CONTROLLABILITY OF HEAT EXCHANGER NETWORKS

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Abstract

Decisions made during *steady-state* process design may put severe limitations on the achievable control performance or *controllability* of chemical plants. In this paper we suggest an appropriate definition of controllability or *dynamic resilience* of heat exchanger networks (HEN) based on the definition of *static* resilience from Saboo and Morari (1984).

We review different controllability *measures* and show how these may be used to *select bypasses and appropriate pairings.* It is then possible to evaluate the controllability or dynamic resilience of HEN. The method is illustrated through some simple examples without stream splitting.

Flow rate dependence of heat transfer coefficients is included in the model. This is found to have a significant effect on control.

The computations and simulations confirm good engineering practice in a qualitative way:
Avoid bypass selections with *two or more downstream paths* to one critical target and designs where both output streams of one exchanger are critical targets. Prefer designs and bypass selections where all critical targets are controlled by either utility streams or bypasses with *a direct effect.*

1 Introduction

During the last decade there have been a large number of papers dealing with *steady-state* optimal design of heat exchanger networks (HEN). However, in practice input temperatures, flow rates, overall heat transfer coefficients etc. vary and we need degrees of freedom for control and on-line optimization. We will refer to the task of keeping the network outlet temperatures at their target values during a *short* time horizon as the controllability or dynamic resiliency problem. When the time horizon is long, the task will be referred to as the operability, flexibility or static resiliency problem.

Quite a few authors have looked at the latter problem. Marselle et al (1982), Saboo and Morari (1984) and Townsend and Morari, (1984) define (static) network resiliency and gave guidelines of how to design a network that has maximum energy recovery under all possible combinations of

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the inlet temperatures and the flow rates of the streams. The solution of this well-defined problem is complicated because both flow rate variations and "pinch jumps" may give non-convexities.

Calandranis and Stephanopoulos (1986) use a similar approach. They introduce disturbance loads and show how these can and should be transferred through the network to a suitable heater or cooler.

Linnhoff and Kotjabasakis (1986) introduce the concept of sensitivity tables and "downstream paths" to describe *structurally* how disturbances propagate in a network, and point out that there is a trade-off between flexibility and installed area. To require maximum energy recovery for all combinations of the disturbances is usually very conservative and leads to expensive designs with too much installed area.

Swaney and Grossmann (1985) and Georgiou and Floudas (1989) present results along the same lines in order to solve the operability problem by mathematical programming.

On the other hand, there is little published on control and controllability of HEN. Control of *single* heat exchangers is addressed in some general textbooks on process control, see for example Shinskey (1979) and Balchen and Mumme (1988). Some recent articles consider nonlinear control (Alsop and Edgar, 1989 and Khambanonda et al, 1991) and adaptive control (i.e. Grimm et al, 1989) of single heat exchangers. Dynamics and control of heat exchangers is also addressed in the area of district heating substations. Jonsson (1990) evaluates dynamical heat exchanger models. Hjorthol (1990) addresses controllability of a single heat exchanger, by selecting valve characteristics which counteract the nonlinearity of the dynamic response of the heat exchanger. Murphy and Bailey (1990) address LQG/LTR control of the feedwater heater train of a nuclear power plant.

Some authors use HEN as examples in more general articles. Nisenfeld (1973) introduce the use of the relative gain array (at steady-state) to evaluate control of a HEN. Holt and Morari (1985) show that controllability of some HEN can be improved by *increasing* the time delay between the exchangers. Daoutidis and Kravaris (1991) include some element of dynamics by considering the order of the response, but this structural approach is generally not too useful in practice since also the numerical values are important.

Reimann (1986) seems to be first author to address specifically controllability of HEN. He suggests some guidelines for how to design a network with good controllability.

Calandranis and Stephanopoulos (1988) extend their previous work on operability (cited above) to include controllability. They discuss the dynamics of HEN briefly, and conclude that bypasses for control purposes should always be placed so that they directly affect the target temperature. However, this may not always be possible or desirable, and in this paper we consider all possible bypass locations.

It is interesting to note that the presence of bypass streams is seldom taken into account during design of optimal HEN.

We use a lumped model where each side of the heat exchanger is modeled as several mixing-tanks in series (normally 6). In this "cell-model" heat is transferred from one mixing-tank on the hot side to the corresponding on the cold as shown in fig. 1. This dynamic model is numerically linearized around the steady-state operating point to get a state-space description of the network as discussed by Wolff et al (1991). The main equations are:

Heat balance around one cell for cold (tube) side:

$$\tau_c \frac{dT_c(i)}{dt} = T_c(i-1) - T_c(i) + \alpha_c \Delta T_{hx}(i) \tag{1}$$

where $\alpha_c = \frac{UA_{cell}}{\rho_c c_{pc} q_c}$, $\tau_c = \frac{V_{cell,c}}{q_c}$, $A_{cell} = \frac{A}{n}$ and $V_{cell,c} = \frac{V}{n}$

It is assumed that the cold fluid is on the *tube* side of the (shell & tube) heat exchanger. The overall heat transfer coefficient U is computed from the simplified expression $1/U = 1/h_h + 1/h_c$,

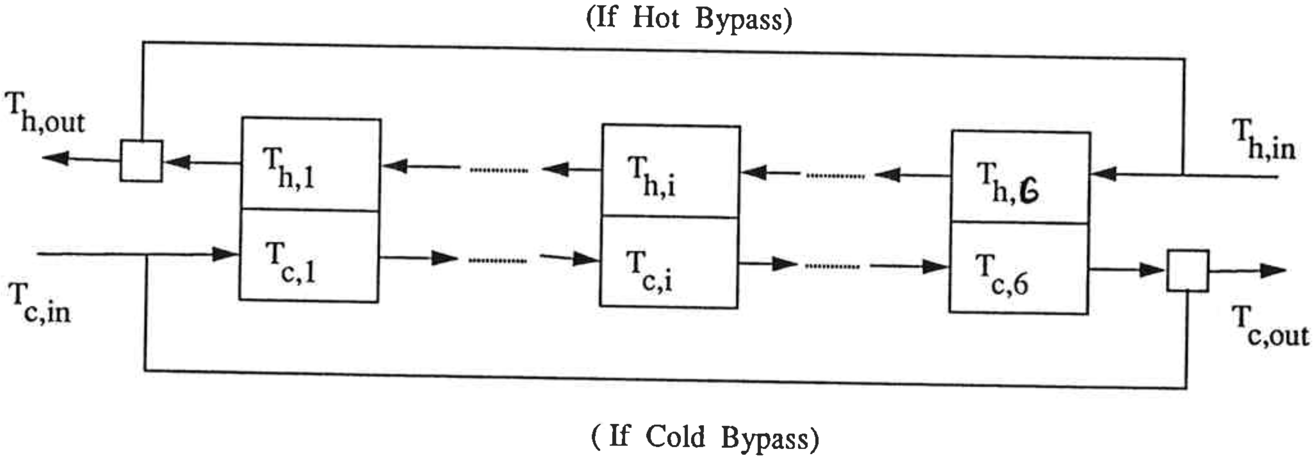

Figure 1: Lumped heat exchanger model used to study controllability of HEN

where h_h and h_c are the film heat transfer coefficents of the hot end the cold fluid, respectively. The film coefficients dependence on the flow rate is taken into accont by: $h_h \sim q^{0.6}$ and $h_c \sim q^{0.8}$.

Heat balance for the hot side assuming 1 tube pass pr shell pass (1,1 heat exchanger) and countercurrent heat exchange:

$$\tau_h \frac{dT_h(i)}{dt} = T_h(i+1) - T_h(i) - \alpha_h \Delta T_{hx}(i)) \tag{2}$$

For cell $1: T_h(i+1) = T_{h,in}$.

Wolff et al use a pure lumped model with the driving force in each cell given by:

$$\Delta T_{hx}(i) = T_h(i) - T_c(i) \tag{3}$$

However, most papers addressing optimization of heat exchanger networks implicitly assume use of ideal countercurrent heat exchangers, and Reimann (1986) and Jonsson (1990) recommend to use the logarithmic mean temperature difference as driving force for the heat transfer in each cell. This represents a hybrid between a lumped and a distributed model. For cell i:

$$\Delta T_{hx}(i) = \frac{(T_h(i+1) - T_c(i)) - (T_h(i) - T_c(i-1))}{ln[(T_h(i+1) - T_c(i))/(T_h(i) - T_c(i-1))]} \tag{4}$$

Although one might argue that one never has ideal countercurrent heat exchange, and that a pure lumped model might be better physically, we have chosen to use this hybrid model to analyze controllability and/or dynamic resiliency of HEN in this paper.

2 Dynamic resilient or controllable HEN

2.1 Degrees of freedom for control and optimization

A single heat exchanger transfers heat from one stream to another, and has only one degree of freedom, which is the heat duty. During *design* of heat exchanger networks, necessary heat exchanger area for each exchanger is calculated from the duties. However, during *operation* one has to vary the heat duty in order to meet the specifications, which typically are to keep certain temperatures constant. In most of the work on static resiliency it has been assumed that this may be done by manipulating the heat exchanger area directly. This may be possible in a few cases, for example for flooded condensers, but in most cases one must install bypass streams and manipulate the bypass fractions in order to change the heat duty.

2.2 Bypass placement

In practice it may be necessary to place bypasses for two reasons:

- Flexibility or static resiliency. Each exchanger must have sufficient area to maintain the specifications for all possible operating points (static disturbances). In a specific operating point this area may be too large and may be effectively reduced by the use of bypass streams.
- Controllability or dynamic resiliency. In a specific operating point one needs degrees of freedom (bypasses) to get satisfactory control behavior in the presence of dynamic disturbances.

The optimal locations for the bypasses depend on whether they are placed to get static or dynamic resiliency. In this paper we mainly address the control aspects and assume that the network is designed and operated at a given operating point. There may then be additional bypasses (or area adjustment) which take care of long-term or static disturbances. These additional bypasses may be used in a hierarchical manner (e.g. using traditional cascade or model predictive control) to reset the control bypass fractions to their nominal values.

Nominal bypass fractions. When evaluating the different examples one ought to decide on nominal bypass fractions. Preferably this should be done from a rigorous optimization where disturbances and performance specifications of the controlled outputs are taken into consideration. For simplicity, we have chosen to compare different alternatives with a constant bypass fraction of 10% and compute the necessary area increase. Note that most of the controllability measures are independent of the input scaling, and thus not critically dependent on the exact values of the bypass fractions.

2.3 Static resiliency

Definitions of static resiliency. Resilient HEN designs are defined as designs with maximum energy recovery (MER) where all combinations of static disturbances can be rejected so that all targets are maintained. (Saboo and Morari, 1984). In order to avoid conservative designs with too much installed area one ought to take the following into consideration:

- Disturbances (supply temperatures and flow rates) are often correlated so that some bad combinations of disturbances represent an operating point that may never occur.
- It is not optimal to require the HEN to have MER for operating points that occur seldom.
- Target temperatures are in practice allowed to vary, they should not be maintained no matter the cost. The optimal target temperatures will be different for different operating points.

Bypass placement to achieve static resilience will not be considered any further in this paper.

2.4 Dynamic resiliency (controllability)

Dynamic disturbance range. To be able to assess controllability generally, the dynamic disturbance range must be known or at least estimated. The dynamic disturbance range is the expected variations at *a given operating point.* This range will be different from, and in most cases smaller than the *steady-state* disturbance range considered in the operability problem.

We have chosen to use temperature disturbances of 10^oC and flow rate disturbances of 20% to evaluate HEN synthesis examples from the literature. This seems reasonable for process control applications if detailed information is unavailable. We assume the disturbances to vary sinusoidally within this range with a frequency ω (which may vary).

Assume there are no other bypasses than those for control. Then a prerequisite for satisfactory controllability of HEN design is that it is static resilient with respect to the dynamic disturbance range.

Performance specification. In order to assess controllability, performance requirements in terms of the allowed dynamic temperature deviations must also be specified. The specification is rough, but nevertheless a *quantitative* expression of how critical the different target temperatures are. The performance weights ought to level off at high frequency. We have chosen to allow target temperature offsets of up to 3^oC at low and medium frequencies to evaluate literature examples. In all practical applications some target temperature are critical while others can be allowed to vary much more. This quantitative discrimination between target temperatures is very important.

Minimum bandwidth that can be achieved in practice. All practical manipulators need some time to respond to a input signal. Typically this limits the the achievable closed-loop bandwidth to about $0.05 - 0.5 rad/sec$ depending on the size of the valves etc.

Definition of dynamic resiliency. Definition of a dynamic resilient or controllable heat exchanger network (HEN):

Given a HEN *design* with nominal operating conditions including bypasses and bypass fractions, defined disturbances (e.g. variations in supply temperatures and flow rates) disturbances and performance requirements (e.g. allowed variations of the controlled target temperatures). *The HEN design is controllable or dynamical resilient if the performance specifications can be met for the specified disturbances by use of feedback control.*

In practice, we must ensure that the closed loop bandwidth requirement for disturbances rejection (and setpoint change) is so low that it can be achieved in practice.

2.5 Number of alternative control configurations

Suppose that N stream temperatures in a HEN are to be controlled. These temperatures are usually the temperatures of the streams leaving the network (*target temperatures*), but some intermediate network temperatures may also be controlled outputs. We further assume that the target temperatures downstream of the $N_{hx,util}$ utility units in the HEN are controlled by manipulating the utility flow.

The remaining problem has $N - N_{hx,util} = N_{byp}$ temperatures to be controlled and we want to use N_{byp} bypasses as manipulators. With N_{hx} *process* heat exchangers the number of different alternatives for selecting bypasses (control configurations) are

$$2^{N_{byp}} \frac{N_{hx}!}{N_{byp}!(N_{hx} - N_{byp})!} \tag{5}$$

In the equation above we have taken into account that one in practice will not place bypasses on both sides of an exchanger. We have not considered "total" bypasses of several heat exchangers or split fractions in HEN where streams are split. In addition, if we use decentralized control, there are N_{byp} different possible pairings for each of the configurations.

The rapid growth of this combinatorial problem with number of process heat exchangers N_{hx} and target temperatures N_{byp} is evident, and this makes it difficult to apply techniques which involve searching over all alternatives. Therefore it is desirable to develop simplified methods and to obtain insights in order to be able to formulate simpler "rules".

3 Measures for evaluating controllability

Controllability measures are used to evaluate the inherent control properties of the process without having to do a controller design. A disadvantage with most measures for analyzing controllability is that they have to be recomputed for each control configuration.

We will use the measures listed below to evaluate controllability or dynamic resilience of HEN. Further justification for their use is given by Hovd and Skogestad (1991).

Scaling. We always assume that the process transfer function matrix $G(s)$ and the disturbance transfer function matrix G_d is scaled so that allowed magnitude of the manipulators ($u's$), disturbances ($d's$) and controlled outputs ($y's$) should vary between 0 and 1 at all frequencies.

Input constraints. When evaluating if a set of N_{byp} bypasses may be an appropriate configuration to control the N_{byp} target temperatures one may first examine $G(s)$. If one row i of $G(s)$ is zero, the set must be discarded because there is no downstream path from any of the manipulators to output i. A rough indicator for a *good* configuration is that, for each output y_i, there is one $|g_{ij}| > 1, \omega < \omega_B$ (with the variables scaled as indicated above). Otherwise we will propably get problems with input constraints when we want to make a change in y_i of magnitude 1. In addition, to get a simple controller design, it is desirable that the other elements $|g_{ij}|$ in row i of $G(s)$ is approximately zero. This does not take into account the magnitude of the disturbances or multivariable effects, and a better indication is easily derived from the requirement of *perfect disturbance rejection.*

$$y(s) = G(s)u(s) + G_d(s)d(s) \tag{6}$$

For square systems:

$$y(j\omega) = 0 \Rightarrow u(j\omega) = G^{-1}(j\omega)G_d(j\omega) \tag{7}$$

One should avoid configurations with elements in $|G^{-1}G_d|$ larger than 1. Specifically if $||G^{-1}G_d||_\infty$ (the largest row sum) in the frequency range important for control, then the nominal bypass fractions (overdesign) must be increased. If that is impossible due to driving force constraints on the exchangers the set of bypasses should be discarded.

Bandwidth limitations, RHP-zeros. A right half plan (RHP) transmission zero of the plant transfer function limits the achievable bandwidth regardless of the controller used (see Morari and Zafirou, 1989). When decentralized control is used, one should also avoid RHP zeros in the elements in order to maintain stability of the individual loops. Bypass selections that give no RHP zeros are preferred.

Interactions, use of RGA. The relative gain array (RGA) is used as a measure of interactions in a general sense, and bypasses that minimize interactions are preferred. In particular, one should avoid cases with large RGA-values at frequencies close to the closed-loop bandwidth because such plants are fundamentally difficult to control (irrespective of the controller)

Pairing, use of RGA. We want to control the HEN with decentralized control loops and use the relative gain array (RGA) as function of frequency to the decide the best pairing, i.e. what bypasses should be used to control what target temperatures. We like to pair so that the RGA-value is close to one around the the expected bandwidth of the system. To ensure stability of individual loops and remaining subsystem when one loop fails, pairing on negative steady-state values should be avoided.

Disturbance rejection, use of G_d and CLDG. The frequency-dependent open-loop disturbance gain matrix (G_d) include both the information in the sensitivity tables of Linnhoff and Kotjabasakis (1986) at steady-state or low frequency and the structural relative order tables of Daoutidis and Kravaris (1991) at high frequency. It can be used to cut downstream paths between large disturbances and critical targets and ensure low relative order between manipulator and target temperature.

For decentralized control some other measures are even more useful to evaluate disturbance rejection. We assume from now on that the manipulators are numbered after the pairing is decided so that u_1 is used to control y_1 etc. Then the controller matrix C is diagonal with elements c_i.

The offset of the targets of the closed loop system is:

$$e(s) = y(s) - r(s) = -S(s)r(s) + S(s)G_d(s)d(s) \tag{8}$$

where S(s) is the sensitivity function $(I + GC)^{-1}$, $r(s)$ is the vector of setpoints and $d(s)$ the disturbances.

At low frequency the offsets may be approximated by

$$y(s) - r(s) \approx -S_{diag}(s)G_{diag}G^{-1}r(s) + S_{diag}(s)G_{diag}G^{-1}G_d(s)d(s) \tag{9}$$

where G_{diag} consists of the diagonal elements (g_{ii}) of G and S_{diag} is defined as $(I+G_{diag}C)^{-1}$, i.e. has elements $1/(1+g_{ii}c_i)$ (Hovd and Skogestad, 1991). We define the closed-loop disturbance gain (CLDG) as $\Delta = G_{diag}G^{-1}G_d$. The elements are denoted δ_{ik} and represents the closed-loop disturbance gain from disturbance k to output i when we do the design for each individual loop.

Since G_d and G are scaled the magnitude $|\delta_{ik}|$ at a given frequency directly gives the necessary loop gain $|g_{ii}c_i|$ at this frequency needed to reject this disturbance. The frequency where $|\delta_{ik}(j\omega)|$ crosses 1 gives the minimum bandwidth requirement for this disturbance. It should be less than the bandwidth that can be achieved in practice, which will be limited by time delays, RHP zeros etc.

Set-point tracking, use of PRGA. In a similar manner the performance relative gain array (PRGA) defined as $\Gamma = G_{diag}G^{-1}$ can be used to evaluate set-point tracking of the system. However, in process control disturbance rejection is often the major concern, and since PRGA (Γ) will generally be small when CLDG ($\Delta = \Gamma G_d$) is small, evaluation of set-point tracking can normally be omitted.

4 Examples

Notation: The input configuration with a *hot* stream bypass on exchanger no 1 and a *cold* stream bypass on exchanger 2 is denoted $1H2C$. With decentralized control case $1H2C$ is the control configuration using bypass $1H$ to control y_1 and bypass $2C$ to control y_2. All other cases are denoted accordingly.

Data: The nominal film transfer coefficients are $200W/m^2K$ for all streams of the example from Townsend and Morari, and $100W/m^2K$ for the example from Gundersen et al. Duties of the heat exchangers are given in the figures and are given in kW.

Scalings: For inputs (bypasses): Unit change corresponds to ± 10% bypass fraction. For outputs (temperatures): Unit change corresponds to ± 3^oC. For disturbances in supply temperatures of the streams: Unit change corresponds to ± 10^oC. For disturbances in flow rates of the streams: Unit change corresponds to ± 20%.

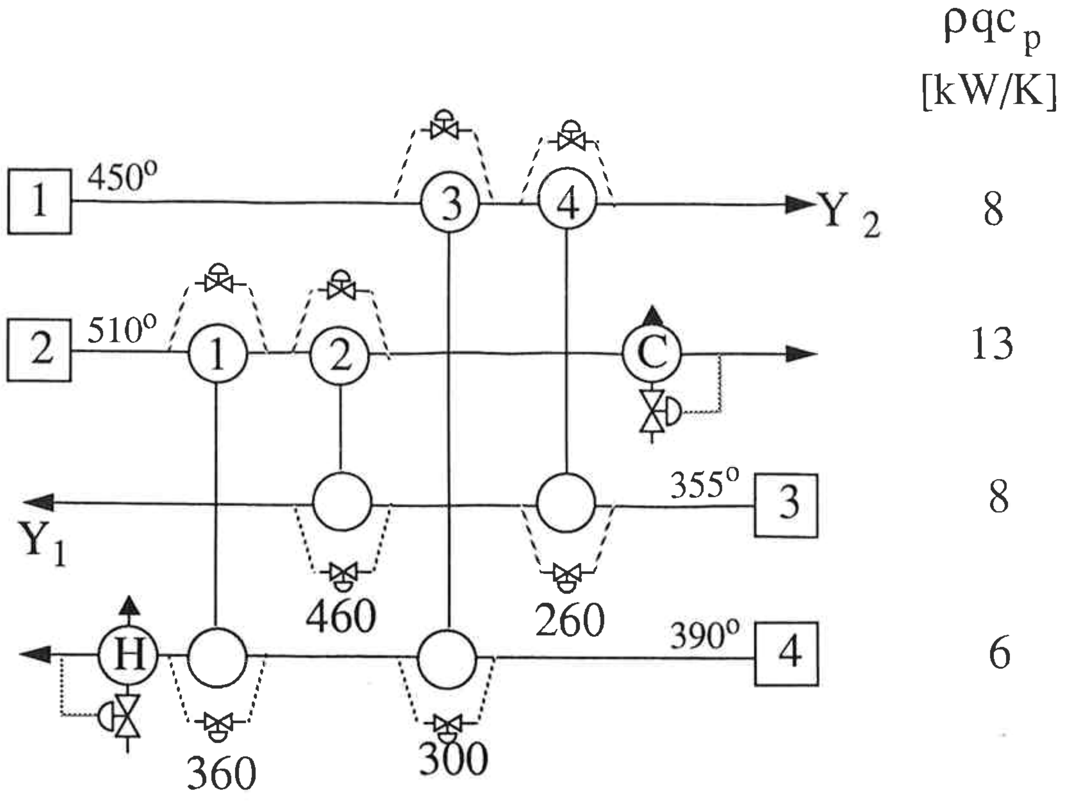

Figure 2: Network from Townsend and Morari (1984) used to study effect of alternative bypass location and pairings

4.1 Network from Townsend & Morari

Consider the network in Fig. 2 from Townsend and Morari (1984) where temperatures $y_1 = T_{3t}$ and $y_2 = T_{1t}$ are to be controlled by introducing two bypasses. As there are 4 process-process heat exchangers (N_{hx}) and 2 bypasses (N_{byp}) there are as many as 24 different pairs of manipulators for this small example. Fortunately it is easy to limit the search. The gain from some of the possible bypass locations to all outputs will be very small and these choices can be omitted right away. These bypasses are often, but not always, those furthest away from the outputs. In this example we will consider all possible bypass locations for illustration.

Analysis of steady-state matrices and input constraints. The steady-state gain from the 8 alternative manipulated variables to the outputs are as follows:

$$G^{all}(0) = \begin{array}{c|cccccccc} & 1H & 1C & 2H & 2C & 3H & 3C & 4H & 4C \\ y_1 & 0.11 & 0.41 & -0.59 & -1.05 & -0.04 & -0.08 & -0.17 & -0.20 \\ y_2 & 0 & 0 & 0 & 0 & 0.16 & 0.34 & 0.45 & 0.50 \end{array}$$

This linear gain matrix was obtained by linearizing around a steady-state with bypasses around all exchangers, but with a nominal bypass fraction of zero. Note the following:

1) The steady-state gains from exchanger 4 to the outputs are not equal even though the heat capacity flow rates are equal for this heat exchanger. This is due to the different flow dependence of the hot and the cold film transfer coefficients. The effect is surprisingly large, the differences are approximately 10 and 15 %.

2) The gain from exchangers 1 and 2 to output y_2 is zero (also dynamically). This eliminates the 4 cases using both these exchangers (e.g. cases $1H2H, 1H2C, 1C2H$ and $1C2C$).

3) The gain from exchanger 3 to output y_1 is negative. Actually there are 2 different downstream paths from exchanger 3 to output y_1, one path through heat exchanger 4 with positive gain, and one through heat exchangers 1 and 2 with negative gain which is dominating. The fact that we have two opposing effects implies that we will get *an inverse response* from bypass on exchanger 3 ($3H$ or $3C$) to y_1 in cases where the path with dominating gain is significantly slower. The

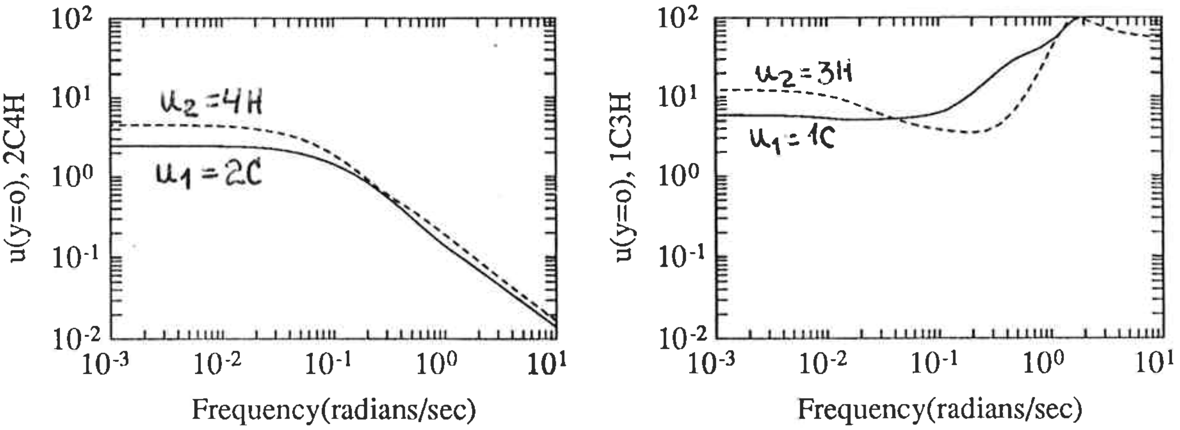

Figure 3: Required manipulations for perfect control $u_{y=0} = G_{41}^{-1} G_d$ for cases $2C4H$ and $1C3H$. Example from Townsend & Morari (1984)

apparent dead times of the different paths will mainly depend on the residence times in the heat exchangers and the connecting pipes.
Opposing effects will always occur when there are downstream paths from both the hot and the cold side of a manipulated exchanger to an output. Increasing the bypass (and reducing the duty) of the manipulated exchanger will introduce a positive temperature "disturbance" on the hot side and a negative on the cold side. These disturbances are dampened downstream through and "across" heat exchangers but they can never change sign. (Easy to prove for temperature-independent physical properties, but this is true in general).
4) Most gains are small compared to one. This signal potential problems with constraints. However, to understand this better we would also have to consider the disturbances.

The disturbance gain matrix is given by

$$G_d(0) = \begin{array}{ccccccccc} & T_{1s} & T_{2s} & T_{3s} & T_{4s} & q_1 & q_2 & q_3 & q_4 \\ y_1 & 0.94 & 1.22 & 0.57 & 0.60 & 0.98 & 2.08 & -2.20 & -1.46 \\ y_2 & 0.54 & 0 & 1.88 & 0.91 & 2.03 & 0 & -0.99 & -0.68 \end{array}$$

For example for output y_2, to reject a unit disturbance in q_1 (corresponds to a 20% change in q_1) by use of bypass $4H$ we need a bypass change of $2.03/0.45 = 4.5$. A unit bypass change corresponds to 10% so this corresponds to a bypass fraction of 45%. Note that this is a linear analysis. Necessary bypass to reject a 20% decrease in q_1 is only 32%, so the linear analysis gives *conservative* estimates of the necessary bypasses.

The magnitude of the inputs (bypasses) needed for perfect control is given by elements of $G^{-1}G_d$. Here G corresponds to 2 selected columns of G^{all}. Unfortunately $G^{-1}G_d$ has to be recomputed for each of the $24 - 4 = 20$ possible configurations. For all 20 configurations required manipulations to reject the worst combination of disturbances are given in fig. 3. The plot indicate that even for the best case $2C4H$ we need a bypass fraction of about 40% bypass. Note that using flow dependent film transfer coefficients favors control as the process transfer function gains increase whereas the disturbance transfer function gains decrease. The combined reduction on the required input for perfect control ($G^{-1}G_d$) is large (typically about 40%).

Bandwidth limitations: RHP-transmission zeros. The 2 opposing effects from exchanger 3

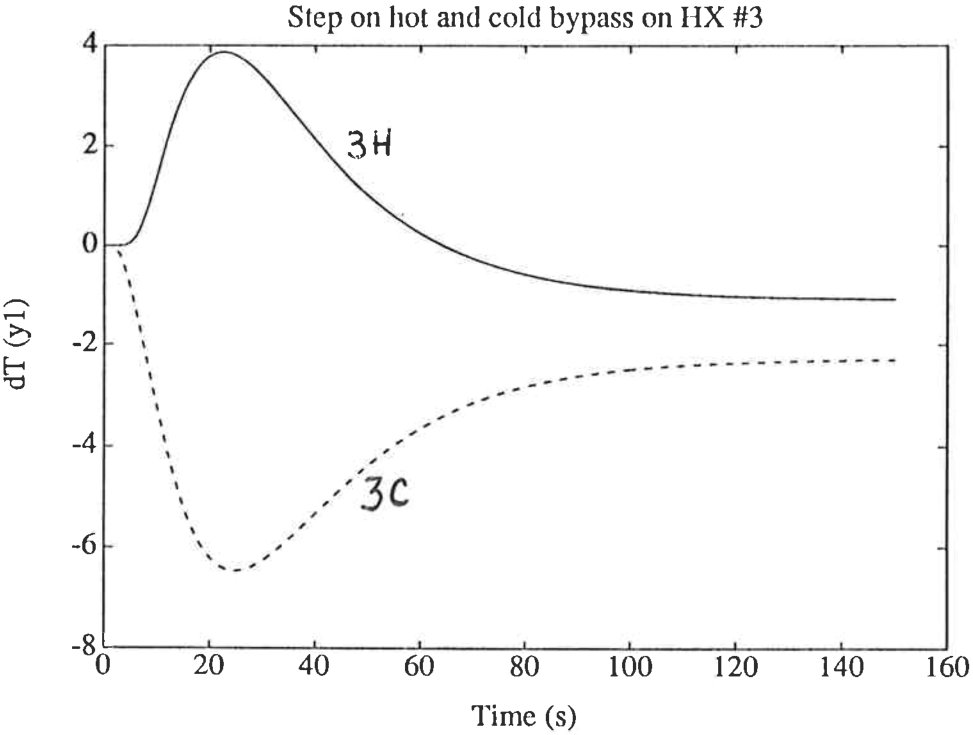

Figure 4: Time simulation (linear model) of step bypass on exchanger 3 for cases $3H4H$ and $3C4C$ showing inverse response and large overshoot.

will limit the achievable bandwidth and the controllability of the HEN for decentralized control. This is confirmed through from computation of the zeros in the elements g_{ii} of G.

All control configurations using bypasses on exchangers 3 and 4 as manipulated variables result in a system with a multivariable RHP-zero. When the cold stream of exchanger 3 is bypassed the RHP-zero is significant (0.004 rad/sec). Thus, cases $3C4H$ and $3C4C$ ought to be disregarded.

Actually, with our dynamic model case $3H4H$ will give an inverse response. A time simulation is shown in fig. 4.

Interaction, pairing: RGA. The 1,1-element of the RGA (λ_{11}) is 1.0 at all frequencies for all cases with the first bypass on either exchanger 1 or 2 and the other bypass on exchanger 3 or 4. This can be seen directly from the network structure, since there is no downstream path from exchangers 1 and 2 to output y_2. As exchangers 1 and 2 can only be used to control output y_1, the best pairing for decentralized control is obvious for all these cases.

Consider in the following the 4 cases with bypasses on exchangers 3 and 4 as the 2 manipulated variables. In this case it is not easy to decide the appropriate pairing. Bode-plot of λ for different choices of manipulated variables is shown in fig.5. Pairing exchanger 4 to output y_2 give λ equal to 1.0 at high frequency in all cases, but *negative* λ *at steady-state* as could be expected from the RHP-transmission zeros. Only case $3H4C$ with reversed pairing (i.e. $4C3H$) seems to be acceptable for decentralized control. From this example it is clear that even in simple cases it may not be obvious how to select bypasses and appropriate pairings, and the conclusion will depend on the operating point. For illustration, in a previous paper (Wolff et al, 1991), we considered the same network structure, but used (due to an error) larger heat capacity flow rates. In that case the RGA values were not negative at steady state, and we concluded that pairing u_1 to y_1 was acceptable in all cases.

Disturbance rejection with decentralized control CLDG. To discriminate between the remaining 16 cases where output y_1 is controlled by a bypass on exchanger 1 or 2 and output y_2 is controlled

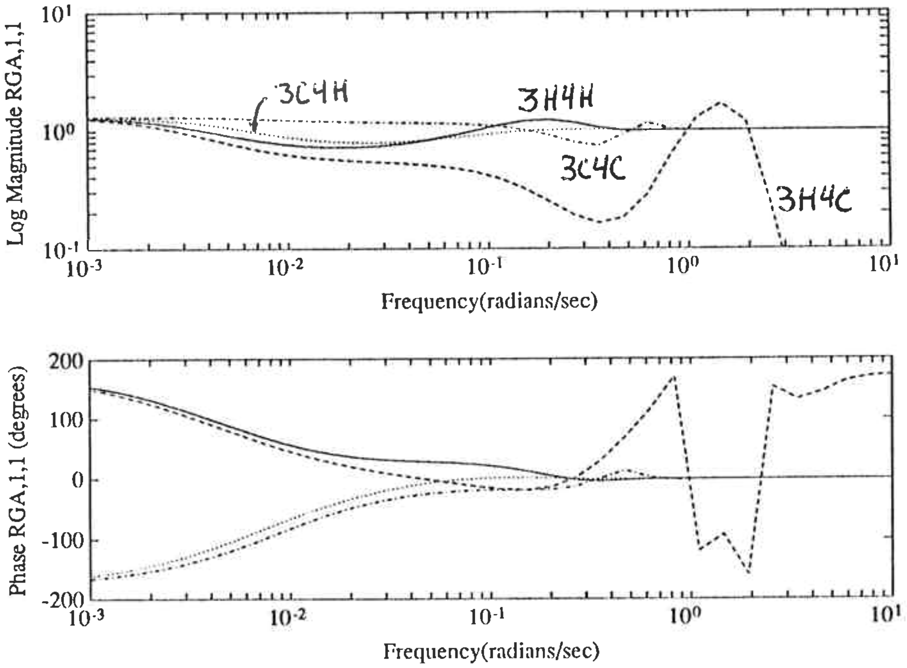

Figure 5: RGA,1,1 for cases $3H4H, 3H4C, 3C4H$ and $3C4C$. Example from Townsend & Morari (1984)

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by a bypass on exchanger 3 or 4, the closed loop disturbance gain may be helpful. The worst disturbances to reject was found to be temperature disturbance on stream 3 on output y_2 and flow rate disturbance on stream 3 on output y_1. (The reason for this is that the first exchanger on stream 3 is the last exchanger on stream 1, which is to be controlled. y_1 is the cold outlet of exchanger 2 and the opposite side of this exchanger is the second exchanger on stream 2 so that a temperature disturbance on stream 2 is dampened before it affects exchanger 2 and the output. Thus, the worst disturbance for output y_1 is the flow rate disturbance of stream 3. In general it is impossible or at least very difficult to anticipate which disturbance that are most difficult to reject without calculating the CLDG).

The most important information from the CLDG-plot is the frequency were the curves crosses 1.0. For all cases and both loops the necessary bandwidth is $\approx 0.2 rad/sec$, see fig. 6. For case $2C4H$ with direct effects from both inputs to the corresponding outputs, the speed of the response will be about 0.05 to 0.5 rad/sec, i.e. about the required. For all other cases the speed of reponse will be slower. For example, for case $2H4H$ there is no direct effect from $2H$ to output y_1 and the response will be slowed down by exchanger 2. The effect will however not be very large because bypass $1H$ affect the hot end of exchanger 2 fast.

4.2 Part of network from Townsend & Morari

Suppose that the cold outlet of exchanger 4 of the network considered above was to be controlled, for example to avoid fouling or corrosion. The 2 outputs, y_1' and y_2 from exchanger 4 must be controlled by adjusting the duty of exchangers 3 and 4. This reduced problem has 4 possible control configurations, and each configuration has 2 alternative pairings, see fig. 7

Analysis of steady-state matrices and input constraints. The steady-state gain from the 4 alternative manipulated variables to the outputs are:

$$G^{all}(0) = \begin{array}{c|cccc} & 3H & 3C & 4H & 4C \\ y_1' & 0.21 & 0.44 & -0.45 & -0.50 \\ y_2 & 0.16 & 0.34 & 0.45 & 0.50 \end{array}$$

Note that the gains from bypass $3C$ are twice as large as those from bypass $3H$.

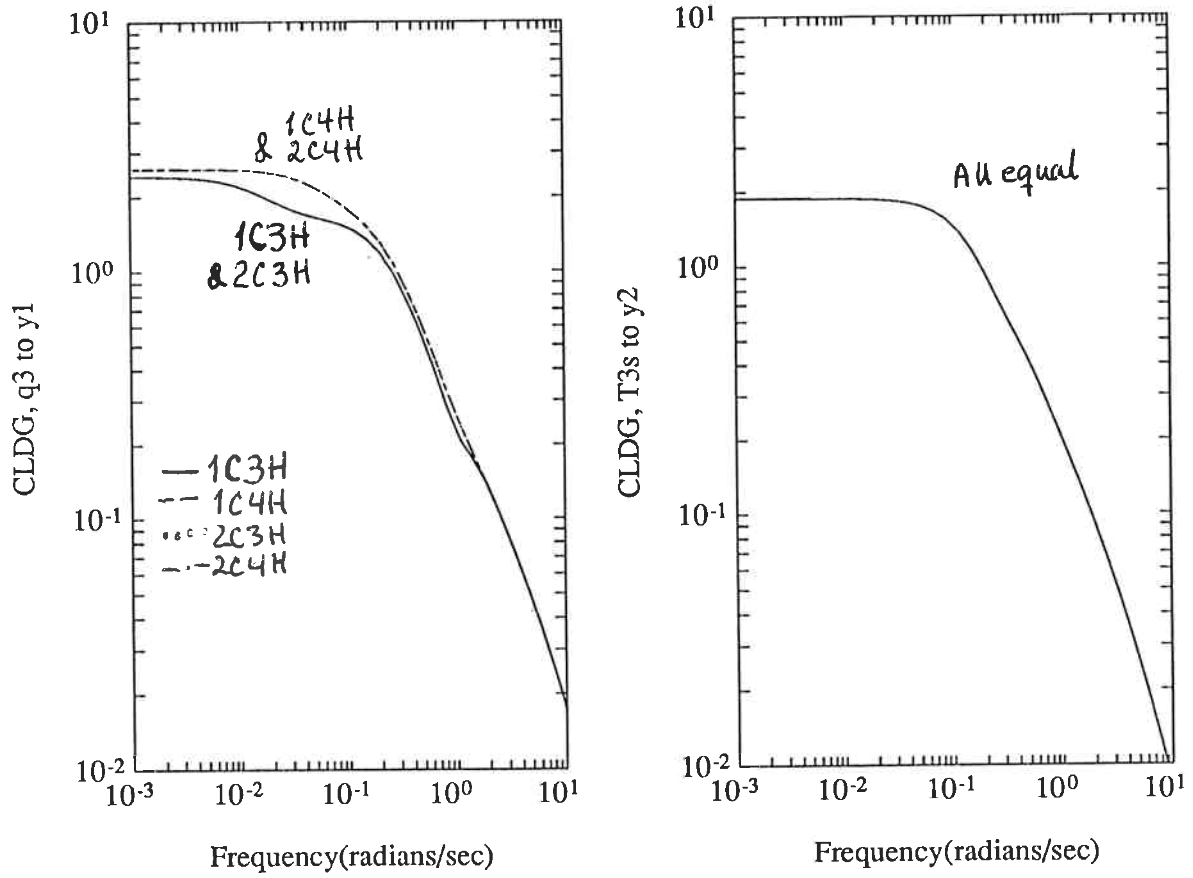

Figure 6: Selected (worst case) elements of CLDG $= G_{diag}G^{-1}G_d)$ for different control configurations. Example from Townsend & Morari (1984).

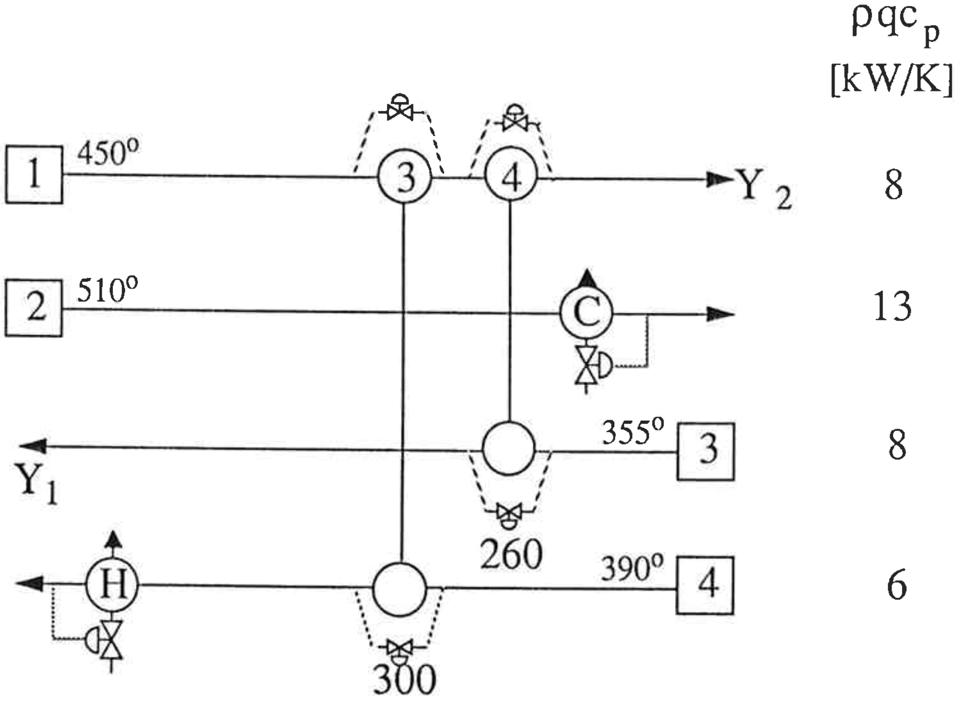

Figure 7: Part of network from Townsend and Morari (1984) used to study effect of alternative bypass location and pairings

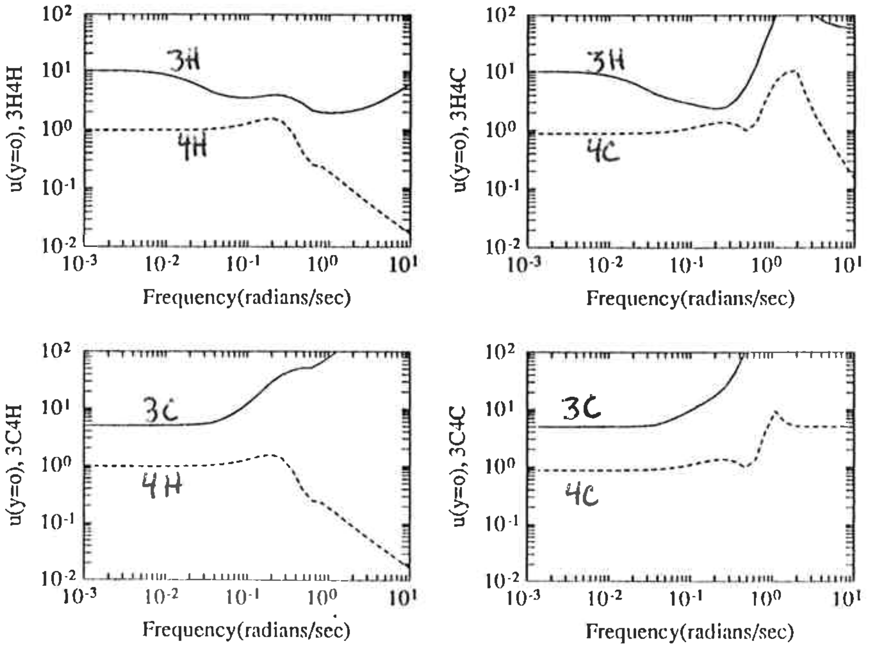

Figure 8: Required manipulations for perfect control $u_{y=0}$ for the 4 alternative cases ($3H4H, 3H4C, 3C4H$ and $3C4C$). Part of example from Townsend & Morari (1984).

The steady-state disturbance transfer function gains are

$$G_d(0) = \begin{array}{c|cccccccc} & T_{1s} & T_{2s} & T_{3s} & T_{4s} & q_1 & q_2 & q_3 & q_4 \\ y_1' & 0.71 & 0 & 1.45 & 1.18 & 1.88 & 0 & -1.18 & -0.88 \\ y_2 & 0.54 & 0 & 1.88 & 0.91 & 2.03 & 0 & -0.99 & -0.68 \end{array}$$

Note that 1) Disturbance of stream 3 is the most difficult temperature disturbance to reject because stream 3 immediately affect the hot outlet of exchanger 4. 2) Flow disturbance of stream 1 is more difficult to reject than flow disturbance of stream 3 because stream 1 goes through exchanger 3 before reaching exchanger 4, altering the inlet temperature of exchanger 4, too. It is important to beware of the fundamental difference between temperature and flow rate disturbances, temperature disturbances are dampened through exchangers in series whereas the temperature effect of flow rate disturbances are enforced.

The magnitude of the inputs (bypasses) needed for perfect control is given by row sums of $G^{-1}G_d$. The 4 alternative cases ($3H4H, 3H4C, 3C4H$ and $3C4C$), are plotted in fig. 8.

The curves indicate that case $3C4C$ is best at steady-state, whereas case $3C4H$ seems to be best at higher frequencies. This is interesting since case input $3C$ is further away from the outputs than $3H$.

Bandwidth limitations: RHP zeros. Both control configurations using bypass $4C$ result in a system with a multivariable RHP-zero at $0.14 rad/sec$. This is a limitation of the control performance.

Interaction, pairing: RGA. The 1,1-element of the RGA (λ_{11}) for control configurations $3H4H, 3H4C, 3C4H$ and $3C4C$ are shown in fig. 9. At steady-state λ_{11} is the same for all cases. This is because we control both streams out of a heat exchanger which has only one degree of freedom. $\lambda_{11}(0)$ is 0.56 illustrating the interaction between the loops. λ_{11} for the 2 cases using bypass $4H$ (solid line) is the same at all frequencies. The reason for this is that both bypass $3H$ and $3C$ must affect output y_1' through the hot side of exchanger 4, which is where the other manipulator is placed. The pairing for these cases is straightforward because λ_{11} is between 0.5 and 2.0 at all frequencies. When $4C$ is used λ_{11} changes sign at $\omega \approx 0.5$ requiring a lower bandwidth. In summary, for all cases we should use exchanger 3 to control y_1'. The reason for this is the

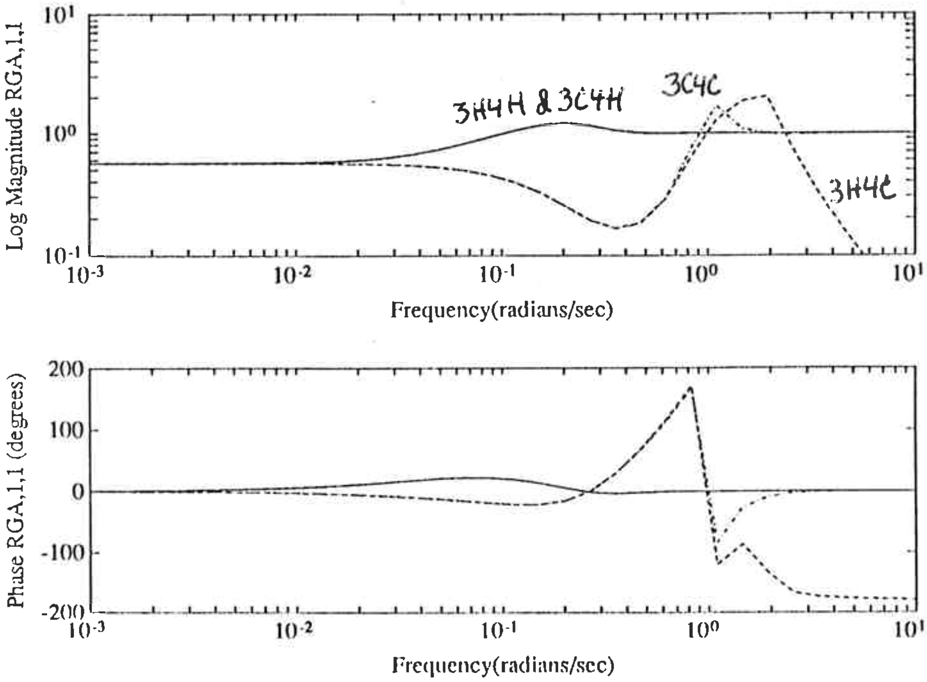

Figure 9: RGA,1,1 for cases $3H4H, 3H4C, 3C4H$ and $3C4C$. Part of example from Townsend & Morari (1984).

"deadtime" through exchanger 4 which occur when exchanger 3 is used to control y_2.

Disturbance rejection with decentralized control CLDG. The closed loop disturbance gains for the disturbance that is most difficult to reject is plotted in fig. 10. For all cases where bypass on exchanger 3 is paired to output y_1' the most difficult disturbances to reject are flow rate disturbance of stream 3 on output y_1' and temperature disturbance of stream 3 on output y_2. The magnitude of these elements of the CLDG (i.e. $|\delta_{17}|$ and $|\delta_{23}|$) are shown in fig. 10a and b, respectively. Conclusion: Output y_1' most difficult, bypass $4H$ best.

However, with reversed pairing the most difficult disturbances to reject are temperature disturbance of stream 1 on output y_1' and flow rate disturbance of stream 1 on output y_2. The magnitude of these elements of the CLDG (i.e. $|\delta_{13}|$ and $|\delta_{25}|$) are shown in fig. 10c and d, respectively.

A possible explanation may be that it is *more difficult* to reject temperature disturbance of stream 1 on y_1' with reversed pairing because y_1' is then controlled by adjusting the temperature of stream 1 at the inlet of exchanger 4, while it is *easier* to reject temperature disturbance of stream 3 on y_2 because y_2 is controlled by exchanger 4 which is placed directly upstream of the target temperature.

For all cases and both pairings flow rate disturbances are worse than temperature disturbance. This fact depend largely on the the relative magnitude (scalings) of disturbances, but to only to a smaller degree on temperature driving forces of the involved exchangers. From the interaction analysis it was concluded that pairing u_1 to y_1' is preferred, and this conclusion is backed by the CLDG. Thus, it may be concluded that designs where both output temperatures of one heat exchanger are to controlled are most sensitive to large flow disturbances in the flow passing through both heat exchangers that are to be adjusted.

To sum up this simple 2 exchanger example:

- Pairing exchanger 3 to output y_1 is preferred (from frequency-dependent RGA, at steady-state they are equal)

- Bypass $4H$ is better than $4C$ (from RHP-zero and frequency-dependent RGA and CLDG, at steady-state they are equal and similar, respectively).

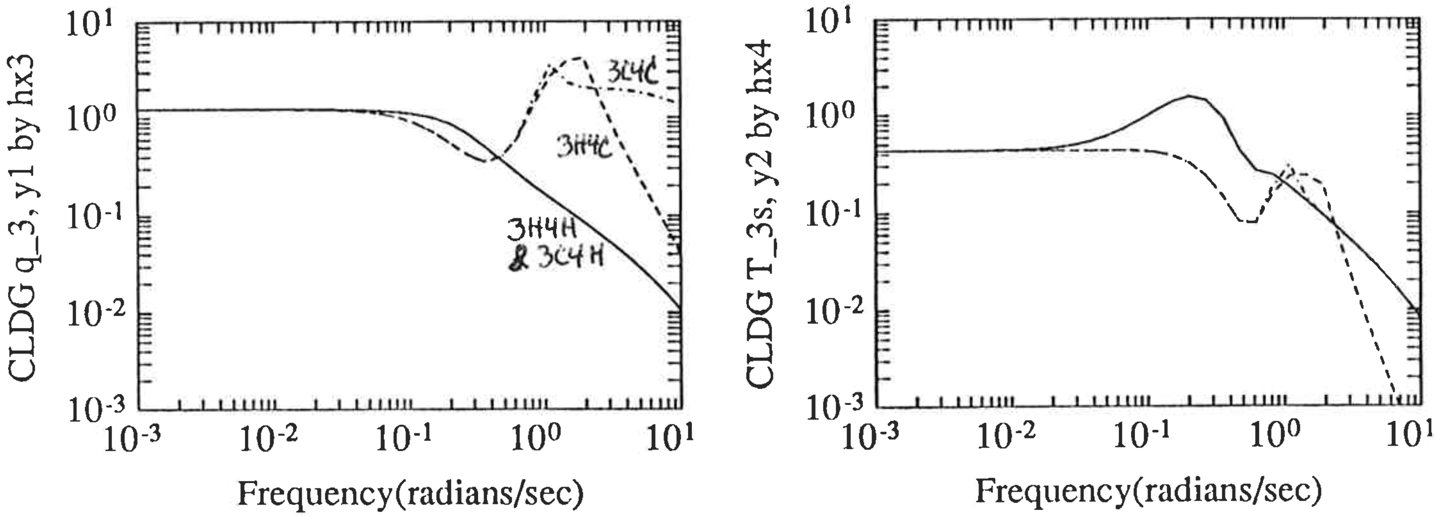

Figure 10: Selected (worst case) elements of the CLDG $= G_{diag}G^{-1}G_d$ for cases $3H4H, 3H4C, 3C4H$ and $3C4C$. Part of example from Townsend & Morari (1984).

- Bypass $3C$ is better than $3H$ at steady-state whereas $3H$ is better than $3C$ dynamically (from input constraints, $G^{-1}G_d$, RGA and CLDG are similar).

Thus, $3H4H$ *or* $3C4H$ is the best control configuration. The trade-off between $3H4H$ and $3C4H$ depend on the controller to be used.

4.3 Network from Gundersen et al

Next we consider the network design in fig. 11. from Gundersen *et al.* (1991). This network design is presented as the global optimal solution of this classic 4 stream problem. Similar to the previous example, two target temperatures is to be controlled by bypasses, but these two temperatures are both outlet temperatures of exchanger no. 1. Because there are 3 process heat exchangers and 2 bypasses, there are 12 different possibilities even for this simple 4 stream problem with a minimum number of units! Note that there is only one downstream path from exchangers 2 and 3 to the controlled outputs. This means that all configurations using bypasses on exchangers 2 and 3 (i.e. cases $2H3H, 2H3C, 2C3H$ and $2C3C$) may be disregarded, because the outputs then cannot be controlled separately.

Analysis of steady-state matrices and input constraints. The steady-state process transfer function matrix is shown below:

$$G^{all}(0) = \begin{array}{c|cccccc} & 1H & 1C & 2H & 2C & 3H & 3C \\ y_1 & 1.84 & 0.91 & 0.03 & 0.12 & -0.12 & -0.20 \\ y_2 & -1.23 & -0.61 & 0.02 & 0.07 & -0.07 & -0.11 \end{array}$$

All gains from bypasses on exchangers 2 and 3 to the outputs are small. The reason for this is that exchangers 2 and 3 are above the pinch, and exchanger 1 below. It will probably be impossible to control this network, but this will depend on the magnitude of the disturbances, too. The steady-state disturbance gain $G_d(0)$ is given by

$$G_d(0) = \begin{array}{c|cccccccc} & T_{1s} & T_{2s} & T_{3s} & T_{4s} & q_1 & q2 & q3 & q4 \\ y_1 & 0.61 & 0.65 & 0.23 & 1.85 & 2.31 & 0.50 & -0.24 & -2.24 \\ y_2 & 1.82 & 0.36 & 0.13 & 1.03 & 2.46 & 0.28 & -0.13 & -3.02 \end{array}$$

Disturbances in the controlled streams (columns 1,4,5 and 8) have a much greater impact on the outputs (rows 1 and 2) than the other disturbances. These disturbance gains are in the range

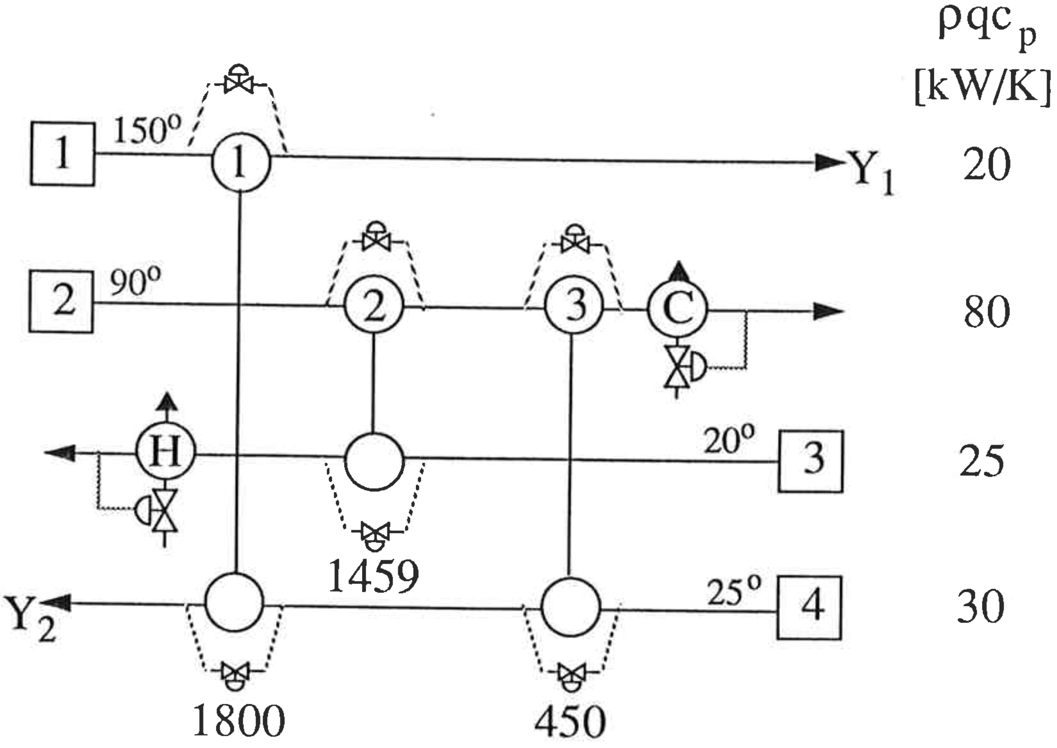

Figure 11: Network from Gundersen *et al* (1991) used to study disturbance rejection

0.6 – 3, which is one to 2 orders of magnitude larger than the effect of manipulating exchangers 2 and 3, so rejection of disturbances is impossible. Computation of the required value of the manipulated variables necessary for perfect control illustrate the poor controllability for this HEN design. Fig. 12 show that disturbance rejection is completely impossible for this system.

Gundersen et al points out that this 4 stream problem have several alternative design with only a marginally higher total annualized cost. Some of these have much better controllability and should be preferred.

It is possible to put one bypass on the hot side of exchanger 1 and one "total" bypass around exchangers 1 and 3. This bypass placement may improve the controllability, but cannot be recommended from steady-state energy considerations.

5 Summary

5.1 Proposed stepwise procedure

. We have looked at the problem of selecting bypasses and appropriate pairings for decentralized control and evaluation of controllability or dynamic resilience of HEN. We suggest the following stepwise procedure (as all matrices are assumed to be scaled, "large" means greater than unity):

1. G: Discard bypass set if one row i of G is zero. (No downstream from any input to output y_i)
2. $G^{-1}G_d$:

   Steady-state: Discard set if large

   Dynamically: Discard set if large within expected bandwidth
3. RHP-zeros: Discard set if significant RHP-zero exist

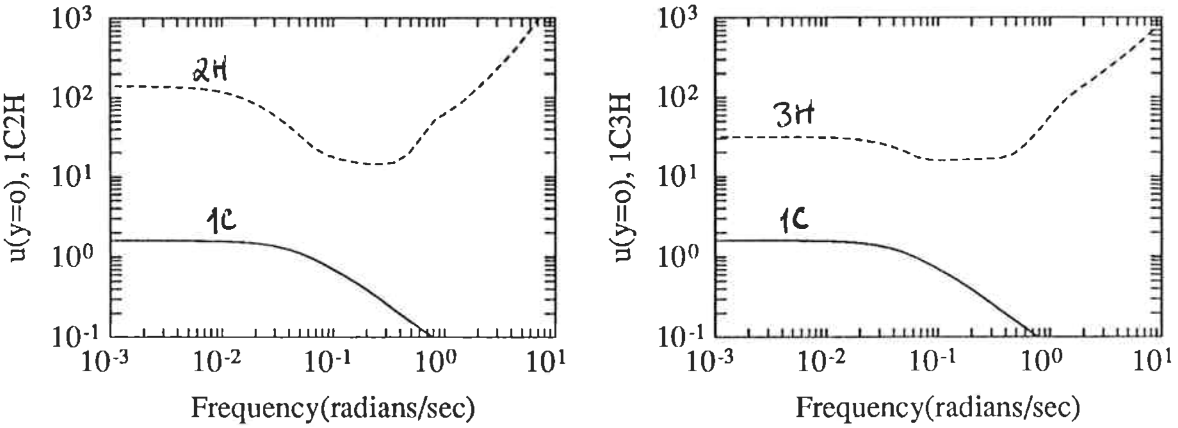

Figure 12: Required manipulation for perfect control $u_{y=0} = G^{-1}G_d$ for cases $1C2H$ and $1C3H$. Example from Gundersen et al. (1991)

4. Interactions RGA:

   Steady-state: Discard set if negative

   Dynamically: Discard set if large within expected bandwidth

5. Decentralized control CLDG: Used to check results above and design controllers (gives expected bandwidth)

6. Decoupler: Consider decoupler if RGA $\neq I$ or PRGA is large

If the HEN design passes tests 1, 2 and 3 for some bypass set, the design is feasible.

5.2 Design for controllability.

So far our work with different HEN example problems indicate that the following controllability "rules" may be recommended:

Selection between different HEN designs:

- Avoid designs with 2 critical targets as the output temperatures of one exchanger
- Avoid, by all means, designs with more critical targets than exchangers in one pinch region.

Selection of manipulated inputs:

- Prefer exchangers with large effect on exactly one output.
- Avoid exchangers with 2 (or more) downstreams paths to outputs.
- Prefer bypasses with a direct effect on an output. If not possible:

  Prefer to place bypass on the opposite stream *or* the upstream exchanger of the opposite stream.

  Avoid placing the bypass on the upstream exchanger of the controlled stream (or further away).

Nomenclature

Concerning the dynamic model:

A - Total heat exchanger area $[m^2]$
A_{cell} - heat exchange area for one cell $= A/n[m^2]$
$c_{p,h}$ - specific heat capacity of hot fluid $[J/kgK]$
$c_{p,c}$ - specific heat capacity of cold fluid $[J/kgK]$
h_h - Film transfer coefficient of hot fluid $[W/m^2K]$
h_c - Film transfer coefficient of cold fluid $[W/m^2K]$
N - No of controlled outputs
$N_{hx,util}$ - No of heaters and coolers in HEN
N_{strm} - No of process streams in HEN
N_{hx} - No of process heat exchangers in HEN
N_{byp} - No of process heat exchangers with bypass = No of outputs controlled by bypasses
n - No of cells in HX model
q_h - Flow rate of hot fluid $[m^3/s]$
q_c - Flow rate of cold fluid $[m^3/s]$
$T_{h,in}$ - temperature on hot side at inlet of heat exchanger
$T_{h,out}$ - temperature on hot side at outlet of heat exchanger
$T_{c,in}$ - temperature on cold side at inlet of heat exchanger
$T_{c,out}$ - temperature on cold side at outlet of heat exchanger
$T_h(i)$ - temperature on hot side of i'th cell in hx model
$T_c(i)$ - temperature on cold side of i'th cell in hx model
T_{is} - supply temperature of i'th stream
T_{it} - target temperature of i'th stream $[K]$
$\Delta T_{hx}(i)$ - Driving force of heat transfer in cell $i[K]$
U - Overall heat transfer coefficient $= h_h h_c/(h_h + h_c)[W/m^2K]$
V - Total heat exchanger volume $[m^3]$
$V_{cell,h}$ - Cell volume on hot side $[m^3]$
$V_{cell,c}$ - Cell volume on cold side $[m^3]$
α_h - Dimensionless parameter defined in eq(1)
α_c - Dimensionless parameter defined in eq(1)
ρ_h - density of hot fluid $[kg/m^3]$
ρ_c - density of cold fluid $[kg/m^3]$
τ_h - residence time in cell of hot fluid $[s]$
τ_c - residence time in cell of cold fluid $[s]$

Concerning control:

$C(s)$ - Diagonal controller transfer function matrix
$c_i(s)$ - Controller element for output i
$d(s)$ - Vector of disturbances.
$e(s) = y(s) - r(s)$ - Vector of output errors
$G^{all}(s)$ - Augmented process transfer function matrix with all possible inputs
$G(s)$ - Process transfer function matrix
$G_d(s)$ - Disturbance transfer function matrix
$g_{ij}(s)$ - ij'th element of $G(s)$
$g_{dik}(s)$ - ik'th element of $G_d(s)$
$r(s)$ - Reference signal (set-point) for outputs
$S(s)$ - Sensitivity function $S = (I + GC)^{-1}$
$u(s)$ - Vector of manipulated inputs.

$u_{y=0}(s)$ - Vector of manipulated inputs necessary for perfect control.
$y(s)$ - vector of outputs

$\Delta(s)$ - Closed loop disturbance gain matrix
$\delta_{ik}(s)$ - ij'th element of $\Delta(s)$
$\Gamma(s)$ - Performance relative gain matrix
$\gamma_{ij}(s)$ - ij'th element of $\Gamma(s)$
$\Lambda(s)$ - Relative gain matrix
$\lambda_{ij}(s)$ - ij'th element of $\Lambda(s)$
ω - Frequency
ω_B - Closed loop bandwidth

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