TOWARDS INTEGRATING DESIGN AND CONTROL: USE OF FREQUENCY-DEPENDENT TOOLS FOR CONTROLLABILITY ANALYSIS

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Abstract

By controllability (dynamic resilience) we generally mean the best closed-loop performance achievable using any controller. Since the controllability can not be altered by change of the control algorithm, but only by design modifications, it follows that the term controllability provides a link between process design and process control. In this paper we focus on two aspects of controllability: The plants sensitivity to disturbances and the limitations imposed by interactions when using decentralized control. We use simple tools such as the RGA, the PRGA (Performance RGA) and the closely related Closed Loop Disturbance Gain (CLDG). For example, if k'th column of the CLDG is large, then this indicates that disturbance k will be difficult to reject. This may pinpoint the need for modifying the process. The PRGA provides a measure of interaction which also includes one-way coupling. In the paper we apply these measures to distillation column control and fluid catalytic cracker (FCC) control.

1 Introduction

In engineering practice, a system is called controllable if it is possible to achieve the specified aims of the control, whatever these may be (Rosenbrock, 1970, p. 171). Unfortunately, in standard state-space control theory the term "controllability" has a rather limited definition in terms of Kalman's state controllability, which mainly has to do with realization theory. In this paper we will use a broader definition:

Controllability (of a plant) is the (best) quality of the response which can be obtained for the plant by use of feedback control.

(Admittedly, this definition is not very precise, since, for example, "best" is not defined.) Closely related (if not identical) terms are "dynamic resilience" (Morari, 1983) and "achievable performance". Our definition of controllability is similar to that used by Perkins (1989). A key idea in the term "controllability" is that it is an *inherent* property of the plant, and is independent of the selected controller parameters (it is assumed that the optimal tunings are used). Of course, one may restrict the class of allowed controllers, and consider, for example, "controllability using linear controllers" (which we do throughout this paper) or "controllability using decentralized control" (which we consider in most parts of this paper). Since

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the controllability can not be altered by change of the control algorithm, but only by *design modifications*, it follows that the term controllability provides a link between process design and process control. "Design modifications" of course include modifications of the process units (eg., adding buffer tanks, increasing the number of trays in a column), but they also include selection of control objectives (eg., control temperature or composition), selection of manipulated variables (eg., placing of bypass streams) and selection of measurements. Stephanopolous (1984, p. 512) give some examples of how process modifications may change the controllability. Seborg et al. (1989, p. 682) give some guidelines for the control structure selection, that is, selection of controlled, manipulated and measured variables.

What limits controllability? Perfect control can only be achieved if the plant is invertible (Morari, 1983). Several authors (eg., Rosenbrock, 1970) discuss the deteriorating effect of *Right-half plane (RHP) zeros* and *time delays* which make it impossible to invert the plant and retain stability of the closed-loop system. *Constraints* make it impossible to implement in practice the inputs that a perfect inversion requires. *Model uncertainty* results in mismatch between the model used by the controller and the actual plant. Large RGA-elements imply sensitivity to model uncertainty for multivariable systems (Skogestad and Morari, 1987b).

In this paper we will focus on two additional aspects of controllability: The plants sensitivity to *disturbances* and the limitations imposed by *interactions* when using decentralized control. We will concentrate on simple measures which depend on the plant model only such as the RGA, PRGA and CLDG. The advent of the computer has largely removed the need to develop simplified tools in order to save computation time. However, there is still a need for simple tools to yield insight and to assist the engineer in prescreening the large number of alternative control structures, and to get initial estimates of a systems controllability. For example, the RGA is an ideal tool in this respect; it may be computed using only limited information and one calculation is sufficient for screening a large number of alternatives.

However, there are of course limitations with such simple tools, and more powerful and exact methods must be used after the initial screening. Such methods generally involve performing a controller design. For example, one method is to obtain an upper bound on performance by adjusting the performance weight such that the optimal controller satisfies the performance objective. One specific approach, which also takes model uncertainty into account, is to adjust the performance weight such that the structured singular value (mu) for robust performance is 1 (eg., see "Approach 2" in Skogestad and Lundström, 1990).

2 Some simple tools: RGA, PRGA and CLDG

Notation. Let $y(s)$ denote the output response and let $e(s) = y(s) - r(s)$ denote the output error. The closed loop response to a setpoint r and a disturbance z becomes

$$e(s) = -S(s)r(s) + S(s)G_d(s)z(s); \quad S = (I + GC)^{-1} \tag{1}$$

The Laplace variable s is often omitted to simplify notation. G is assumed to be a $n \times n$ square matrix, but G_d may be nonsquare. In most of this paper we consider decentralized control, and the controller $C(s)$ is diagonal with entries $c_i(s)$ (see Fig. 1). This implies that after the variable pairing has been determined, the order of the elements in y and u has been arranged so that the plant transfer matrix $G(s)$ has the elements corresponding to the paired variables on the main diagonal.

RGA. The RGA was first introduced by Bristol (1966) as a measure of interaction and as a tool for pairing selection for decentralized control. However, later it has become clear that the RGA is a useful controllability measure also when decentralized control is not used (eg., Skogestad and Hovd, 1990). The RGA was originally defined at steady-state, but it may easily be extended to higher frequencies (Bristol, 1978). Shinskey (1967,1984) has demonstrated practical applications of the steady-state RGA. The book by McAvoy (1983) yields a good introduction to interactions in multivariable systems, and demonstrates the usefulness of the frequency-dependent RGA. For $n \times n$ plants the RGA is defined by the ratio of the "open-loop" and "closed-loop" gains between input j and output i

$$\lambda_{ij}(s) = \frac{(\partial y_i/\partial u_j)_{u_{l \neq j}}}{(\partial y_i/\partial u_j)_{y_{l \neq i}}} = g_{ij}(s)[G^{-1}(s)]_{ji} \tag{2}$$

Thus, a RGA matrix can be computed using the formula

$$\Lambda(s) = G(s) \times (G^{-1}(s))^T \tag{3}$$

where the $\times$ symbol denotes element by element multiplication (Hadamard or Schur product). The RGA matrix has some interesting algebraic properties (Bristol, 1966):

a) It is scaling independent (eg., independent of units chosen for u and y). Mathematically, $\Lambda(D_1 G D_2) = \Lambda(G)$ where D_1 and D_2 are diagonal matrices.

b) All row and column sums equal one.

c) Any permutation of rows or columns in G results in the same perturbations in the RGA.

d) If $G(s)$ is triangular then $\Lambda(G) = I$.

PRGA. One inadequacy of the RGA (eg., McAvoy, 1983, p. 166) is that it, because of property d, may indicate that interactions is no problem, but significant one-way coupling may exist. To overcome this problem we introduce the performance relative gain array (PRGA). The PRGA-matrix is defined as

$$\Gamma(s) = \tilde{G}(s)G(s)^{-1} \tag{4}$$

where $\tilde{G}(s)$ is the matrix consisting of only the diagonal elements of $G(s)$, i.e., $\tilde{G} = diag\{g_{ii}\}$. This matrix was originally introduced at steady-state by Grosdidier (1990) in order to understand the effect of directions under decentralized control. The elements of Γ are given by

$$\gamma_{ij}(s) = g_{ii}(s)[G^{-1}(s)]_{ij} = \frac{g_{ii}(s)}{g_{ji}(s)}\lambda_{ji}(s) \tag{5}$$

Note that the diagonal elements of RGA and PRGA are identical, but otherwise PRGA does not have all the nice algebraic properties of the RGA. PRGA is independent of *input* scaling, that is, $, (GD_2) = , (G)$, but it depends on output scaling. This is reasonable since performance is defined in terms of the magnitude of the outputs. Note that PRGA $= G_s^{-1}$ where G_s is obtained by input scaling of G such that all the diagonal elements are 1 (at all frequencies).

CLDG. A closely related measure, the closed loop disturbance gain (CLDG), was recently introduced by Skogestad and Hovd (1990). For a disturbance k and an output i, the CLDG is defined by

$$\delta_{ik}(s) = g_{ii}(s)[G(s)^{-1}G_d(s)]_{ik} \tag{6}$$

The reason for the name CLDG will become clear later. A matrix of CLDG's may be computed from

$$\Delta = \{\delta_{ik}\} = \tilde{G}G^{-1}G_d = , G_d \tag{7}$$

The CLDG is scaling dependent, as it depends on the expected magnitude of disturbances and outputs. The CLDG is closely related to the relative disturbance gain (RDG), denoted β_{ik}, introduced by Stanley et al. (1985). We have $\delta_{ik}(s) = \beta_{ik}(s)g_{dik}(s)$.

Note that the PRGA and CLDG have to be recomputed whenever another choice of pairings is selected, whereas the RGA need only be rearranged in accordance with the rearrangement of G (because of property c).

3 Scaling

The RGA has the advantage of being scaling dependent, but for the other measures it is crucial that the variables are scaled properly. In general, the variables should be scaled to be within the interval -1 to 1, that is, their desired or expected magnitudes should be normalized to be less than 1. Recommended scalings:

- Inputs (u): An u_j of magnitude 1 should correspond to the allowable input signal (eg., the input reaching its constraint).

- Outputs (y): An e_i of magnitude 1 should correspond to the largest allowed control error.

- Disturbances (z): A z_k of magnitude 1 should correspond to the largest expected disturbance.

The measures depend on scaling as follows: RGA: independent of scaling; PRGA: depends on scaling of y; G_d, CLDG and RDG: depends on scaling of z and y; Condition number and Disturbance condition number: depends on scaling of u and y. All interpretations and examples in this paper assume that appropriate scaling has been performed.

Comment: Note that the outputs, y_i, have been scaled in terms of the allowed control error, e_i. For use of the PRGA this implicitly assumes that also the setpoint changes, r_i, are of the same magnitude as the allowed control error. If this is not the case, then one should use the diagonal matrix D_r to scale the setpoints such that they all are of magnitude 1, and use the matrix $PRGA_r = \tilde{G}G^{-1}D_r$ to evaluate the performance for setpoint tracking.

4 Performance requirements imposed by disturbances

Some plants have better "built-in" disturbance rejection capabilities than others, that is, their controllability with respect to disturbance rejection is better. For SISO systems, the sensitivity to disturbances is directly given by $G_d(s)$. We have when we consider only disturbance rejection ($r = 0$)

$$e(s) = y(s) = S(s)G_d(s)z(s) \tag{8}$$

If we assume that scaling has been applied to G_d such that at each frequency the expected $z(j\omega)$ is less than 1 in magnitude, then the requirement to achieve $e(j\omega)$ less than 1 in magnitude is that at each frequency

$$|S(j\omega)G_d(j\omega)| < 1 \quad \text{or} \quad |S(j\omega)| < 1/|G_d(j\omega)| \tag{9}$$

This is a performance requirement on the sensitivity function S imposed by disturbance rejection. If $|G_d|$ is large (larger than 1) then feedback is needed to reject this disturbance. A plant with a small G_d is preferable (better controllability) since the need for feedback control then is less, or alternatively, with a given feedback controller (given S) the effect on e of the disturbance is less.

Example: Assume that the appropriately scaled $G_d(s) = k_d/(1+\tau_d s)$, and assume $k_d > 1$. Then the required bandwidth, ω_B, imposed by the requirement of disturbance rejection, is the frequency at which the asymptote of $|G_d(j\omega)|$ is 1. We get $\omega_B = k_d/\tau_d$. We want the required ω_B to be small. That is, we get the obvious result that a "large" (k_d large) and "fast" (τ_d small) disturbance requires a large bandwidth and is difficult to reject.

For multivariable systems we get

$$e = SG_d z \approx (GC)^{-1}G_d z \tag{10}$$

where the approximation holds at low frequencies where control is effective and S is small. In this case we cannot only consider G_d, but need also consider the directions of G_d relative to those of S or GC. These issues are discussed by Skogestad and Morari (1987a) who introduced the disturbance condition number. There are two cases when things are relatively simple: 1) When $C(s)$ is diagonal (discussed in the next section on decentralized control), 2) When a perfect decoupler is used such that the responses in all channels are identical, i.e. $C(s) = k(s)G(s)^{-1}$ where $k(s)$ is a scalar transfer function (note that such a controller cannot be used for plants with large RGA-elements). Then $GC(s) = k(s)I$ and at low frequencies $e \approx \frac{1}{k}G_d z$, and similar to the SISO case the magnitude of the elements of $G_d(j\omega)$ (when appropriately scaled) directly gives us the necessary loop gain, $k(j\omega)$, needed for disturbance rejection.

5 Performance relationships for decentralized control

Assume that G and G_d have been scaled such that 1) the expected disturbances, $|z_k(j\omega)|$, are less or equal to one at all frequencies, and 2) the outputs y_i are scaled such that the allowed errors, $|e_i(j\omega)|$, are less or equal to one at all frequencies (we do not here scale separately the setpoints, r_i, and therefore implicitly assume that these are of the same magnitude as the allowed errors).

Consider the effect of a setpoint change r_j and a disturbance z_k on the offset e_i. With all loops closed the closed-loop response becomes (Fig. 1)

$$e_i = -[S]_{ij}r_j + [SG_d]_{ik}z_k \tag{11}$$

For $\omega < \omega_B$ we may usually assume $S = (I+GC)^{-1} \approx (GC)^{-1}$. Provided the corresponding cofactor of G is nonzero,[1], and c_i is sufficiently large (decentralized control), this approximation will also hold for individual elements

$$[S]_{ij} \approx \frac{[G^{-1}]_{ij}}{c_i}; \quad [SG_d]_{ik} \approx \frac{[G^{-1}G_d]_{ik}}{c_i}; \quad \omega < \omega_B \tag{12}$$

With this approximation (11) becomes

$$e_i \approx -[G^{-1}]_{ij}\frac{1}{c_i}r_j + [G^{-1}G_d]_{ik}\frac{1}{c_i}z_k; \qquad \omega < \omega_B \tag{13}$$

If $g_{ii}(s) \neq 0$ the definitions of the PRGA and CLDG yield

$$e_i \approx -\frac{\gamma_{ij}}{g_{ii}c_i}r_j + \frac{\delta_{ik}}{g_{ii}c_i}z_k; \quad \omega < \omega_B \tag{14}$$

Using $\tilde{S} = (I+\tilde{G}C)^{-1} \approx diag\{1/(g_{ii}c_i)\}$ this may be written on matrix form

$$e \approx -\tilde{S}\tilde{G}G^{-1}r + \tilde{S}\tilde{G}G^{-1}G_d z = -\tilde{S}, r + \tilde{S}\Delta z; \quad \omega < \omega_B \tag{15}$$

From (14) we see that the ratio $\gamma_{ij}/(g_{ii}c_i)$ gives the magnitude of the offset in output i to a setpoint change in output j. This ratio should preferably be small. That is, on a conventional magnitude Bode plot, the curve for $|\gamma_{ij}|$ should lie below $|g_{ii}c_i|$ at frequencies where we want small offsets.

For process control disturbance rejection is usually more important than setpoint tracking. From (14) we see that the ratio $\delta_{ik}/(g_{ii}c_i)$ gives the magnitude of the offset in output i to a disturbance z_k. That is, the curve for $|\delta_{ik}|$ should lie below $|g_{ii}c_i|$ at frequencies where we want small offsets. A plot of $|\delta_{ik}(j\omega)|$ will give useful information about which disturbances k are difficult to reject.

Note that for input disturbances $G_d = G$ and we get $\delta_{ik} = g_{ii}$. Thus, large diagonal elements in G (when appropriately scaled) may imply difficulties rejecting input disturbances.

Comparison with all loops open. To get a better physical interpretation of the RGA and CLDG consider the response $\tilde{e}_i$ to a setpoint change r_i and a disturbance z_k when all the other loops are open. We get

$$\tilde{e}_i = -(1+g_{ii}c_i)^{-1}r_i + (1+g_{ii}c_i)^{-1}g_{dik}z_k \tag{16}$$

At low frequencies we have $|g_{ii}c_i| \gg 1$ and derive

$$\tilde{e}_i \approx -\frac{1}{g_{ii}c_i}r_i + \frac{g_{dik}}{g_{ii}c_i}z_k; \qquad \omega < \omega_B \tag{17}$$

Comparing (17) and (14) we see for a setpoint change r_i in loop i that $\gamma_{ii} = \lambda_{ii}$ gives the approximate change in offset caused by closing the other loops. Similarly, for loop i and disturbance z_k we see that the the open-loop disturbance gain, g_{dik}, is replaced by the closed-loop disturbance gain, δ_{ik}. Also note that the relative disturbance gain (RDG) $\beta_{ik} = \delta_{ik}/g_{dik}$ gives the approximate change in offset caused by closing the other loops.

Limitations of (14). The main limitation with (14) is that it applies only to lower and intermediate frequencies. Furthermore, the issue of stability is not adressed. Another limitation is the assumption that all diagonal elements in $G(s)$ are nonzero. These issues are addressed in more detail by Hovd and Skogestad (1991).

[1] Cofactors of G identically equal to zero are relatively rare except for the offdiagonal zero elements of triangular transfer matrices. However, for these zero elements both $[S]_{ij}$ and $[(GC)^{-1}]_{ij}$ are zero and the approximation holds even though the cofactor is zero.

6 Summary

Let us at this point summarize some results we shall use in the examples (see Hovd and Skogestad (1991):

Pairing Rule 1. Avoid pairings ij with negative values of the steady-state RGA, $\lambda_{ij}(0)$.

Pairing Rule 2. Prefer pairings which yield the RGA-matrix close to identity (the rule follows from considering overall stability, note that it is not sufficient to check only if the diagonal elements are close to 1).

Pairing Rule 3. Prefer pairings ij where $g_{ij}(s)$ puts minimal restrictions on the achievable bandwidth for this loop (the rule follows from (14) above).

Rule 3 is the conventional rule of pairing on variables "close to each other". Rules 1-3 will in many cases determine the best choice of pairings for decentralized control. To evaluate controllability we shall use:

Controllability Rule 1. Avoid plants (designs) with large RGA-values (in particular at frequencies near cross-over). This rule applies for any controller, not only to decentralized control (Skogestad and Morari, 1987b).

Controllability Rule 2. For decentralized control avoid control structures (an entire *set* of pairings) with large values of PRGA ($|\gamma_{ij}|$) or CLDG ($|\delta_{ik}|$) at frequencies close to the desired ω_B, and in particular if the achievable bandwidth for the corresponding loop i is restricted (because of $g_{ii}(s)$, see pairing rule 3) (the rule follows from (14) above).

7 Some simple examples

Example 1. Given

$$G = \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix} \tag{18}$$

We get

$$RGA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad PRGA = \begin{pmatrix} 1 & 0 \\ -10 & 1 \end{pmatrix} \tag{19}$$

From PRGA we see that severe one-way interactions will occur, even though RGA=I.

Example 2. Given

$$G = \begin{pmatrix} 1 & -10 \\ 10 & 1 \end{pmatrix} \tag{20}$$

We get

$$RGA = \begin{pmatrix} 0.01 & 0.99 \\ 0.99 & 0.01 \end{pmatrix}; \qquad PRGA = \begin{pmatrix} 0.01 & 0.1 \\ -0.1 & 0.01 \end{pmatrix} \tag{21}$$

This shows that we may have all elements in PRGA small compared to 1. Also note that the elements in PRGA are small even though there obviously are large interactions. The reason is that the interactions in this case increase the effective loop gain. Thus, from a performance point of view (controllability rule 2) pairing on the diagonal elements of G seems to be a good choice, even though we see clearly from G and the RGA that the opposite pairing must be better. This apparent inadequacy of the PRGA arises because it is a performance measure only and does not take into account what happens at frequencies close to ω_B. If we consider stability (eg., pairing rule 2) then we would arrive at the opposite pairing for which we have

$$G' = \begin{pmatrix} -10 & 1 \\ 1 & 10 \end{pmatrix}; \quad RGA' = \begin{pmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{pmatrix}; \quad PRGA' = \begin{pmatrix} 0.99 & -0.1 \\ 0.1 & 0.99 \end{pmatrix} \tag{22}$$

More details for a very similar example, including closed-loop simulations and sensitivity plots, are given by Hovd et al. (1990).

8 Example 3: Distillation Control

From a control point of view distillation columns may be considered as a 5×5 system with 5 inputs u and 5 outputs y (see Fig.2).

$$u = (L, V, D, B, V_T)^T \tag{23}$$

$$y = (y_D, x_B, M_D, M_B, p)^T \tag{24}$$

The number of possible pairings for single-loop control is very large. However, in most cases the condenser duty, V_T, is used to control pressure, p, and we have a 4×4 control problem. The 4×4 RGA-matrix for this case has a 2×2 identity matrix in the lower right corner, and a 2×2 full matrix in the upper left corner. Intuitively, one would then expect the pairings suggested by the order of (23) and (24) to be preferred. This yields the LV-configuration where L and V are used for composition control which is studied below. However, industrial experience (Shinskey, 1984) have suggested that other options may be preferable, and this has been confirmed by controller design and frequency dependent RGA-analysis (Skogestad et al., 1990).

8.1 Composition control with LV configuration

In order to demonstrate the use of the frequency dependent RGA and CLDG for evaluation of expected control performance, consider a binary distillation column with 40 theoretical trays plus a total condenser. This is "column A" studied by Skogestad et al. (1990). We use a rigorous model which includes liquid dynamics in addition to the composition dynamics. The model has a total of 82 states. Disturbances in feed flowrate F (z_1) and feed composition z_F (z_2) are included. The LV configuration is used, that is, the manipulated inputs are reflux L (u_1) and boilup V (u_2). Outputs are the product compositions y_D (y_1) and x_B (y_2). The model then becomes

$$\begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix} = G(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} + G_d(s) \begin{pmatrix} dz_1 \\ dz_2 \end{pmatrix} \tag{25}$$

The steady state gain matrices are

$$G(0) = \begin{pmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{pmatrix}; \quad G_d(0) = \begin{pmatrix} 11.8 & 17.6 \\ 17.6 & 22.4 \end{pmatrix} \tag{26}$$

The disturbances and outputs have been scaled such that a magnitude of 1 corresponds to a change in F of 30%, a change in z_F of 40%, and a change in x_B and y_D of 0.01 molefraction units. We get at steady-state ($s = 0$):

$$RGA = \begin{pmatrix} 35.1 & -36.1 \\ -36.1 & 35.1 \end{pmatrix}; \quad PRGA = \begin{pmatrix} 35.1 & -27.6 \\ -43.2 & 35.1 \end{pmatrix}$$

$$CLDG = \begin{pmatrix} -72.7 & -2.0 \\ 107.2 & 24.9 \end{pmatrix} \tag{27}$$

In this case RGA and PRGA are quite similar. We note from the CLDG that disturbance 2 (feed composition) has a very small effect on output 1 (top composition) at steady-state. This also holds at higher frequencies as we shall see below.

Pairings. Rule 1 dictates that one should use u_1 to control y_1 and u_2 to control y_2, as indicated by (25). This is in agreement with industrial practice.

Analysis of the model. Fig. 3 shows the open-loop disturbance gains, g_{dik}, as a function of frequency. These gains are quite similar in magnitude and rejecting disturbances z_1 and z_2 seems to be equally difficult. However, this conclusion is incorrect.[2] The reason is that the *direction* of these two disturbances is quite different, that is, disturbance 2 (z_F) is well aligned with G and is easy to reject, while disturbance 1 (F) is not. This is seen from Fig. 4 where the closed-loop disturbance gains, δ_{i2}, for z_2 are seen to be much smaller than δ_{i1} for z_1. Also note that the requirement of rejecting disturbance 1 results in a bandwidth requirement of about 4 rad/min for both loops (considering the frequency where $|\delta_{i1}|$ crosses 1).

Observed control performance. To check the validity of the above results we used the single-loop PI controllers by Skogestad et al. (1990). The loop gains, $|g_{ii}c_i|$, with these controllers are also shown in Fig. 4. The loop gain for loop 1 is smaller than the closed-loop disturbance gain, $|\delta_{11}|$ at higher frequencies (thus, relatively poor performance is expected for output 1), while for loop 2 it is larger than $|\delta_{2k}|$ at all frequencies (thus, better performance is expected for output 2). Closed-loop simulations with these controllers are shown in Fig. 5. The simulations confirm that disturbance 2 is much easier rejected than disturbance 1. They also confirm that with these controller settings, disturbance 1 (in F) has a larger effect on output 1 (y_D) than on output 2 (x_B). In summary, there is an excellent correlation between

[2]This conclusion would be correct if a decoupling controller was used (Section 4), but this kind of controller should not be used for this plant with large RGA-elements.

the analysis based on $|\delta_{ik}|$ in Fig. 4 and the simulations. This is not surprising when one considers Fig. 6 which shows the accuracy of the approximation $[S(s)G_d(s)]_{ik} \approx \frac{\delta_{ik}(s)}{g_{ii}(s)c_i(s)}$ which was used to derive Eq.(14) and which formed the basis for the analysis in Fig. 4. The approximation is very good at low frequencies, but as expected poorer at frequencies around the closed loop bandwidth.

9 Example 4: Control of a Fluid Catalytic Cracker

The Fluid Catalytic Cracking (FCC) process is an important process for upgrading the heavy components of crude oil in refineries. A overview of a typical FCC is shown in Fig. 7. Typically, the control problem is to use as manipulated inputs

1. F_a - flow rate of air to the regenerator
2. F_s - flow rate of regenerated catalyst to the riser reactor

to control the outputs

1. T - some temperature (T_1 or T_{rg})
2. $\Delta T_{rg} = T_{cy} - T_{rg}$ - Temperature rise from regenerator dense bed to cyclones.

Either the riser exit temperature T_1 or the regenerator temperature T_{rg} is used to control the cracking reaction in the riser in order to get the desired product split. Advanced Model Predictive Control (MPC) schemes are installed on many FCC units, but the "basic control" is based on decentralized control.

9.1 Modeling of the FCC process

We shall use the model of Lee and Groves (1985), as we believe this is the model which best describes modern FCC's. The only addition we have made is to include the simple afterburning model in Eq. B.14 in Kurihara (1967), which assumes that the temperature rise from regenerator dense bed to the cyclones is proportional to the concentration of oxygen in the gas leaving the regenerator dense bed. This addition to the model is necessary, as the Lee and Groves model only has the concentration of oxygen in the gas leaving the regenerator dense bed as an output. The model has been implemented in a simulation program and numerical differentiation is used to obtain a linear model.

9.2 Choice of control structure

Two control structures that have been proposed for control of FCC's are (Kurihara, 1967):[3]

1. Conventional control structure: F_s is used to control T_1, and F_a is used to control ΔT_{rg}.

2. Kurihara control structure: F_a is used to control T_{rg} and F_s is used to control ΔT_{rg}.

Note that the term "control structure" as used above includes both the choice of controlled outputs ($T = T_1$ for conventional structure and $T = T_{rg}$ for Kurihara) *and* the choice of pairings.

For the model used in this work, the transfer function matrix from manipulated variables to outputs for the *conventional* control structure has RHP (transmission) zeros at 0.02 rad/min and 0.2 rad/min, whereas the transfer function matrix for the *Kurihara* control structure has a RHP zero at 0.2 rad/min. Since the smallest (in magnitude) RHP-zero limits the achievable closed-loop bandwidth, the Kurihara structure is preferable from this point of view.

We found that the conventional control structure corresponds to pairing on negative steady state relative gains, that is, it does not satisfy *pairing rule 1*. This means that for the conventional control structure, one of the control loops must be unstable by itself for the whole control structure to be stable. In contrast, the Kurihara control structure corresponds to pairing on positive steady state relative gains. Both the RHP transmission zeros and the relative gains indicate that the Kurihara control structure is preferable to the conventional control structure. We will therefore only consider the Kurihara control structure in the following.

9.3 Analysis using frequency dependent RGA and CLDG

In the following consider the Kurihara case and let $y_1 = T_{rg}$, $y_2 = \Delta T_{rg}$, $u_1 = F_a$ and $u_2 = F_s$. The outputs (i) are scaled by allowing the following maximum control errors:

1. Regenerator temperature: 3 K.

2. Temperature rise from regenerator dense bed to the cyclones: 5 K.

The disturbances (k) are scaled by allowing the following maximum variations:

1. Temperature of feed oil: 5 K.

[3] There are also other options. Specifically, a common industrial choice today is to use T_1 and T_{cy} as controlled outputs. With our model this control structure yields no multivariable RHP zeros.

2. Temperature of air: 5 K.
3. Flowrate of feed oil: 4 kg/s. (ca. 10% relative to original value)
4. Rate of formation of coke: 2.5% relative to original value.

Figure 8 shows the magnitude of $\gamma_{ii}(j\omega) = \lambda_{ii}(j\omega)$, $\delta_{ik}(j\omega)$ and $g_{ii}c_i j\omega$ (the loop gain) for both loops. For δ_{ik} the subscript i refers to the output and the subscript k refers to the disturbance as listed above. Note that $\lambda_{11} = \lambda_{22}$ as we have a 2×2 system. It is clear that with the chosen scalings, disturbance 3 (the oil flowrate) is the most difficult to reject, and disturbance 2 (the air temperature) is the easiest to reject in both outputs. The requirement of rejecting disturbance 3 results in a bandwidth requirement of about 0.3 rad/min for loop 1 and 0.04 rad/min for loop 2 (considering the frequency where $|\delta_{i3}|$ crosses 1).

In this example we use two PI controllers with integral times of 17 and 167 minutes, respectively. It was necessary to make the bandwidth in loop 2 smaller than in loop 1, because the RHP transmission zero at $0.2 min^{-1}$ lies mainly in the direction of u_2. Fortunately, as noted above the required bandwidth for disturbance rejection is also smaller in loop 2. Comparing the loop gains $g_{ii}c_i(j\omega)$ (dashed lines) to the $\delta_{ik}(j\omega)$'s (solid lines) indicates that disturbance 3 should have some effect on both y_1 and y_2. Since the bandwidth in loop 1 is higher, we expect y_1 to return more quickly to its original than y_2. The other disturbances appear easy to reject.

9.4 Comparison with simulation results

Figure 9 shows a simulation where disturbance 2 (an increase in the air temperature of 5 K) enters at time 60 minutes and disturbance 3 (an increase in the oil flowrate of 4 kg/s) enters at time 180 minutes. The predictions based on the CLDG are in excellent agreement with the results from the simulation. It is clear that disturbance 3 is much worse than disturbance 2. Also, as predicted by the CLDG's, disturbance 3 is rejected more quickly in y_1 than in y_2.

10 Conclusions

In the paper we have derived performance relationships for decentralized control systems in terms of the individual loops. Importantly, the relationships depend on the model of the process only, that is, are independent of the controller. This means that frequency-dependent plots of γ_{ij} and δ_{ik} may be used to evaluate the achievable closed-loop performance (controllability) under decentralized control. Plants with small values of these measures are preferred. Furthermore, the values of δ_{ik} may tell the engineer which disturbance k will be most difficult to handle using feedback control. This may pinpoint the need for using feedforward control, or for modifying the process. For example, in process control adding a feed buffer tank will dampen the effect of disturbances in feed flowrate and composition. Plots of δ_{ik} may be used to tell if a tank is necessary and what holdup (residence time) would be needed.

The bounds may be used to obtain a first guess of the controller parameters. However, as the derivation of the bounds depends on approximations which are valid at low frequencies only, undesirable effects may occur at frequencies around the closed loop bandwidth. Thus the behavior of the closed-loop system must be checked using other methods, and the controllers possibly redesigned.

Generalizations are also possible. Consider controllers on the form $C(s) = H(s)K(s)$ where $K(s)$ is a *diagonal* matrix (for example, consisting of PID controllers), and $H(s)$ is a multivariable precompensator, for example, a constant matrix (steady-state or high-frequency decoupler), a simple dynamic decoupler or a one-way decoupler. The performance results on decentralized control may be generalized to include decouplers, $H(s)$, by replacing G by GH when evaluating PRGA and CLDG.

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Nomenclature

$e = y - r$ - vector of control errors
$g_{ij} = [G]_{ij}$ - ij'th element of G
$G(s)$ - plant model for effect of u on y
$\tilde{G}(s)$ - matrix consisting of diagonal elements of G
$g_{dik} = [G_d]_{ik}$ - ik'th element of G_d
$G_d(s)$ - disturbance model for effect of z on y
r - vector of reference outputs (setpoints)
$S = (I + GC)^{-1}$ - sensitivity function
u - vector of manipulated inputs

y - vector of outputs
z - vector of disturbances

β_{ik} - ik'th element of RDG matrix (Stanley et al., 1985)
$\delta_{ik}(s) = g_{ii}(s)[G^{-1}(s)G_d(s)]_{ik}$ - Closed Loop Disturbance Gain (CLDG)
$\Delta(s) = \tilde{G}(s)G^{-1}(s)G_d(s)$ - CLDG-matrix
$\gamma_{ij}(s) = g_{ii}(s)[G^{-1}(s)]_{ij}$ - ij'th element in PRGA-matrix
$\Gamma(s) = \tilde{G}(s)G^{-1}(s)$ - PRGA matrix
$\lambda_{ij}(s) = g_{ij}(s)[G^{-1}(s)]_{ji}$ - ij'th element in RGA-matrix
$\Lambda(s)$ - RGA matrix
ω - frequency [rad/min]
ω_B - closed loop bandwidth, frequency at which asymptote of $|S(j\omega)|$ first reaches 1

Subscripts
i - index for outputs or loops
j - index for manipulated inputs or setpoints
k - index for disturbances

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FIGURES

1. **Figure 1**. Block diagram of decentralized control structure.
2. **Figure 2**. Control of distillation column using LV-configuration.
3. **Figure 3**. Open loop disturbance gains, g_{dik}, for distillation column.
4. **Figure 4**. Closed loop disturbance gains, δ_{ik}, and loop gains, $g_{ii}c_i$, for distillation column.
5. **Figure 5**. Disturbance rejection for distillation column using PI control. Responses to a unit step in z_1 at $t = 0$ and unit step in z_2 at $t = 50$min.
6. **Figure 6**. Check of approximation (12) for Example 3. The figure shows the magnitude of $[SG_d]_{ik}/(\frac{\delta_{ik}}{g_{ii}c_i})$
7. **Figure 7**. Overview of typical FCC plant.
8. **Figure 8**. Closed loop disturbance gains, δ_{ik}, relative gain λ_{ii} and loop gain $g_{ii}c_i$ for FCC example.
9. **Figure 9**. Disturbance rejection for FCC example using Kurihara control structure. Responses to a unit step in z_2 at $t = 60$ min and a unit step in z_3 at $t = 180$ min.