An Essay on the Modeling of Uncertainty in Control Systems

Sigurd Skogestad

Dept. of Chemical Engineering
Norwegian Institute of Technology
Trondheim, Norway

Control is an engineering discipline, and the problem formulation and solution are usually closely interlinked. That is, we usually want to minimize the effort spend on problem formulation, unless it turns out to be strictly necessary to get acceptable control performance.

Therefore, it seems to me that we probably are on the wrong track if we want a complete problem description including detailed model, performance specifications and uncertainty models before we start the design. Rather, it seems that in most cases we want to use the robust control tools etc. for analysis (what if...) as described by Roy Smith in his "Rationale for the Workshop". We then start with a simple model, do a reasonable control design, and check whether it is acceptable for various possible sources of model uncertainty. If it happens that the design seems very poor ($\mu$ is high) then a more careful analysis in many cases reveals that the assumed uncertainty is unrealistic.

Of course, there are difficult cases where model uncertainty is very important, and where standard methods fail to give a good design, and in such cases $H_\infty$ or $\mu$-optimal controller designs may be useful. Nevertheless, I would probably be very reluctant of implementing such a controller. Rather, I would be more happy if I was able to design a controller with another method (possibly after a lot of parameter tweaking), but with a value of the $H_\infty$ norm or $\mu$ reasonably close to the optimum. The reason is that any optimization tends to take advantage of the problem formulation (model, performance, noise, uncertainty), and I simply would not trust that that anyone is able to give a problem formulation where "everything" is included — certainly, in such cases an actual implementation would be needed. On the other hand, if you design the controller by some other method, and get a "second opinion" from a $\mu$- analysis, then I would as an engineer feel much more comfortable of going about and implement the solution.

In summary, what I am saying is that I am not all that unhappy about the fact that the new robust control methods are used mainly for analysis. On the other hand, we still
need to learn a lot more about the needs for models for the purpose of controller design — this also includes uncertainty modeling.

About modeling, I also have the strong feeling that there will be a limitation on how far we can get with only identification, and that we need to combine identification with fundamental models. I have a simple process control model of a heat exchanger, which is very ill-conditioned, which I propose as a benchmark problem for identification (developed with Elling W. Jacobsen). For this problem I believe identification is difficult if there is uncertainty or noise on the input signals.

A last issue, which so far has been neglected in formal controller design, is the cost of problem formulation and controller design, versus the improvement we are getting in terms of control performance. As engineers we are of course aware of this trade-off, but as a control theoreticians it is an non-issue. However, as we are requiring more detailed problem formulations, this will eventually become an important issue.

In practice it may go as this: We start with a simple controller design, analyze this, and if it is not acceptable we get some more data (e.g., uncertainty model) or try a better design (e.g., \( \mu \)-optimal). We may then continue like this for a few iterations. Generally the cost increases for each iteration. Thus, even if it in theory were possible to get an acceptable controller design (imagine the competitor has told us that he got it to work), we would as engineers after a certain number of iterations "give up", and probably conclude that is cheaper to redesign the plant or otherwise change the control problem (this is in most cases an option).