ii} c_i|$ at this frequency needed to reject this disturbance. The frequency where $|\delta_{ik}(j\omega)|$ crosses 1 gives the minimum bandwidth requirement for this disturbance. It should be less than the bandwidth that can be achieved in practice, which will be limited by time delays, RHP zeros etc.
4.1 Disturbance rejection

1. *Perfect disturbance rejection.* Assuming perfect control is possible \((y = 0)\), the required magnitude of the manipulated variable is:

\[
u = G^{-1}G_d d
\]  

(6)

Correspondingly for non square plants, \(u = G^+G_d d\) where \(G^+\) denotes the pseudoinverse of \(G\).

*Maximum disturbance rejection.* If perfect control is not possible (ex. more disturbances than manipulated variables or input constraints) the issue is to minimize the offset in \(y\). This corresponds to minimizing \(\|y\|_\infty\). The resulting problem may then be stated as (at each frequency):

\[
\max_d \left( \min_u \left\| y \right\|_\infty \right) \\
\text{s.t.} \quad \|d\|_\infty \leq 1 \\
\quad \|u\|_\infty \leq 1 \\
\quad y = Gu + G_d d
\]

(7)

The answer gives the worst disturbance direction given any possible best input direction. We look at this condition frequency-by-frequency. *Comment:* May alternatively evaluate for one disturbance at a time, then the \(\max_d\) is not needed and the optimization problem is much simpler:

\[
\min_u \left\| y \right\|_\infty \\
\text{s.t.} \quad \|u\|_\infty \leq 1 \\
\quad y = Gu + G_d d \\
\quad d = 1
\]

(8)

This has the additional advantage of yielding information about which disturbances are most difficult to reject. The individual effect of each disturbance is not additive because of the constraints; one can have perfect control of each single disturbance, but together they cause the manipulated inputs to meet the constraints. Conversely multiple disturbances may cancel each other while the individual effects exceed the counteracting effect of the manipulated variables.

*Disturbance rejection with minimum use of control inputs.* Demanding that \(y \approx 0\) is usually not optimal, since this corresponds to high peaks in the manipulated variables. Constraints on \(u\) may also degrade the possibility of perfect tracking. A performance specification is introduced for \(y\) and a maximization over \(d\) is performed to ensure that the 'worst case' is considered. In light of this, the optimization of the manipulated variables may be formulated as:

\[
\max_d \left( \min_u \left\| u \right\|_\infty \right) \\
\text{s.t.} \quad \|d\|_\infty \leq 1 \\
\quad \|y\|_\infty = \left\| Gu + G_d d \right\|_\infty \leq 1
\]

(9)

The difference between the two measures given by eq. 8 and eq. 9 are illustrated in fig. 4.1
Figure 4: Comparison of disturbance rejection measures given by eq. 8 and eq. 9

Figure 5: Illustration of how a manipulated input to one unit is a disturbance to another unit

We see that eq. 8 corresponds to "optimizing you subproblem", that is to use the inputs to their fullest extent, whereas eq. 8 corresponds to "meeting you specification, but minimize the effect on the environment (other subproblems). The last statement follows since inputs in one subproblem usually are disturbances to another subproblem (see fig. 4.1).

5 Controllability analysis

To investigate closer the effect of recycle on controllability, a smaller example was designed. The system consists of a mixer, reactor, flash tank and splitter and is shown in fig. 6. The reaction is $A \rightarrow B$ and some inert (I) is present in the feed in addition to the reactant. The model is based on material balances only for simplicity.

- Controlled outputs are $y_1 = P$ (product stream) and $y_2 = x_I$ (purge inert content).
- Manipulators are $u_1 = F$ (feed flow) and $u_2 = \text{purge or recycle rate}$.
- Disturbances are $d_1 = \text{feed composition (feed inert fraction, } z_F) \text{ and } d_2 = \text{reaction rate constant (e.g., caused by temperature disturbances)}$.

Additional information is given in appendix 1.

There are several choices of manipulators for control for the splitter between recycle and purge:

I. Split fraction $f = S/(R+S)$
II. Purge stream flow S ("S" denotes side stream or secondary product)

III. Recycle stream flow R

This problem also appears in control discussions for other equipment types; for distillation S may be the product (D or B) R the reflux or boilup. Important questions related to these alternatives are:

- Is there any difference?
- Which manipulator is best?

At first it may seem like there is no difference, as a change in S will immediately give a change in R. More precisely, let V denote the flow leaving the flash tank and entering the splitter, then a material balance yields:

\[ V = R + S \]  \hspace{1cm} (10)

However, some more careful thinking and use of intuition will probably lead us to use the smallest stream as the manipulator (at least for the case when there is a large difference in flow rate between R and S). For example, assume the purge stream is small and that we select R as our manipulator which we adjust to meet some specifications, such as inert content, \( x_I \). Now if there is a disturbance in V so that it suddenly drops, then since S is small it is likely that it drops to zero. Thus we get a (undesired) drop also in R! Consequently, it is better to use S as the manipulator when S is small.

These intuitive arguments are mainly related to constraints, However, there is actually other differences between the three options I, II and III. These are related to 1) the fact that disturbances in V are redirected differently as just noted, and 2) that changes in R or S yield changes in V.

Other considerations may be the need for a "stable" (small variations) recycle or purge stream for purposes such as dilution of feed prior to reaction or use of purge as subsequent feed stock.

To evaluate more carefully the alternatives we shall look at the dynamic behaviour.

Let us first consider the disturbance responses for the following three cases:
Figure 7: Open-loop step responses of disturbances to measurements

I. Fixed F and recycle ratio f
II. Fixed F and purge S
III. Fixed F and recycle R

The dynamic responses to step disturbances in \( d_1 = z_{F,1} \) and \( d_2 = k \) are shown in fig. 7. Initially the responses are identical. The reason for the difference in dynamic response is that the change in \( z_{F,1} \) or \( k \) changes the composition in the reactor, which subsequently changes the vapor flow rate \( V \). Now depending on the control configuration, this change in \( V \) is redirected back to the reactor (II), out of the plant (III) or is distributed (I). For case II the response to a step in feed composition (\( x_F \) increases from 0.10 to 0.20) causes a build-up in the recycle stream \( R \), with a subsequent decrease in reaction rate. This is why only part of the response is shown.

Now let us consider changes in the manipulated variable \( u_1 = F \) and \( u_2 = f, S \) or \( R \).

Assume we make a step change in \( u_2 \) (c, S or R) which initially yields the same change in \( R \) (\( \Delta R = -0.2 \)). Initially, for a real system, all three configurations would respond in the same manner (this is not the case for our example where we have not included dynamics in the flash etc. so that there is some direct effect), but there will then be a secondary change in \( V \) that will be redirected differently for the three alternatives. The secondary change in \( V \) is caused both by a direct effect on the flow rates and an indirect effect caused by changes in the reactor conditions (compositions). The responses in fig. 8 (lower part) show that when using the recycle stream as a manipulator (III), then the secondary change in \( V \) dominates and the
resulting effect is small.

Investigating the effect of a step in input $u_1$ (the feed flow $F$ increases from 1.0 to 1.5), the secondary effect of changing vapor flow $V$ will also here influence the response. In case II with constant purge ($S$), the reactor effluent product fraction decreases towards zero due to a large build-up in recycle $R$ and the reactor feed flow, and subsequent slow reaction. This is why only part of the response is shown. In fact in case (II) one may get an inverse response as shown more clearly in fig. 9. The curve shows the response after first being excited with an increase in the manipulated variable $u_2$ as opposed to the "base case" response in fig. 8.

Figure 9: Inverse response by step in feed flow. Case II.
Figure 10: 1,1-RGA-element (for the pairing $u_1$ to $y_1$)

Figure 11: 2,1 CLDG element for both pairings.

We will subsequently do a more careful analysis using RGA and disturbance measures to show the effects of interaction and choice of pairing by the different configurations.

**Pairing** Intuitively, one will expect to pair feed rate with product rate ($u_1$ with $y_1$) and the manipulator for recycle/purge with purge composition ($u_2$ with $y_2$). However a more careful analysis reveals that there is considerable interaction, and at high frequency the best pairing is the opposite. The 1,1-RGA-element (for the pairing $u_1$ to $y_1$) is shown as a function of frequency in fig. 10. We note that it changes from close to 1 at low frequencies to 0 at high frequencies, indicating serious interaction in the intermediate frequency range and also that the preferred pairing changes from low to high frequency.

In fig. 11 is shown the closed loop disturbance gain for feed composition disturbance to purge inert content for the two different pairing selections. All configurations show interaction, considering both open- and closed loop, the choice of pairing thus depends on the closed-loop speed of response (bandwidth).

In fig. 12 we show the minimum control error given constraints on the input variable for all cases and decomposed for for individual disturbances for case III. The model used for calculating $\max_{y} \min_{u} \| y \|_\infty$ is linear, but through the constraints the effects of the two disturbances are not additive. In fig. 12 the sum of the single disturbances does not equal the result as calculated by eq. 8, although the difference is small. The figure illustrates the problems using only single loop gain and single
Figure 12: Input variable constrained minimum control error.

step responses to evaluate disturbance rejection. It is interesting to note that the individual disturbance effects are dominating at different frequencies.

It is possible to use this measure to establish at which frequency the system is most sensitive to disturbances. By investigating the effect of the different disturbances at this frequency limits for open-loop disturbance amplitude may be set in case of control failure.

6 Conclusion

- We have introduced a new controllability measure that gives the limiting performance. This measure shows some promise in making a choice based upon a broad basis of disturbances.

- Choosing manipulators on the regulatory control level must take into account the disturbance propagation. When recycles are present secondary effects must be evaluated.

- Correct use of combinations of analysis tools enhances the possibility of finding the best control configuration.

Specific to recycle systems:

- Split fraction control (I) gives the lowest variation in product flow, but may lead to large variations in overall flows and reactor effluent conditions.

References


Appendix 1. Model for Recycle example

The system consists of a mixer, reactor, separator and a tee. The conditions in the reactor are described by three ordinary differential equations:

\[
\begin{align*}
\frac{dC_A}{dt} &= \frac{F_i}{V}(C_{A_i} - C_A) - kC_A \\
\frac{dC_B}{dt} &= \frac{F_i}{V}(C_{B_i} - C_B) + kC_A \\
\frac{dC_I}{dt} &= \frac{F_i}{V}(C_{I_i} - C_I)
\end{align*}
\]

The volume of the reactor is constant; \( V = 2m^3 \). The nominal feed = 1 kmol/s, consisting of 90 \% A and 10 \% I. The nominal values of the full stream set is given below:

<table>
<thead>
<tr>
<th>Stream name</th>
<th>Flow</th>
<th>( x_A )</th>
<th>( x_B )</th>
<th>( x_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>1.00</td>
<td>0.90</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Reactor feed</td>
<td>2.78</td>
<td>0.64</td>
<td>0.01</td>
<td>0.35</td>
</tr>
<tr>
<td>Reactor effluent</td>
<td>2.78</td>
<td>0.35</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>Flash bottoms</td>
<td>0.80</td>
<td>0.006</td>
<td>0.989</td>
<td>0.005</td>
</tr>
<tr>
<td>Flash top</td>
<td>1.98</td>
<td>0.50</td>
<td>0.02</td>
<td>0.48</td>
</tr>
<tr>
<td>Purge</td>
<td>0.20</td>
<td>0.50</td>
<td>0.02</td>
<td>0.48</td>
</tr>
<tr>
<td>Recycle</td>
<td>1.78</td>
<td>0.50</td>
<td>0.02</td>
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</tbody>
</table>
The reaction constant $k = 0.05$. The flash is essentially an ideal separator between components A and I (top product) and component B (bottom product). The actual K-values used in the flash calculations were $A = 80$, $B = 0.02$ and $I = 100$. A density of $900 \text{kg/m}^3$ and an average moleweight of $40 \text{kg/kmol}$ were used in the conversion between mole flow and volume flow.

The mixer, splitter and flash all consist of algebraic equations, giving a total problem of index 1.