A new reconfigurable fault tolerant control design based on Laguerre series

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Abstract—In this paper a new design method is presented for reconfigurable fault tolerant control of multivariable nonlinear time delay systems. The control reconfiguration algorithm comprises two main parts: fault detection and diagnosis and model reference adaptive control. First, a combination of generic residual generation method and its Laguerre approximation is proposed due to the problem of residual similarity in fault diagnostic process. Second, a novel approach for adaptive reconfigurable control synthesis is proposed using a nonlinear system identification approach based on the approximation of systems via Laguerre series. The Laguerre-based reconfiguration method is implemented and tested on a time delay form of COSY benchmark model as a preliminary study of reconfigurable control applied to a nonlinear time-delayed model. Actuator faults have been implemented in the COSY benchmark and used to evaluate the control reconfiguration schemes. Simulation results showed acceptable level of fault tolerance.

Keywords—Fault tolerant control systems, fault detection and diagnosis, residual generation, model reference adaptive control, Laguerre series.

I. INTRODUCTION

Fault Tolerant Control Systems (FTCS) design is becoming the key issue in technical systems due to rapidly increasing demands for higher system performance, product quality, productivity and cost efficiency. As it is well-known, FTCS are control systems that possess the ability of accommodating system component failures automatically. In General, FTCSs can be classified into two types: Passive Fault Tolerant Control Systems (PFTCS) and Active Fault Tolerant Control Systems (AFTCS). The design objective of AFTCS is to 1) design a Fault Detection and Diagnosis (FDD) scheme providing as precise as possible information about a fault and 2) design a new reconfigurable controller in response to and compensating for the fault-induced changes in the system, such that the stability and acceptable close-loop system performance can be maintained.

A successful control system reconfiguration relies on a real-time FDD scheme to provide the most up-to-date information about the system. Model-based methods of fault-detection can be used in presence of input and output signals and applying dynamic process models. These methods are based, e.g., on parameter estimation, parity equations or state observers [1, 2]. The goal of fault detection is to generate several symptoms indicating the difference between nominal and faulty status. Based on different symptoms, fault diagnosis procedure follows, determining the fault by applying classification or inference methods. In the FDD module, both the fault parameters and the system state variables need to be estimated online in real time. In the case of nonlinear systems one approach is to linearize nonlinear model using system identification methods. Several fault diagnosis methods have been proposed for systems linearized around an operating equilibrium point (see [3] and [4]). Some of these techniques have also been extended to nonlinear systems. In [5] nonlinear observers are synthesized to detect tank leakage.

The strategy of reconfigurable control is based on inherent information of nominal and faulty systems so as to make these systems consistent in some proper sense. This strategy fits the model following/matching scheme well if we regard the nominal systems as the reference models. Model Reference Adaptive Control (MRAC) is a mathematical approach which can be used to design a model-based reconfigurable control. Research on the application of adaptive methods to reconfigurable control has been extensively studied during past years. Poirot and co-authors in [6] have proposed a framework for constructing adaptive and reconfigurable control system based on the control loop paradigm. In [7] three adaptive algorithms for multivariable model reference control in flight control applications have been compared. A Failure Detection and Identification (FDI) and Adaptive Reconfigurable Control (ARC) procedure based on a decentralized FDI-ARC scheme integrating separate modules for aerospace applications has been carried out by Boskovic and co-authors in [8]. They also have done an adaptive fault tolerant control based on the estimation of damage-related parameter and switching among multiple controllers for flight control in [9]. Furthermore, Boskovic and co-authors proposed a new failure parameterization that models a large class of failures in terms of a single parameter in [10]. In order to accomplish the model following /matching scheme, MRAC is an appropriate approach. A number of investigators have proposed MRAC techniques for nonlinear systems [11], [12] and [13].

In practice, systems with time delays are frequently encountered (e.g., process control systems). Time-delayed linear systems have been intensively investigated in [14]. Lyapunov design has been proven to be an effective tool in controller design for nonlinear systems. Adaptive control has also proven its capability in controlling nonlinear time-delayed
systems [15], [16], [17]. The FDD problem in time delay systems has also been investigated recently in [18] and [19].

A common topic that has been gaining interest is nonlinear systems. Model linearization is the key issue in the case of nonlinear systems. The idea we used in this paper is approximation of systems via basis functions. This representation is much more parsimonious than the FIR or IIR ones, but contrary to the transfer function model does not require assumptions about the order and time delay of the process. Although it is implemented in a state-space form, it is straightforward to use in system identification. Furthermore, orthogonal polynomials can be used to make the polynomial coefficients uncorrelated, to minimize the error of approximation, and to minimize the sensitivity of calculations to round-off error. Laguerre models suggested here provide feasibility and stability for the closed-loop system. Laguerre lattice filter identifier counts for 1) the identification of the system’s dynamics, that are mapped into the filter’s structure through the utilization of estimated reflection coefficients and 2) the system’s order identification. In [20] the usage of Laguerre parameterization of input sequences in model predictive control has been investigated. Also application of Volterra series to the modeling of static and dynamic nonlinear systems is investigated in [21]. A model predictive control strategy based on Laguerre functions has also been proposed in [22].

This paper looks at the problem of reconfigurable fault tolerant control systems in time-delayed nonlinear systems using Laguerre series in case that system is noise free and disturbances are dispensable. This work on the reconfigurable controller synthesis is under assumption that the faulty system information is known by using Laguerre model. After achieving Laguerre model, MRAC is applied on the basis of this linearized model to have better control. At next step, in order to have an accurate FDD, we suggest a novel approach to the problem of residual similarity. A combination of residual generation methods and parameter estimation using Laguerre series would simplify decision making. We employed this concept to generate more detailed residuals. However, because increasing residual signals may leads to diagnosis complexity, classification methods will be helpful. The effectiveness of the Laguerre-based MRAC has been shown via simulation results on a time delay form of COSY benchmark.

II. PROBLEM STATEMENT

Let us consider the following nonlinear multivariable system subjected to input delay

\[ \dot{X}(t) = f_1(X(t), U(t - \tau), F(t), d(t), w(t)) \]
\[ Y(t) = f_2(X(t), U(t - \tau), F(t), d(t), w(t)). \] 

in which \(X \in \mathcal{R}^N\), \(U \in \mathcal{R}^m\) and \(Y \in \mathcal{R}^N\) are state vector, input and output signals respectively, \(\tau \in \mathcal{R}^+\) is the known input delay, \(F_{\text{sys}}\) is the vector of occurred faults (nf is the number of faults), \(d_{\text{sys}}\) is the disturbance vector and \(w_{\text{sys}}\) represents uncertainty or noise vector.

We can approximate (1) using an Nth order Laguerre model

\[ Y(t) = \sum_{i=1}^{N} C_i(f)L_i(q, \alpha)U(t) + \sum_{i=1}^{N} d_i(t)L_i(q, \alpha)Y(t) \] 

where

\[ L_i(q, \alpha) = \sqrt{1 - \alpha^2} \left(1 - \alpha q \right)^{i-1} \]

\(|\alpha| < 1\) is Laguerre parameter and Laguerre coefficients \(C_i(f)\) and \(d_i(t)\) could be calculated using recursive least square algorithm. For complicated systems, high orders of Laguerre polynomials give a simple realization of the system and would decrease the approximation error.

Assuming \(d(t) = w(t) = 0\) - the system is noise free and disturbances are dispensable - we propose the MRAC scheme for multivariable plant model

\[ Y(t) = C(f)L(q, \alpha)U(t) \]

where \(Y(t) \in \mathcal{R}^N\) and \(U(t) \in \mathcal{R}^N\) are output and control input vectors. Moreover

\[ L(q, \alpha) = \begin{bmatrix} L_1(q, \alpha) & 0 & 0 \\ \vdots & \vdots & \vdots \\ L_p(q, \alpha) & 0 & 0 \\ \end{bmatrix} \]

is Laguerre matrix and

\[ C(f)_{nxnp} = \begin{bmatrix} C_{11}(f) & \cdots & C_{1p}(f) \\ \vdots & \ddots & \vdots \\ C_{n1}(f) & \cdots & C_{np}(f) \end{bmatrix} \]

is Laguerre coefficients matrix under the effect of fault .

The problem is to design an adaptive reconfigurable fault tolerant controller for this system such that all signals of the closed-loop system are bounded and the plant output \(y(t)\), asymptotically exact, follows the output \(Y_m(t)\) of the reference model

\[ Y_m(t) = W_m(s)r = C_mL(q, \alpha) r \]

where \(W_m(s) \in \mathcal{R}^N\) is a stable rational transfer matrix, \(r \in \mathcal{R}^N\) is a bounded reference input signal and \(C_m\) is Laguerre coefficients matrix.

Let us write (4) as

\[ Y_p = G(s)U_p \]
where $G(s) = C(f)L(q, \alpha)$ is the transfer matrix of the linear system. It should be noted that since the relative degree of the Laguerre model is $n^* = 1$

$$L_i(s, \alpha) = \frac{\sqrt{2\alpha}}{s + \alpha} \left(\frac{s}{s + \alpha}\right)^{i-1}$$

(9)

the relative degree of $G(s)$ is also $n^* = 1$.

To meet the control objective the following assumptions are made about $G(s)$ [23]:

(A1): $G(s)$ is strictly proper and has full rank.

(A2): The transmission of zeros of $G(s)$ have negative real parts.

(A3): An upper bound $\bar{v}_0$ on the observability index $v_0$ of $G(s)$ is known.

(A4): There exists a known matrix $S_p(f)$ such that $K_p(f)S_p(f) = (K_p(f)S_p(f))^T > 0$.

In (A4), the high frequency gain matrix $K_p(f)$ of the model is defined as

$$K_p(f) = C_p(f)C_m^{-1}$$

which is finite and nonsingular. It should be noted that the system dynamics may have some inputs which are not control inputs, but only play a leading role in modeling. In this situations, to achieve appropriate $K_p(f)$, matrixes $C_p(f)$ and $C_m^{-1}$ must be defined such that all of their elements refer to control inputs.

Without loss of generality, it is assumed that the transfer function of the reference model is diagonal SPR and of the form

$$W_m(s) = \text{diag} \left(\frac{1}{s + a_i}\right), a_i > 0, i = 1, ..., N$$

(11)

III. FAULT DETECTION AND DIAGNOSIS METHOD

Early fault detection, which reduces the possibility of catastrophic damage, is possible by comparing the measured signals with a database that contains characteristic signals for machines operating with or without faulty conditions. To obtain the most up-to-date and comprehending database for fault diagnosis simplicity, we need to consider all possible conditions such as types and severities of faults.

In some cases, decision making is difficult because of similarly behaved residuals resulted from faults with different types or different severities. This is the motivation to search more residuals to distinguish these faults. Hence, a combination of residual generation approaches is suggested to overcome this weakness and to make the database better suited for the FDD. The new approach proposed in this paper, combines residual generation methods with parameter estimation using Laguerre series. In fact, residuals from basic FDD method which are weak to have a good decision are expanded via Laguerre series which results in Laguerre coefficients. These coefficients can be used as supplementary residuals to have a well-informed database and to facilitate the decision making easier. This approach is formulized as follows. Let

$$\text{Residue}(t) = \sum_{i=1}^{n} r(i, q) \text{Residue}(t - i)$$

(12)

be the expansion of resulting residuals from basic FDD method where $q$, $r(i, q)$ and $n$ denote respectively delay operator, coefficients and model order. Coefficients can be expanded in Laguerre model

$$r(i, q) = \sum_{j=1}^{p} a_{ji}L_j(q, \alpha)$$

(13)

which leads to

$$\text{Residue}(t) = \sum_{i=1}^{n} \sum_{j=1}^{p} a_{ji}L_j(q, \alpha) \text{Residue}(t - i)$$

(14)

resulting in Laguerre coefficients $a_{ji}$'s which can be used as supplementary residuals to make an easier fault detection.

Reconfigurable controller

In general, the existing reconfigurable controller design methods fall into one of the following approaches: Linear quadratic regulator, Eigen structure assignment, multiple-model, adaptive control, pseudo-inverse, perfect model following and feedback linearization [22]. Our objective is to design a reconfigurable control strategy to accommodate system component faults.

In this paper, a nonlinear MRAC strategy based on Laguerre model is presented. A Laguerre model is used to synthesize a linear controller that provides a satisfactory closed-loop performance near the nominal operating point.

The proposed control law is considered as

$$U = \theta^T w_1 + \theta_2^T w_2 + \theta_3^T = \theta^T w$$

(15)

where

$$\theta^T = [\theta_1^T \theta_2^T \theta_3^T]$$

(16)

$$w = [w_1^T w_2^T r]^T$$

(17)

$$w_1 = \frac{A(s)}{A(s)} u , w_2 = \frac{A(s)}{A(s)} y , A(s) = [s^{\nu_0-1} I s^{\nu_0-2} ... s I]^T$$

(18)

and $A(s)$ is a monic Hurwitz polynomial of degree $\nu_0$ (A3).

**Proposition 1**: Consider the system (1) with the assumptions (A1) – (A4) and the reference model (7). Then the adaptive control (15) with update laws

$$\dot{\theta}^T = -P_\theta(f)e w^T$$

(20)

assures that all closed-loop signals are bounded and the tracking error $e_1 = Y_p - Y_m$ converges to zero asymptotically.
In equation (20) \( \epsilon = Y(t) - D_p(f)L(q, \alpha) \tilde{\theta}^T w \) is the output error, \( Y(t) = D_p(f) L(q, \alpha) U(t) \) and \( \tilde{Y}_i(t) = D_p(f) L(q, \alpha) \tilde{\theta}^T w \) are output vector and its estimate, respectively.

IV. SIMULATION STUDY

A time delay form of the COSY benchmark is considered for simulation studies. The benchmark is composed of three identical tanks with identical area of cross section. The measured variables are the levels of the first tank \( h_1 \), second tank \( h_2 \) and third tank \( h_3 \) respectively. Inflows of the first tank \( q_{1in1} \) and third tank \( q_{3in2} \) are chosen as known inputs. Inflows vary between 0 and 1 and the status of inter connecting pipes switches between 0 and 1, therefore the system can be considered as a hybrid system [4].

When the valve \( V_{12} \) operates with delay \( T = 10 \) seconds, the objective is to control levels of the first and second tanks. The third tank is hardware redundant which may be used in the presence of faults. Control inputs are Inflow of first tank \( q_{1in1} \) and status of main inter connecting pipe \( V_{12} \).

COSY benchmark without time delay has been investigated by many researchers [25]. But in order to apply Proposition 1 to its time delay form, the nonlinear model is linearized via Laguerre series with sampling time \( T_s = 0.1 \) seconds.

A. FDD method

In this paper the preliminary FDD approach considered is parity equations. A straightforward model-based method of fault detection is to take fix model parity equations. A straightforward model-based method of and

\[
\begin{align*}
\text{Residue}_1(t) &= h_1(t) - \sum_{i=0}^{i=2} C_i(\alpha)L_i(z, \alpha) u_i(t) \\
\text{Residue}_2(t) &= h_2(t) - \sum_{i=0}^{i=2} C_i(\alpha)L_i(z, \alpha) u_i(t).
\end{align*}
\]

When faults occur with similar residuals, Laguerre model of order 4 is used to derive Laguerre coefficients of residuals (output errors) to obtain secondary residuals. Preliminary residuals are expanded as

\[
\text{Residue}_1(t) = \sum_{i=1}^{4} a_{i1} L_i(q, \alpha) \text{Residue}_1(t - 1)
\]

and

\[
\text{Residue}_2(t) = \sum_{i=1}^{4} a_{i2} L_i(q, \alpha) \text{Residue}_2(t - 1).
\]

The difference between these coefficients \( a_{ij} \) and those of normal condition \( a_{ijN} \)

\[
e_{ij} = a_{ij} - a_{ijN}.
\]

are used as supplementary residuals.

For diagnosis, classification methods such as Artificial Neural Networks, Fuzzy clustering, Rule-based reasoning and Statistical methods are widely used [2]. The rule-based reasoning method is the one selected for this paper.

B. Case one: Normal operation

We first start our study by considering the Laguerre-based MRAC strategy in the case with no faults. Hence we simulated the three tank model with \( V_{12}(t) = 0 \), which means that the third tank is not under control. The resulting response is shown in Fig.1.

Leakage in first tank and blockage of main pipe between first and second tank are two common kinds of faults considered in three tank system. The severity of these faults can also be an important factor to take a proper action. Here we simulate blockage fault to show how severity of the fault affect decision making.

C. Case Two: Blockage Fault Occurrence

Next, we simulate the system by assuming that the inter-connecting pipe \( V_{12} \) is blocked completely. Responses of \( h_1 \) and \( h_2 \) in this case are shown in Fig. 2. Blocking \( V_{12} \) results in an undesirable response, which emphasizes the necessity of control reconfiguration. In this situation the third tank enters as a hardware redundancy. The above fault can be easily detected by means of two preliminary residuals. The resulting responses are shown in Fig. 3. Fig. 4 also shows residuals. In this case to detect blockage we have no problem using these residuals; but when partial blockage occurs, the residuals would be as shown in Fig. 5. As it is clear, it is somehow difficult to detect the right fault from difference between Fig. 4 and Fig. 5. Therefore, supplementary residuals \( e_{ij} \) shown in Fig. 6-9 would be helpful.

V. CONCLUSIONS

In this paper, we employed the concept of parameter estimation based on Laguerre series to solve the problem of reconfigurable fault tolerant control of nonlinear time delay systems. To achieve fast and reliable fault detection, diagnosis and correct reconfiguration, a new parameter estimation method using Laguerre series has been proposed and is combined with other residuals generation approaches. For this purpose, a model reference adaptive fault tolerant control based on Laguerre model for nonlinear time delay systems is proposed. Simulation results for a time delay form of COSY benchmark system in the presence of blockage fault has shown the effectiveness of proposed Laguerre-based FDD-FTCS approach. Note that, the proposed method is also simulated for the leakage and the other faults. Although the obtained simulation results show again its effectiveness, but these simulation results have been omitted due to the lack of space.
However, the robustness of the proposed method to parameter uncertainty and/or its extension for real-world case will be investigated in our future works.
REFERENCES


