Modelling and Experimental Investigation of a Current to Pressure Converter

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Abstract— A Current-to-Pressure (I/P) converter is a device which converts a current input, typically between 4mA and 20mA, to a proportional pressure output. Such devices are often used in valve positioners and regulators to provide accurate and flexible control. This paper describes a physics based mathematical model of an I/P converter implemented using Matlab which is capable of simulating response under dynamic conditions. Detailed physical conditions such as thermal conduction and convection (and their effects on components), compressible orifice flow and inertial movement of components have been incorporated into the model using an explicit time-stepping lumped-parameter scheme. Parameters were obtained for the model using a series of novel experimental procedures. Experimental tests were carried out on a set of I/P converters over a range of operating temperatures and input current sequences. It is shown that the correlation between the simulated results (based on only measured physical characteristics of components) and the experimental test results is quantitatively accurate. It is further shown that the simulation allows the effects of significant design changes to be predicted, and that comparison between experimental and simulated results reveals areas where complex flow behaviour modifies pressure output significantly.

Keywords: Current-to-Pressure converter, Time-stepping, compressible flow

I. INTRODUCTION

Current-to-pressure converters are used to control the pressure of many applications in a wide range of industries because of their capacity for precise control and their dynamic behaviour. The dynamic modelling of a voice-coil I/P converter relies on the information obtained from actual testing done on existing devices and eventually can be used to validate the simulation model for various parameters which the system might endure during its operation.

The mathematical model has been developed based on governing equations which are determined by the flow, mechanical and electromagnetic behaviours of the system. The following sequential steps were used for the model: calculation of mass flow rate through each compartment, determination of the pressure in each compartment by considering thermal effects and finally the determination of the dynamics of mechanical elements. By means of various tests on specific components, key parameters were found which could be fed to the mathematical model and by making sensible approximation of non-critical parameters (for which information proved difficult to obtain), a working simulation was developed on Matlab to predict the dynamic behaviour of the I/P converter. The simulation is based on an explicit time-stepping lumped-parameter scheme.

There is a limited number of reported researches on the modelling and simulation of current-to-pressure converters; however the following references have proved valuable. There are different methodologies for developing and simulating pneumatic actuators which are controlled by proportional valves [1][3]. Simulation based on polytropic and bondgraph modelling are two different methods which can allow analysis of the dynamics of pneumatic actuators. The polytropic method does not take into consideration the effect of thermal heat transfer [2]. Validation of a simulation of a developed model is important. By means of experiments, it is possible to investigate how accurate the dynamics of a theoretical model is compared with pneumatic actuators [4].

A non-linear mathematical modelling which considers the 1) mass flow rate through restrictions, 2) pressure and temperature evolution in each compartment and 3) dynamics of the mechanical elements has been used in this paper [5]. Effects of non-linear flow through valves, and compressibility, are critical elements for accurate modelling [6]. To validate the results of simulation, testing was carried out on an actual I/P converter and the results were compared for quantitative analysis.

II. MATHEMATICAL MODEL

An analysis of subsystems at a fundamental level was applied to derive the mathematical model; fluid, thermal and electromagnetic interactions were incorporated as differential equations of motion. Considering the flow through the compartments of the I/P converter, there are three points in the system which actively restrict the mass flow rate as illustrated in Fig.1; modelling must include the effects of compressibility as the pressures involved can exceed 5 bar.

![Figure 1: Simplified Schematic of I/P converter](image)

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The governing equation for mass flow rate through a restrictor for a compressible fluid is given by (1).

\[
\dot{m} = \frac{C_0 \rho_0}{\sqrt{R_0 T_0}} A_1^{1/\gamma} \left[ \frac{2}{\gamma+1} \left( \frac{P_1}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right]^{(\gamma+1)/2(\gamma-1)}
\]

for a Critical Pressure ratio >0.528, and

\[
\dot{m} = \frac{C_0 \rho_0}{\sqrt{R_0 T_0}} A_1^{1/\gamma} \left[ 1 - \left( \frac{P_1}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right]
\]

for a Critical Pressure ratio ≤0.528,

where \( \dot{m} \) is the mass flow rate, \( C_0 \) is the discharge coefficient, \( P_0 \) is the upstream pressure, \( T_0 \) is the upstream temperature and \( A_1 \) is the cross sectional area of the throat.

The density, pressure and temperature in each compartment are linked by the ideal gas law:

\[ P = \rho R T \] (3)

Where P is the Pressure, \( \rho \) is the density and T is temperature.

The characteristic of the pressure, volume and density in each volume varies as a function of time, thus giving the following expression for the differential pressure:

\[
\frac{dP}{dt} = \frac{d\rho}{dt} RT + \rho R \frac{dT}{dt}
\] (4)

Heat transfer will have an effect on the overall system, especially considering the operating range of the I/P converter varies between -40°C and +85°C.

Heat transfer through conduction is modelled by considering the overall heat transfer coefficients, the contact area between pairs of components, and the temperature differences. This allows a calculation of heat flow rate between any given pair of components (the word ‘components’ is here used to include the external air or the flowing pressurized gas). Reasonable assumptions have been made for the heat transfer coefficients based on the types of material. Direct application of Fourier’s law of heat conduction by considering thermal conductivity proved quite difficult since many of the components within the I/P converter have an uneven shape and a varied surface finish. The rate of change of temperature from heat transfer follows from Newton’s Law:

\[
\frac{dT_i}{dt} = \frac{1}{m_i C_i} \sum_j h_{ij} A_{ij} \Delta T_{ij}
\] (5)

Where \( m_i \) is the mass of individual component \( i \), \( C_i \) is the specific heat capacity of the component (or air), \( h_{ij} \) is the overall heat transfer coefficient between adjacent components (or fluids) \( i \) and \( j \), \( A_{ij} \) is the area of component \( i \) in contact with \( j \) and \( \Delta T_{ij} \) is the temperature difference between the components.

Hence the temperature of each component, \( i \), evolves in time according to

\[
T_i(t + \Delta t) = T_i(t) + \frac{dT_i}{dt} \times \Delta t,
\]

where \( \Delta t \) is the time step used in the simulation. Each volume contains mechanical elements which regulate the flow. A poppet valve controls the flow going into the booster region from a high pressure air supply and its distance of travel determines the throat area and thus the flow rate. The distance \( X \) it moves is dependent upon the balancing force created by the control volume pressure and the booster volume pressure. The mass of the poppet valve is assumed to be negligible and considering the elastic component controlling the movement, the following equation was derived:

\[
x = b(P_c - P_b) + x_o
\] (7)

Where \( x_o \) is the initial clearance of the ball valve and \( b \) is the opening per Pascal difference in the control and booster pressure, measured using a force gauge.

A voice coil is used to regulate the flow from the control volume which eventually controls the control pressure \( (P_c) \). An electromagnetic force, \( F_e \) which is created by the voice coil is used to push a flexure towards a nozzle and by varying the input signal the amount of force can be controlled.

The electromagnetic force is given by:

\[ F_e = d.I, \]

Where \( d \) is the force per unit current of the solenoid (measured experimentally) and \( I \) is the input current.

The flexure behaviour is quite similar to a spring and obeys Hooke’s law.

\[ F_x = k.Ax_2 \]

Where, \( k \) is the stiffness of the flexure element and \( x_2 \) is the distance travelled by the flexure.

By considering the system in balance condition, the opening of the flexure from the nozzle is a function of the force of the pressure pushing against the flexure through a nozzle, the electromagnetic force and the mechanical (spring) force resulting from the flexure.

Ignoring spring stiffness of the flexure and aerodynamic effects, the (idealized) steady-state pressure of the control
volume is given by a force balance between the magnetic force of the voice-coil pulling the flexure towards a nozzle ($d I$) and the control volume’s pressure within the nozzle that pushes against this force:

$$P_c \frac{\pi D^2}{4} = d I,$$

where $P_c$ is the control volume pressure and $D$ is the diameter of the nozzle.

III. SIMULATION

The mathematical model was implemented in Matlab to allow the simulation of the response under dynamic conditions. This was achieved by using a forward-difference method to time-step the differential equations. Various physical parameters have to be defined in the simulation model for accurate response; these physical parameters were obtained by means of experiments on crucial components.

The Matlab model has been defined such that it will generate graphs of the booster pressure ($P_b$) and the control pressure ($P_c$) with respect to time and input current. The following step inputs current have been used:

- From 0-1s: 0mA
- From 1-2s: 1mA
- From 2-3s: 2mA
- From 3-4s: 4mA
- From 4-5s: 10mA
- From 5-6s: 20mA

A. Simulation Results

The initial values of the temperatures of each compartment and component were set at 293K. The fluid supply temperature set at 283K. Initial values of the booster volume pressure and the control volume pressure were set at 1bar (absolute) and the supply pressure at 6bar (absolute). Fig 1 shows the simulation result and Fig 3 shows the simulation results which takes into consideration the volumes of the connecting pipes to the pressure sensors.

IV. EXPERIMENTAL SET-UP AND RESULTS

Dynamic test was carried out at Norgren Ltd. where the booster pressure and control pressure were recorded. As illustrated in Fig 2, the I/P converter and a volumetric load cell from which the flow was supplied, were mounted inside an environmental chamber to maintain uniform temperature during the test. The current signal was generated from a programmable D.C. current supply. The pressure data from the pressure transducers were captured and processed by an oscilloscope.

Fig 4 shows the test results carried at a constant temperature of 293K.
V. DISCUSSION

Table 1 and Table 2 show the test results versus the simulation results for the booster volume pressure and the control volume pressure respectively. From Table 1 and Table 2, it shown that the simulation results varies up to 14.3% for the booster volume pressures and up to 26.2% for the control volume pressures. Considering the normal range of input current

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Figure 3: Simulation including volumes of connecting pipes

Figure 4: Test results for Pb and Pc
which is 4- to 20-mA, the variation of the control volume pressures is within reasonable range.

TABLE 1: BOOSTER PRESSURE WITH PERCENTAGE VARIATION

<table>
<thead>
<tr>
<th>I/mA</th>
<th>Physical Test</th>
<th>Simulation</th>
<th>% variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.2</td>
<td>1.372</td>
<td>14.3</td>
</tr>
<tr>
<td>1</td>
<td>1.43</td>
<td>1.581</td>
<td>10.5</td>
</tr>
<tr>
<td>2</td>
<td>1.7</td>
<td>1.818</td>
<td>7.1</td>
</tr>
<tr>
<td>4</td>
<td>2.23</td>
<td>2.148</td>
<td>-3.6</td>
</tr>
<tr>
<td>10</td>
<td>3.71</td>
<td>3.527</td>
<td>-4.9</td>
</tr>
<tr>
<td>20</td>
<td>5.98</td>
<td>5.879</td>
<td>-1.7</td>
</tr>
</tbody>
</table>

TABLE 2: CONTROL PRESSURE WITH PERCENTAGE VARIATION

<table>
<thead>
<tr>
<th>I/mA</th>
<th>Physical Test</th>
<th>Simulation</th>
<th>% variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.04</td>
<td>1.312</td>
<td>26.2</td>
</tr>
<tr>
<td>1</td>
<td>1.21</td>
<td>1.521</td>
<td>25.7</td>
</tr>
<tr>
<td>2</td>
<td>1.47</td>
<td>1.756</td>
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<td>4</td>
<td>2.01</td>
<td>2.087</td>
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<tr>
<td>10</td>
<td>3.49</td>
<td>3.465</td>
<td>-0.7</td>
</tr>
<tr>
<td>20</td>
<td>5.71</td>
<td>5.812</td>
<td>1.8</td>
</tr>
</tbody>
</table>

It should be noted that from Fig. 1 and Fig. 4, that the response times for both booster volume pressures and the control volume pressures are quite different, this can be explained by the fact that the connecting pipes to the pressure sensors increase the volumes and eventually increases the response time. The additional volumes of the connecting pipes were included in the simulation run to give better accuracy of the results. This is shown in Fig. 3. It is important to highlight that the simulation will not give complete accuracy as there are assumptions and approximations made in the modelling, however comparison between simulation and experiment can lead to greater understanding of both.

VI. CONCLUSION

This paper analyses how a mathematical model based on domains of fluid, mechanical and electromagnetic of a current-to-pressure can be developed in Matlab to simulate the dynamic response of the system. The simulation results show that the mathematical modelling of the system is effective over the operating range of input current. In addition, experiments were carried out in a controlled environment to validate the simulation model. Within acceptable levels of accuracy, the simulation has been shown to predict important behaviour of the pressures in the booster and control chambers with regards to changes made to the physical I/P converter.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_i</td>
<td>Contact area</td>
<td>m^2</td>
</tr>
<tr>
<td>A_t</td>
<td>Cross sectional area of orifice</td>
<td>m^2</td>
</tr>
<tr>
<td>b</td>
<td>Displacement per unit pressure</td>
<td>m/Pa</td>
</tr>
<tr>
<td>C_d</td>
<td>Coefficient of discharge</td>
<td></td>
</tr>
<tr>
<td>C_i</td>
<td>Specific heat capacity</td>
<td>J/(kgK)</td>
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<tr>
<td>d</td>
<td>Electromagnetic force constant</td>
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</tr>
<tr>
<td>γ</td>
<td>Heat capacity ratio</td>
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</tr>
<tr>
<td>h_i</td>
<td>Overall heat transfer coefficient</td>
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<tr>
<td>I</td>
<td>Current</td>
<td>A</td>
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<td>m</td>
<td>Mass flow rate</td>
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<tr>
<td>P_{01}</td>
<td>Upstream Pressure</td>
<td>Pa</td>
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<tr>
<td>P_b</td>
<td>Booster volume pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>P_c</td>
<td>Control volume pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>P_t</td>
<td>Pressure at orifice</td>
<td>Pa</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
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<tr>
<td>T_{01}</td>
<td>Upstream temperature</td>
<td>K</td>
</tr>
<tr>
<td>x_0, x</td>
<td>Displacement</td>
<td>m</td>
</tr>
</tbody>
</table>

ACKNOWLEDGEMENT

This work is supported by Knowledge Transfer Partnership (KTP) programme number 8693 and by Norgren Ltd (Watson Smith), a subsidiary of IMI plc.

REFERENCES


