A Comparison of Various Control Structures for Continuous Bioreactors: Exact Linearization Control

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Abstract

In this paper we study the application of exact linearization control to a class of continuous bioreactors described by a simple two-state model. Different from previous studies, we put emphasis on the restrictions of exact linearization control that limit its applicability, that is, the possibility for unstable zero dynamics and singular points. The control inputs are the dilution rate and the feed substrate concentration, the controlled output are the cell concentration, the substrate concentration and the productivity. A complete analysis of zero dynamics, singular points and disturbance decoupling property is done for each of the six SISO and a multivariable control structures. Different control structures are evaluated and compared based on their potential difficulties. For this class of bioreactors, unstable zero dynamics is not a severe problem in general. The zero dynamics are globally stable for all control structures studied though there may be unstable zero dynamics in other control structures. Besides the singular points identified before at which the steady state gain is zero, we identified singular points at which the steady state gain is not zero. Singular points is a severe problem for some of the control structures since they are within or close to the desired operating range. This study shows the importance of control structure selection in exact linearization control.

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1 Introduction

It is well-known that the high nonlinearity of processes is one of the challenging control problems[2]. In the case of mild nonlinearity, linear control, based on local linearization, may provide satisfactory performance. However, in the case of high nonlinearity, linear control may lead to very poor performance and nonlinear control may be necessary. Inspired by this realization, there have been considerable interests in nonlinear control in the past years. As a result, exact linearization control theory has emerged[9]. By using nonlinear coordinate transformations and state feedback, a wide class of nonlinear processes can now be "globally" linearized in either an input-state or an input-output sense. Exact linearization control provides a promising method to the control of nonlinear systems.

Many application studies to various chemical processes[6] as well as other highly nonlinear plants have already been done. Among them, more than ten are applications to continuous bioreactors (for details, see [6]). Although most of the studies are very successful, some do meet problems which limit the applicability of exact linearization control. Dochain and Perrier[4] noticed unstable zero dynamics in an anaerobic digestion process. Lien and Wang[11] and Henson and Seborg[8] identified singular points just in the desirable operating range (the optimal steady state) in the control of the productivity using the dilution rate D and the feed substrate concentration s_f, respectively. In both cases, as they have pointed out, the steady state gain is zero, and a singularity is not unexpected. It is well-known that the applicability of exact linearization techniques relies on the existence of stable zero dynamics and the strong relative degree. Disturbance decoupling property is also very important since there is not yet an explicit way to deal with disturbance rejection in feedback design.

In this paper, these potential difficulties with the application of exact linearization technique to a class of continuous bioreactors is examined. We concentrate on input-output linearization since input-state linearization can be viewed as a special case of input-output linearization. Input-state linearization can have all the same problems except unstable zero dynamics (no zero dynamics with input-state linearization). Various control structures are compared. Similar studies have been done before. Agrawal and Lim's evaluation[1] based on local controllability, local stability, the steady state gain and input multiplicity. Menawat and Balachander[12] noticed the better interaction property with feed substrate concentration s_f as control input than that with dilution rate D. Henson and Seborg[8] also evaluated different control structures using both input-state and input-output linearization. The difference is that besides the classes of continuous bioreactors considered are quite different, we emphasize on analysis of zero dynamics, singular points and disturbance decoupling property. In Section 2 we introduce the process and the control objective. In Section 3 we briefly review the exact input-output linearization theory. In Section 4 we study single loop control structures with the dilution rate D, and in Section 5 the single loop control with feed substrate concentration s_f, and in Section 6 the multivariable control. Finally in Section 7 we summarize the results.

2 Description of the Control Problem

2.1 The Process and Its Nonlinear Model

A continuous bioreactor in its simplest form is described by the following state equations:

$$\frac{dx}{dt} = (\mu - D)x \tag{1}$$

$$\frac{ds}{dt} = D(s_f - s) - \frac{\mu}{Y}x \tag{2}$$

where x and s are the state variables representing the cell mass concentration and the substrate concentration, respectively, D is the dilution rate, and s_f is the substrate concentration in the feed stream. The kinetics of the cell mass production is defined in terms of the specific growth rate, μ, and the yield of cell mass, Y. Equations (1) and (2) are the results of the material balances on the cell mass and the substrate in a constant-volume stirred tank reactor. Although this is a very simple model, it is the most commonly used in the literature and it does represent the dynamical behavior of many important biological processes or one stage of them which are characterized by the growth of a single cell population from a single limiting substrate.

Here we assume a constant yield and assume that the specific growth rate depends only on substrate concentration. Two commonly used kinetic models are:

1. Monod

$$\mu = \frac{\mu_m s}{K_m + s} \tag{3}$$

2. Substrate inhibition

$$\mu = \frac{\mu_m s}{K_m + s + s^2/K_i} \tag{4}$$

In the Monod law, the specific growth rate $\mu(s)$ increases monotonely with s and hence does not take account of any substrate inhibitory effects at high concentrations. The second law is a modification of the Monod law with one additional term accounting for possible substrate inhibitory effects.

In this study, we assume both the dilution rate D and the feed substrate concentration s_f can be used as manipulated variables. We focus on the following three outputs:

$$y1 = x \tag{5}$$

$$y2 = s \tag{6}$$

$$y3 = \beta\mu x \tag{7}$$

$y1$ and $y2$ are simply the cell concentration and the substrate concentration, respectively. $y3$ can be interpreted as the productivity of a purely growth associated product.

From equations (1) and (2) we can get the following steady state model:

$$\bar{\mu} = \bar{D} \tag{8}$$

$$\bar{x} = Y(\bar{s_f} - \bar{s}) \tag{9}$$

where $^-$ denotes steady state value. These steady state relationships are very useful in the following analysis. In particular, equation (8) shows that steady state specific growth rate, and hence the steady state substrate concentration, is only determined by the dilution rate, and is independent of the feed substrate concentration.

2.2 Linear Model

We are studying nonlinear control of bioreactors, a linear model which approximates the plant around a steady state is not necessary. However, as we will see, such a linear model can still provide much important information.

Let $X = [x - \bar{x}, s - \bar{s}]^T$, and $U = [D - \bar{D}, s_f - \bar{s_f}]^T$, where $^-$ denotes steady state value. By using Tailor series expansion, we have

$$\frac{dX}{dt} = AX + BU \tag{10}$$

where

$$A = \begin{bmatrix} 0 & \mu' x \\ -\frac{\mu}{Y} & -\mu - \frac{\mu' x}{Y} \end{bmatrix} \tag{11}$$

$$B = [b_1 \; b_2] = \begin{bmatrix} -x & 0 \\ x/Y & \mu \end{bmatrix} \tag{12}$$

where the prime $'$ denotes derivatives with respect to s. All the variables in the A and B matrices, and subsequently in the following transfer matrix, denote the steady state values. For simplicity we have omitted $^-$. The input-state transfer matrix is

$$G(\lambda) = \begin{bmatrix} \frac{-x}{\lambda + \frac{\mu' x}{Y}} & \frac{\mu \mu' x}{(\lambda + \mu)(\lambda + \frac{\mu' x}{Y})} \\ \frac{x/Y}{\lambda + \frac{\mu' x}{Y}} & \frac{\mu \lambda}{(\lambda + \mu)(\lambda + \frac{\mu' x}{Y})} \end{bmatrix} \tag{13}$$

where in order to distinguish from the substrate concentration s we have used λ to denote the complex variable.

The two open-loop poles are $-\mu$ and $-\mu' x/Y$. The former is always stable , while the latter is stable if and only if $\mu' > 0$. For bioreactors with monoincreasing cell growth rate, for example, the Monald model, μ' can never be less than zero, hence this kind of bioreactors will theoretically be open loop stable at any steady states. However, for any bioreactor whose cell growth rate has a maximum, it will be open loop unstable at substrate inhibition level ($\mu' < 0$). The plant is locally state uncontrollable at any steady state with only one control variable D, but still stabilizable with a fixed mode $-\mu$ which disappears from the first column of the transfer matrix. When s_f is used as the only control variable, the plant is locally state controllable everywhere except those satisfying $\mu' = 0$. Since B has full rank, multivariable control is always controllable.

2.3 Control Objective

The control objective is to attain and maintain the desired steady state. The set-point usually comes from quality control specifications or a steady state optimization of some performance index. In the latter case, the set-point can also be updated on-line by a top-level optimizer.

3 Exact Input-Output Linearization

We give a brief review of the exact input-output linearization theory. For simplicity we consider control affine SISO systems:

$$\frac{dx}{dt} = f(x) + g(x)u \tag{14}$$

$$y = h(x) \tag{15}$$

where $u \in \mathcal{R}$ is the control input, $x \in \mathcal{R}^n$ is the state vector, $y \in \mathcal{R}$ is the controlled output, $f(x)$ and $g(x)$ are n-dimensional smooth functions on $\mathcal{R}^n$, $h(x)$ is a smooth function on $\mathcal{R}^n$.

The nonlinear system is said to have a relative degree of r at the point x_0 if for all x in a neighbourhood of x_0

$$L_g L_f^i h(x) = 0, \quad \forall\, 0 \leq i < r-1 \tag{16}$$

$$L_g L_f^{r-1} h(x) \neq 0 \tag{17}$$

where $L_f h$ is the Lie derivative, i.e. $L_f h = \frac{\partial h}{\partial x} f(x)$.

If a nonlinear system has a finite relative degree, one can always construct a nonlinear state coordinate transformation $\eta = \phi(x)$ such that

$$\phi_i(x) = L_f^{i-1} h(x), \quad 1 \leq i \leq r \tag{18}$$

$$L_g \phi_i(x) = 0, \quad r+1 \leq i \leq n \tag{19}$$

This transforms the nonlinear system into the normal form:

$$\frac{d\eta_i}{dt} = \eta_{i+1}, \quad 1 \leq i \leq r-1 \tag{20}$$

$$\frac{d\eta_r}{dt} = \alpha(\eta) + \beta(\eta)u = L_f^r h(x) + L_g L_f^{r-1} h(x) u, \tag{21}$$

$$\frac{d\eta_i}{dt} = \gamma(\eta), \quad r+1 \leq i \leq n \tag{22}$$

$$y = \eta_1 \tag{23}$$

It is then obvious that the nonlinear state feedback control law

$$u = \frac{1}{L_g L_f^{r-1} h}(v - \sum_{k=0}^{r-1} a_{k+1} L_f^k h - L_f^r h) \tag{24}$$

will make the system linear from v to y, i.e.

$$y^{(r)} + \sum_{k=0}^{r-1} a_{k+1} y^{(k)} = v \tag{25}$$

A linear controller can then be designed for the linearized system. If the objective is to track a setpoint y_{sp}, one simple way is to let

$$v = a_0 \int (y_{sp} - y) d\tau \tag{26}$$

Note that this linearizing control law makes the last $n - r$ state variables of η unobservable from the output. Internal stability requires those unobservable modes to be stable. To be precise, we need the concept of zero dynamics, which is a generalization of the concept of zeros to nonlinear systems. Let us partition the state vector as

$$\zeta = [\eta_1 \; \dots \; \eta_r]', \quad z = [\eta_{r+1} \; \dots \; \eta_n]' \tag{27}$$

Then eq. (22) can be rewritten as

$$\frac{dz}{dt} = \gamma(\zeta, z) \tag{28}$$

Zero dynamics of a nonlinear system is defined as

$$\frac{dz}{dt} = \gamma(0, z) \tag{29}$$

This is equivalent to the dynamics with the output $y(t)$ constrained to identically zero. Exact input-output linearization is, in fact, a nonlinear analog of placing poles at plant zeros, hence cancels the zero dynamics and leads to z unobservable. It is obvious now that the zero dynamics must be stable to guarantee internal stability.

Remark: The applicability of exact input-output linearization depends on the existence of relative degree and on the stability of zero dynamics. However, both relative degree and stability of zero dynamics are local properties of a nonlinear system. This local nature greatly complicates the applicability problem. It is no longer so simple as whether or not applicable to a system. Zero dynamics of a nonlinear system may be stable in some operating regions but unstable in others. Similarly, a nonlinear system may has singular points where the relative degree cannot be defined. So applicability only applies to specific operating region of a nonlinear system.

The concept of singular point can also be precisely defined. A point x_0 is a singular point if there exist a k and a point $x \neq x_0$ such that

$$L_g L_f^k h(x) \neq 0 \tag{30}$$

$$L_g L_f^k h(x_0) = 0 \tag{31}$$

Disturbance Decoupling

For a nonlinear system with disturbance of the form

$$\frac{dx}{dt} = f(x) + g(x)u + p(x)d \tag{32}$$

$$y = h(x) \tag{33}$$

The relative degree of disturbance d can be defined analogously, i.e. a disturbance d has a relative degree of ρ at the point x_0 if for all x in a neighbourhood of x_0

$$L_p L_f^i h(x) = 0, \quad \forall\, 0 \leq i < \rho - 1 \tag{34}$$

$$L_p L_f^{\rho-1} h(x) \neq 0 \tag{35}$$

It has been shown in [3]:

- Disturbance d can be decoupled from the output using only feedback control if $\rho > r$.
- Disturbance d can be decoupled from the output using feedforward/feedback control if $\rho = r$.
- Disturbance d cannot be decoupled from the output unless derivatives of disturbance is used in control law if $\rho < r$.

Using derivatives for control is generally not practical. Hence if $\rho < r$, disturbance rejection must be considered in feedback control design. However disturbance rejection cannot be handled explicitly yet in exact linearization control. Neither can model uncertainty.

Remark: Though we review only the results of SISO systems here, it does introduce the relevant concepts and issues which will be discussed in this paper. For more details about MIMO systems, readers may refer to [10]. Exact input-output linearization even applies to general nonlinear systems which are not control and (or) disturbance affine[7], though only numerical solutions are available in general.

4 Single Loop Control with the Dilution Rate D

Here we have

$$f = \begin{bmatrix} \mu x \\ -\frac{\mu}{Y} x \end{bmatrix}, \qquad g = \begin{bmatrix} -x \\ s_f - s \end{bmatrix}$$

Zero Dynamics

Zero dynamics usually depends on the choice of both control variable and controlled variable. However, as we will show, zero dynamics will almost not depend on the specific controlled variable when the dilution rate D is used as the control variable. Since D appears on the right side of each of the state equations, the relative degree is 1 no matter x or s is the controlled variable. In fact, the relative degree is 1 for all of the controlled variables which are both controllable and functions of state variables only (namely not including the input D). Since the relative degree is 1, the zero dynamics is of order 1. By eliminating D from (1) and (2) we have

$$\frac{dz}{dt} = -\mu z \tag{36}$$

where

$$z = \frac{s_f - s}{x} - \frac{1}{Y} \tag{37}$$

It can be easily shown that z is uncontrollable. Hence z and any of the controllable outputs constitute a new coordinate basis of the state space. Indeed equation (36) is the zero dynamics for all the controllable outputs and it is also in the normal form. The cause for a common zero dynamics is the state uncontrollability. Since there must be some "zero-pole" cancellation in an uncontrollable system, the one-order zero dynamics must be the same as the uncontrollable dynamics which is independent of the choice of output.

From equation (36) we see that this zero dynamics is globally stable. This is indeed a very desirable property. The relative stability, however, depends on operating points. It is very poor at substrate limiting range (low substrate concentration). Finally we must point out that, if the output function involves control input D, the relative degree is zero. Then the zero dynamics is of order 2. Besides equation (36), there is an additional differential equation of the zero dynamics which may be unstable.

4.1 Output $y1$

When the dilution rate D is used as the only control variable and the cell concentration x is the output, this control scheme is called turbidostat. Although this is not a very good control scheme as have pointed out by many authors, this is the first one which have ever been studied and this is also the one which have been most extensively studied. The reason may be that the dilution rate D is the direct and easy way for control and that the measurement of cell concentration is relatively reliable.

In this case, we have

$$y1 = h = x \tag{38}$$

$$L_g h = -x \tag{39}$$

$$L_f h = \mu x \tag{40}$$

The control law

$$D = -\frac{1}{x}(a_0 \int (y1_{sp} - x)d\tau - a_1 x - \mu x) \tag{41}$$

will make the closed loop system input-output linear with a transfer function

$$T(s) = \frac{a_0}{s^2 + a_1 s + a_0} \tag{42}$$

Singular Point

$L_g h = -x = 0$ means wash-out, so it is not possible to have singular points under normal operation condition.

Disturbance Decoupling

Disturbance s_f has a relative degree of 2 (see section 5.1), which is larger than that of the control input, so disturbance will be completely decoupled by the feedback control law. This means that the controlled output will not be affected by disturbance s_f at all. This is verified by simulation result.

4.2 Output $y2$

This control scheme is the nutristat. It has also been studied very early. In this case, we have

$$y2 = h = s \tag{43}$$

$$L_g h = s_f - s \tag{44}$$

$$L_f h = -\frac{\mu}{Y} x \tag{45}$$

The control law

$$D = \frac{1}{s_f - s}(a_0 \int (y2_{sp} - s) d\tau - a_1 s + \frac{\mu}{Y} x) \tag{46}$$

will make the closed loop system input-output linear with a transfer function of the form of (42).

Singular Point

Similarly, $L_g h = s_f - s = 0$ means wash-out, so it is not possible to have singular points under normal operation condition, either.

Disturbance Decoupling

In this control scheme, the relative degree of disturbance s_f is 1 since s_f appears on the right side of equation (2). Since the relative degree of control input is also 1, feedback control is no longer sufficient for the disturbance decoupling. However, we can still completely decouple the disturbance by using a feedforward-feedback control. The disturbance s_f must be measured on-line and the measured value should be used in the control law (46) instead of the nominal value. This result is also verified by simulation.

4.3 Output $y3$

In this case, we have

$$y3 = h = \beta \mu x \tag{47}$$

$$L_g h = -\beta \mu x + \beta \mu' x (s_f - s) \tag{48}$$

$$L_f h = \beta\mu^2 x - \beta\frac{\mu\mu'}{Y}x^2 \tag{49}$$

The control law

$$D = \frac{1}{-\beta\mu x + \beta\mu' x(s_f - s)}(a_0\int(y3_{sp} - y3)d\tau - a_1\beta\mu x - \beta\mu^2 x + \beta\frac{\mu\mu'}{Y}x^2) \tag{50}$$

will make the closed loop system input-output linear with a transfer function of the form of (42).

Singular Point

$$L_g h = -\beta\mu x + \beta\mu' x(s_f - s) = 0 \tag{51}$$

is equivalent to

$$x = 0 \tag{52}$$

$$or \quad -\mu + \mu'(s_f - s) = 0 \tag{53}$$

So, in this control scheme, besides the trivial singular point $x = 0$, we do have nontrivial one which satisfies equation (53). For both Monald law and substrate inhibition law, equation (53) has one solution. The linearizing feedback control will fail in this singular point. More unfortunately, this singular point is within the desirable operating range as we can show that this singular point coincides with optimal steady state, i.e. the maximum steady state productivity. By noting the steady state relationships (10) and (11), we have

$$\frac{d\bar{h}}{d\bar{D}} = \frac{Y}{\bar{\mu}'}[\bar{\mu}'(\bar{s}_f - \bar{s}) - \bar{\mu}] \tag{54}$$

Since the steady state gain is zero at optimal steady state, an infinite control gain seems necessary to make the feedback system globally linear. Hence this singularity is not unexpected.

This singularity was firstly identified by Lien and Wang[11] for a plant with Monald law. They also proposed a modified feedback linearizing control law, i.e. using $\frac{L_g L_f^{r-1} h}{\epsilon + (L_g L_f^{r-1} h)^2}$ instead of $\frac{1}{L_g L_f^{r-1} h}$, where ϵ is a small positive parameter. This modified linearizing control applies only to nonlinear systems which are open loop stable at singular points. By adjusting the parameter ϵ, one can arbitrarily approach the original feedback linearizing law as close as one wishes in everywhere except singular points. The feasibility of the modified control law for this control scheme was shown by their simulation results.

Disturbance Decoupling

The relative degree of disturbance s_f is also 1. So a feedforward-feedback control is necessary and sufficient for a complete disturbance decoupling.

5 Single Loop Control with the Feed Substrate Concentration s_f

Only recently has the feed substrate concentration s_f been used as control variable. Menawat and Balachander[12] have noticed the harmful interaction when dilution rate D is used as the control variable. Here the harmful interaction means that the interaction does not enhance but resists the response of the controlled variable, hence makes the response sluggish and require large control action. The interaction will enhance the response of the controlled variable if the feed substrate concentration s_f is used as the control variable and hence can get better robustness in the sense to meet the control objective in minimum effort.

Here we have

$$f = \begin{bmatrix} (\mu - D)x \\ -Ds - \frac{\mu}{Y}x \end{bmatrix}, \qquad g = \begin{bmatrix} 0 \\ D \end{bmatrix}$$

5.1 Output $y1$

In this case, we have

$$y1 = h = x \tag{55}$$

$$L_g h = 0 \tag{56}$$

$$L_f h = (\mu - D)x \tag{57}$$

$$L_g L_f h = \mu' x D \tag{58}$$

$$L_f^2 h = (\mu - D)^2 x + \mu' x(-Ds - \frac{\mu}{Y}x) \tag{59}$$

The control law

$$s_f = \frac{1}{\mu' x D}(a_0 \int (y1_{sp} - x)d\tau - a_1 x - a_2(\mu - D)x - (\mu - D)^2 x + \mu' x(Ds + \frac{\mu}{Y}x)) \tag{60}$$

will make the closed loop system input-output linear with a transfer function

$$T(s) = \frac{a_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{61}$$

Zero Dynamics

Since the relative degree is 2, there is no zero dynamics.

Singular Point

$L_g L_f h = 0$ means $x = 0$ or $\mu' = 0$. $x = 0$ is the trivial singular point. $\mu' = 0$ is only possible for substrate-inhibited plants. So, for plants with monoincreasing specific growth rate, for example, Monald law, it is impossible to have nontrivial singular point. However, for substrate-inhibited plants this control scheme does have singular point at the substrate concentration with maximum specific growth rate. More unfortunately, this singular point is often within or close to the desirable operating range since in most cases a sufficiently high specific growth rate is necessary.

Unlike the singular point in section 4.3, here the steady state gain at the singular point is not zero as it can be easily seen from (9) or (13) that the steady state gain is always equal to the yield Y. The physical reason will be explained later.

Note that the plant is not asymptotically stable at this singular point. Hence even the modified linearizing control proposed by Lien and Wang is not applicable.

Disturbance Decoupling

Since the relative degree of the disturbance D is 1 which is smaller than that of the control input, disturbance decoupling is impossible. Hence disturbance rejection must be considered in feedback design.

5.2 Output $y2$

Though we can get good interaction by using s_f as the control variable, the control scheme using s_f to control $y2$ is not feasible. The reason is that the controlled output $y2 = s$ is independent of the feed substrate concentration s_f at steady states.

5.3 Output $y3$

In this case, we have

$$y3 = h = \beta\mu x \tag{62}$$

$$L_g h = \beta\mu' x D \tag{63}$$

$$L_f h = \beta\mu(\mu - D)x - \beta\mu' x(Ds + \frac{\mu}{Y}x) \tag{64}$$

The control law

$$s_f = \frac{1}{\beta\mu' x D}(a_0 \int (y3_{sp} - y3)d\tau - a_1\beta\mu x - \beta\mu(\mu - D)x + \beta\mu' x(Ds + \frac{\mu}{Y}x)) \tag{65}$$

will make the closed loop system input-output linear with a transfer function of the form of (42).

Zero Dynamics

Since the relative degree is only 1, the zero dynamics is of order 1. In fact, equation (1) is the zero dynamics. To transform it to the normal form, we must let $y3 = \bar{y3}$. By noting $\bar{y3} = \beta\bar{\mu}\bar{x} = \beta D\bar{x}$, we have

$$\frac{dx}{dt} = (\mu - D)x = D(\frac{y3}{\beta D} - x) = D(\bar{x} - x) \tag{66}$$

Let

$$z = x - \bar{x} \tag{67}$$

we get the normal form zero dynamics

$$\frac{dz}{dt} = -Dz \tag{68}$$

This zero dynamics is also globally stable. Moreover, the relative stability is independent of the operating points. However, when the feed substrate concentration s_f is used as the control variable, we may have unstable zero dynamics if a different output is chosen. For example, we can easily seen from transfer matrix (13) that we will have unstable zero dynamics if $y = x - s$ is chosen as the output.

Singular Point

The singular point here are exactly the same as in section 5.1. For substrate-inhibited plants this control scheme has one nontrivial singular point at $\mu' = 0$. It can be easily shown that the steady state gain at this nontrivial singular point is not zero, either. Indeed it is βDY. Henson and Seborg[8] also noticed a singular point at $\mu' = 0$ in this control scheme for a more complicated plant with three state variables (product concentration is included). Though the singular points are in the same place by chance, they are physically different. In their case the singular point is not unexpected since the singular point is also the optimal steady state and the steady state gain is zero at the singular point as in section 4.3. Here the singular point is not an optimal steady state since the productivity $y3$ increases monotonely with the feed substrate concentration s_f, and the steady state gain at singular point is not zero but βDY.

Disturbance Decoupling

Since the relative degree of the control input s_f and that of the disturbance D are both equal 1, a feedforward-feedback control is necessary and sufficient for a complete disturbance decoupling.

5.4 More on Singular Points

We identified a singular point at $\mu' = 0$ in both section 5.1 and 5.3. We also pointed out that the steady state gain at this singular point is not zero. Then what is the physical reason? Though it is not yet proved, it seems related to output controllability. A nonlinear system is output controllable at $\bar{x}$ if and only if the system matrices A, B and C from the local linearization at $\bar{x}$ satisfies

$$rank[CB\ CAB\ ...\ CA^{n-1}B] \;=\; p, \quad p\ number\ of\ the\ outputs \tag{69}$$

With the A and B matrices in Equations (11) and (12), and the C matrix as

$$C = \frac{\partial h}{\partial x} \tag{70}$$

it can be shown that the bioreactor is output uncontrollable at all the (trivial and nontrivial) singular points met in this paper.

When s_f is the control input, we are still able to find an output in which control structure there is no singular point at $\mu' = 0$ though the singular point at $\mu' = 0$ exists no matter $y1$ or $y3$ is used as controlled output. For example, we can choose $y = x + s$, then at $\mu' = 0$ the system does not have singularity and is output controllable.

An infinite control action seems necessary in exact linearization control strategies when the output is uncontrollable. This suggests that output uncontrollability might be a sufficient condition for singularity though unproved. It is likely that output uncontrollability is also necessary.

6 Multivariable Control

Here we use both the dilution rate D and the substrate concentration s_f as control inputs, and choose the state variable x and s as the controlled outputs. Note that the plant is no longer control linear when both inputs are used for control. This difficulty can be avoided by using $F = Ds_f$ instead of s_f as one control input. Then we have

$$u = \begin{bmatrix} D \\ F \end{bmatrix}, \qquad y = \begin{bmatrix} x \\ s \end{bmatrix}$$

$$f = \begin{bmatrix} \mu x \\ -\frac{\mu}{Y}x \end{bmatrix}, \qquad g = \begin{bmatrix} -x & 0 \\ -s & 1 \end{bmatrix}$$

and

$$L_g^0 h = h = y \tag{71}$$

$$L_g h = \begin{bmatrix} -x & 0 \\ -s & 1 \end{bmatrix} \tag{72}$$

$$L_f h = \begin{bmatrix} \mu x \\ -\frac{\mu}{Y}x \end{bmatrix} \tag{73}$$

The control law

$$u = (L_g h)^{-1}[A_0 \int (y_{sp} - y)d\tau - A_1 y - L_f h] \tag{74}$$

will make the closed loop system input-output linear with a transfer function matrix

$$T(s) = (s^2 I + sA_1 + A_0)^{-1} A_0 \tag{75}$$

If we choose diagonal $A_0 = diag\{a_{011}, a_{022}\}$ and $A_1 = diag\{a_{111}, a_{122}\}$, we will also have input-output decoupling, i.e.

$$T(s) = \begin{bmatrix} \frac{a_{011}}{s^2 + a_{111}s + a_{011}} & 0 \\ 0 & \frac{a_{022}}{s^2 + a_{122}s + a_{022}} \end{bmatrix} \tag{76}$$

From (74) we are able to derive the control law in original control inputs D and s_f. In the case of input-output decoupling, we have

$$D = -\frac{1}{x}(a_{011} \int (y_{sp1} - x)d\tau - a_{111}x - \mu x) \tag{77}$$

$$F = Ds + (a_{022} \int (y_{sp2} - s)d\tau - a_{122}s + \frac{\mu}{Y}x) \tag{78}$$

$$s_f = s + \frac{1}{D}(a_{022}\int(y_{sp2} - s)d\tau - a_{122}s + \frac{\mu}{Y}x) \tag{79}$$

(77) and (79) are the real control law used in implementation. This control law can also be derived directly by using the method in [7] for general nonlinear systems.

Zero Dynamics

Both outputs have relative degree of 1, the sum is 2. So there is no zero dynamics.

Singular Point

If D and F are the control inputs, we see from (72) that there is only a trivial singular point $x = 0$. However, if D and s_f are used instead, we see from (79) that there is one more singular point at $D = 0$. This singular point can also be identified explicitly using the method in [7]. $D = 0$ is a nontrivial singular point as this can happen in normal operation. Though $D = 0$ is possible, it is unlikely for a well-designed control system. So this singular point is not very important in practice. Moreover, we can eliminate this singular point by setting $D_{min} = \epsilon > 0$.

The reason there is a singular point at $D = 0$ when s_f is used instead of F is that F is not a completely independent variable. When $D = 0$, s_f can still vary independently, but F must equal to 0. However this constraint is not considered.

Disturbance Decoupling

We no longer have disturbance since both inputs are used for control.

7 Conclusions

The potential difficulty with the application of exact linearization technique to a class of continuous bioreactors is studied. A complete analysis of zero dynamics, singular points and disturbance decoupling is done for each control structure. Results can be summarized as following:

1. When D is the only control input, the zero dynamics is globally stable and independent of the specific controlled output as long as it is a function of state variables only. When s_f is the only control input, the zero dynamics is globally stable if the output is x or $\beta\mu x$. However unstable zero dynamics is possible if other outputs are selected.

2. Besides the singular point already identified by Lien and Wang[11] in the control of productivity using D, we also find a singular point at the substrate concentration with maximum specific growth rate for substrate-inhibited processes when s_f is the only control input and when either the cell concentration x or productivity is the controlled output. This singular point is often within or closer to the desirable operating range. Unlike the former, this singular point is not the optimal steady state, and the steady state gain at this singular point is not zero. The physical reason seems related to local output controllability. The bioreactor is output uncontrollable at all the (trivial and nontrivial) singular points met in this paper.

The singular point at the substrate concentration with maximum specific growth rate is not inherent to control input s_f, i.e. we are still able to find outputs which do not have singularity and are controllable at this point.

3. For disturbance decoupling, feedback control is sufficient in the control of x using D; feedforward-feedback control is necessary and sufficient for all the others except the control of x using s_f. It is impossible to have disturbance decoupling in the control of x using s_f, disturbance rejection must be considered in feedback design.

4. Besides the improvement of dynamics performance, the multivariable controller does not suffer from the unstable zero dynamics, singular point and disturbance decoupling.

This study shows the importance of control structure selection in the application of exact linearization control. By choosing an alternative control structure we may eliminate or relieve the problems with exact linearization control. By relieve we mean, for example, moving the singular points far away from the desirable operating range and/or having stable zero dynamics in a larger range.

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