Stabilizing gain design for PFC (Predictive Functional Control) with estimated disturbance feed-forward

K. Zabet, R. Haber, K. Mocha

Department of Process Engineering and Plant Design, Laboratory of Process Automation, Cologne University of Applied Science, D-50679 Köln, Betzdorfer Str. 2, Germany

Abstract: PFC (Predictive Functional Control) is a simple but very effective SISO (Single-Input Single-Output) predictive controller. A disturbance estimator is proposed based on PFC algorithm in case of non-measurable disturbances. The algorithm considers both external disturbance signal and internal, structural disturbances (process/model mismatch). Estimated disturbance feed-forward control can lead to instability because of the additional closed-loop of the estimator. Stabilizing technique by filtering the estimated disturbance is investigated for aperiodic processes including dead time.

Keywords: Predictive functional control, disturbance estimator, feed-forward control, stability

1. INTRODUCTION

PFC (Predictive Functional Control) is a simple predictive control algorithm (Richalet, et al., 1978) which does not require any matrix inversion or numerical minimization of a cost function. PFC algorithms can be easily realized for all types of SISO (Single-Input, Single-Output) processes. Furthermore the manipulated variable and/or its increment, as well other variables can be constrained. PFC can be extended by feed-forward control in the case of measurable disturbances.

The idea of IMC (Internal Model Control) is to estimate the disturbance (either an external one or a model mismatch between process and model used in the controller) and to use this signal in the control algorithm. Dr. Richalet recommended a different scheme to estimate the disturbance with PFC and to use the estimated signal for feed-forward control. The advantage of the method is that it considers both external not-measurable disturbances and internal, structural model mismatch. For this reason the expression "global disturbance estimator". In the case of external disturbances the method works in a similar way as if the disturbance would have been measured and feed-forward controlled.

Feed-forward control with measured disturbance is an open-loop control. However a feed-forward control with estimated disturbance has an additional closed loop and influences the stability. Therefore some modification must be done to enhance the stability. The simplest technique is using an attenuation factor multiplied by the estimated disturbance (Richalet and O‘Donavan, 2009).

The paper is structured as follows. In Section 2 the PFC algorithm is shortly shown. In Section 3 the disturbance estimation using PFC algorithm is presented. In Section 4 the estimated disturbance feed-forward control is shown. Section 5 presents a sufficient stability criterion for the feed-forward gain for both using and not using a first-order stabilizing filter. A case study of a mixer with heating jacket illustrates the theoretical results in Section 6.

2. PREDICTIVE FUNCTIONAL CONTROL

The principle of PFC is that the controlled variable $y$ achieves the reference trajectory at the target point (or points) using one change (or minimal number of changes) in the manipulated variable $u$. The desired change in the controlled variable $y$ during the prediction horizon $n_p$ (from the actual time $k$) is calculated from the desired change of the reference trajectory and the predicted change of the model output $y_m$.

The manipulated variable $u$ can be calculated easily from the change of the reference trajectory and the predicted change of the model output during the prediction horizon, see Fig.1.

![Fig.1: PFC principle](image)

The desired changes in the controlled variable $y$ during $n_p$ prediction step can be defined supposing that $y$ reaches the reference trajectory at the target point ($n_p$ step ahead):
\[
\dot{y}(k+n_p | k) - y(k) = \epsilon(k) - \hat{\epsilon}(k+n_p | k) = 0 \tag{1}
\]
where \( \epsilon(k) = y_r - y(k) \) and \( y_r \) is the assumed constant reference signal.

The reference trajectory can be chosen an exponential function for simplicity. Then the control error is decreasing monotonously:

\[
\dot{\epsilon}(k+1 | k) = \lambda_r \epsilon(k) \\
\dot{\epsilon}(k+2 | k) = \lambda_r \dot{\epsilon}(k+1 | k) = \lambda_r^2 \epsilon(k) \\
\ldots
\]

\[
\dot{\epsilon}(k+n_p | k) = \lambda_r^n \epsilon(k) \tag{2}
\]
where \( \lambda_r \) is the reduction ratio of the trajectory’s error.

The reference trajectory provides the settling time \( t_{95\%} = \frac{3\Delta t}{\lambda_r} \) for the closed loop control system if \( \Delta t \) is the sampling time.

From (1) and (2), the desired change in \( y \) is defined as follows:

\[
\Delta \hat{y}(k+n_p | k) = \dot{y}(k+n_p | k) - y(k) = (1 - \lambda_r^n) \epsilon(k) \tag{3}
\]

The changes of \( y \) can be predicted using a proportional, first-order model equation without dead time (chosen for simplicity) in discrete-time as

\[
y_m(k) = -a_m y_m(k-1) + K_m(1+a_m)u(k-1) \tag{4}
\]
where \( y_m \) is the model output, \( u \) is the model input, \( a_m \) is the discrete-time model parameter and \( K_m \) is the static gain of the model.

Supposing that the actual input signal \( u \) is kept constant during the prediction horizon, the predicted model output becomes after \( n_p \) steps:

\[
\hat{y}_m(k+n_p | k) = -a_m y_m(k) + K_m(1+a_m)u(k) \\
\hat{y}_m(k+2 | k) = -a_m y_m(k+1 | k) + K_m(1+a_m)u(k) \\
= (-a_m)^2 y_m(k) + (-a_m + 1)K_m(1+a_m)u(k) \\
= (-a_m)^2 y_m(k) + K_m[1 - (-a_m)^2]u(k) \\
\ldots
\]

\[
\hat{y}_m(k+n_p | k) = (-a_m)^n y_m(k) + K_m[1 - (-a_m)^n]u(k) \tag{5}
\]
Then, the predicted change in \( y_m \) becomes:

\[
\Delta \hat{y}_m(k+n_p | k) = \hat{y}_m(k+n_p | k) - y_m(k) = [1 - (-a_m)^n] [K_m a(k) - y_m(k)]
\]
Simple comparison between the predicted change of the reference trajectory in (3) and the predicted change of \( y_m \) in (5) results in the manipulated variable:

\[
u(k) = k_0 (y_r - y(k)) + k_1 y_m(k) \tag{6a}
\]
where:

\[
k_0 = \frac{1 - \lambda_r^n}{K_m[1 - (-a_m)^n]} , \quad k_1 = \frac{1}{K_m} \tag{6b}
\]
If the process has dead time \( d = d_m \) then \( \hat{y}(k+d_m | k) \) in (6a) has to be replaced by \( \hat{y}(k+d_m | k) \):

\[
\hat{y}(k+d_m | k) = y(k) + y_m(k-d_m) - y_m(k-d_m) \
\]
(7)
In case of higher-order aperiodic processes the transfer function can be partitioned in parallel connection of first-order processes

\[
\hat{y}_{m,i}(k) = -a_{m,i} \hat{y}_{m,i}(k-1) + K_{m,i}(1+a_{m,i})u(k-1) \tag{8}
\]
with the corresponding parameters \( K_{m,i} \) and \( a_{m,i} \) of the \( i \)-th sub-process. (If the process has multiple poles then different but very similar poles have to be assigned to each multiple pole.)

The basic algorithm can be easily extended for this case, as well (Khadir and Ringwood, 2008):

\[
u(k) = k_0 [y_r - y(k)] + \sum_{i=1}^{n} k_i y_{m,i}(k) \tag{9a}
\]
where \( n \) is the order of the process,

\[
k_0 = \frac{1 - \lambda_r^n}{\sum_{j=1}^{n} K_{m,j}[1 - (-a_{m,j})^{n_p}]} , \quad k_i = \frac{1 - (-a_{m,i})^{n_p}}{\sum_{j=1}^{n} K_{m,j}[1 - (-a_{m,j})^{n_p}]} \tag{9b}
\]

3. PFC WITH GLOBAL DISTURBANCE ESTIMATOR

The following global disturbance observer was developed by Dr. Richalet based on PFC, see the scheme in Fig. 2. The main advantage of this scheme is that it uses same tools as the already installed PFC, which can be easily implemented.

A simulated process model is controlled by a fast PFC in the estimator. Both the “real” controller and the estimator controller use the same process model without dead time. The controlled variable \( y \) is applied as the reference signal of the estimator PFC.

As the estimator control loop is not disturbed the difference between both manipulated/control signals are equal to the external disturbance acting on the process input if the process model and the controllers are perfect. Consequently this difference is the estimated disturbance acting to the
process’s input. A detailed description of the disturbance estimator is given in (Richalet and O’Donavan, 2009).

The estimator controller’s tuning parameters are: \( T_{ce} \) and \( n_{pe} \). The principle of the estimator in this scheme is to define the control signal \( u_k(k) \) which force the estimator process output \( y_k(k) \) to reach the controlled variable \( y \) during the prediction horizon \( n_{pe} \) after the dead time \( d_w \).

The desired changes in \( y_k \) during \( n_{pe} \) steps are:

\[
\Delta y_{em}(k + d_m + n_{pe}) = (1 - \lambda_{re}) \Delta y(k) - \lambda_{re}(k + d_m) \\
= (1 - \lambda_{re}^n)(y(k) - y_{em}(k))
\]

where \( \lambda_{re} = \exp(-3\Delta t/T_w) \) is the reduction ratio of the bias between \( y(k) \) and \( y_{em}(k) \). Here \( y_{em} \) is the non-delayed model output of the estimator loop \( y_{em}(k - d_m) = y_{e}(k) \).

The desired estimator closed loop settling time \( t_{95\%}=T_{ce} \) has to be chosen very small to provide a fast estimation of the disturbance.

The predicted change of the estimator model output \( y_{em} \) is:

\[
\Delta y_{em}(k + d_m + n_{pe}) = u_k(k) \sum_{j=1}^{n} K_{m,j} [1 - (-a_{m,j})^{n_{re}}] y_{em,j}(k)
\]

Comparing (10) and (11) results in

\[
u_k(k) = \frac{1 - \lambda_{re}^n}{\sum_{j=1}^{n} K_{m,j} [1 - (-a_{m,j})^{n_{re}}]} \sum_{j=1}^{n} k_{re} y_{em,j}(k)
\]

where \( n \) is the order of the model and

\[
k_{re} = \frac{1 - (-a_{m,j})^{n_{re}}}{\sum_{j=1}^{n} K_{m,j} [1 - (-a_{m,j})^{n_{re}}]}
\]

The estimated disturbance is

\[
dist(k) = u_k(k) - u(k - d_m - n_{pe})
\]

In the following simulations the following processes and PFC tuning parameters are supposed, see Table 1.

<table>
<thead>
<tr>
<th>Process/Model</th>
<th>( T_s )</th>
<th>( n_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delayed first-order</td>
<td>25 s</td>
<td>1</td>
</tr>
<tr>
<td>Delayed third-order</td>
<td>25 s</td>
<td>10</td>
</tr>
</tbody>
</table>

Delayed first-order model of these processes is assumed in the PFC algorithm with model gain \( K_m = 2 \), time constant \( T_w = 10 \text{s} \), and dead time \( T_{dm} = 10 \text{s} \). A first-order approximating model was used with the third-order process, in order to show the robust behaviour of the algorithm.

Using a third-order model with a third-order process - as usual - would lead to a better control behaviour.

Both external disturbance signal and process gain variation are assumed in the simulations. The control scenario is:

- sampling time \( \Delta t=1\text{s} \) and simulation time = 460 s,
- at \( t=10\text{s} \) stepwise increase of \( y_r \) from 0 to 1,
- at \( t=160\text{s} \) stepwise external disturbance \((0 \to -0.5)\).
- at \( t=310\text{s} \) process gain increase by 50%.

Fig. 3 shows the estimation of the disturbance for the given processes (without feed-forward control). The estimator parameters are: \( T_{ce} = 1\text{s} \) and \( n_{pe} = 1 \) for first-order process, \( T_{ce} = 5\text{s} \) and \( n_{pe} = 3 \) for third-order process.

The plots in Fig. 3/a and 3/b show that the real external disturbances are estimated well, of course only after the dead time. The structural disturbance coming from the processes gain change is estimated as an external disturbance signal at the process input. The estimated disturbance has the same effect as the process parameters change. The plot in Fig. 3/b shows estimated structure disturbance at about \( t=20\text{s} \), mainly because of the mismatch between the third-order process and its approximating first-order model.
4. PFC WITH ESTIMATED DISTURBANCE FEED FORWARD CONTROL

To compensate the disturbance effects on the process, PFC with estimated disturbance feed-forward algorithm can be used, see Fig. 2.

PFC control equation with disturbance feed-forward is:

\[ u(k) = k_0[y_y(k) - y_m(k) + y_n(k - d_n)] + \sum_{i=1}^{n} k_i y_{m,i}(k) - \text{dist}(k) \]  

(15)

where \(\text{dist}(k)\) represents either measured or estimated disturbance signal:

\[ y_m(k) = \sum_{i=1}^{n} a_{m,i} y_{m,i}(k-1) + [u(k-1) + \text{dist}(k-1)]\sum_{i=1}^{n} K_{m,i} (1 + a_{m,i}) \]  

(16)

The plot shows that the external disturbance signal is compensated completely. The (usually non-measurable) structure disturbances are compensated as slowly as the external disturbances by the control without disturbance feed-forward algorithm.

Fig. 5 shows the simulation of PFC with estimated disturbance feed-forward algorithm.

Fig. 4 shows the simulation of PFC with feed-forward algorithm based on the measured disturbance.

Plot 5/a shows that the estimated external and structural disturbances are compensated in case of the first-order process, however with high oscillation when the structural disturbance occurred. Plot 5/b shows instability of the process control resulting from the mismatch between the third-order process and its first-order approximating model. It is emphasized that if a correct third-order model was used with the third-order process then the control and manipulated signals would not oscillate so much. In the next section we shall show how the control behaviour can be improved even in the case of such a model mismatch.

Stabilizing the process control with estimated disturbance feed-forward algorithm can be fulfilled by applying the estimated disturbance through a filter as recommended by Richalet and O’Donavan (2009).
5. STABILIZING PFC WITH ESTIMATED DISTURBANCE FEED-FORWARD CONTROL

To enhance the stability of the closed-loop system control with estimated disturbance feed-forward algorithm, the estimated disturbance is fed forward through a filter.

Zero-order filter with gain $K_f$ is assumed first in the following analysis. The control signal equation using $K_f$ is:

$$u(k) = k_0[y_r - y(k) - y_m(k)] + \sum_{i=1}^{n} k_i y_m(k) - K_f \cdot \text{dist}(k-1)$$

(17)

Assuming nominal dead time $d_p = d_w$, and process gain variation as a structure disturbance $K_p = (1 + \delta_k)K_m$, the process pulse-transfer function has the form: $q^{-d}G_m(q^{-1})$.

Pulse-transfer function of the control signal with estimated disturbance fed-forward (see the Appendix 1) is:

$$u(k) = \frac{k_0v_r - k_0q^{-d}G_m(p^{-1})\text{dist}(k)}{1 + k_0q^{-d}G_m(p^{-1}) + \sum_{i=1}^{n} [k_i(1-q^{-d}) - k_i]G_m(q^{-1})}$$

(18)

The sufficient stability condition of the closed-loop system can be introduced in the frequency domain for $|q|^{-1} = |e^{-\beta\Omega}| = 1$ as:

$$K_f < \frac{1 + k_0q^{-d}G_m(p^{-1})}{1 + \sum_{i=1}^{n} [k_i(1-q^{-d}) - k_i]G_m(q^{-1})}$$

(22)

The frequency response of the process can be defined parametrically according to the identified process model, or defined non-parametrically according to the impulse response $y_I$ (or the step response $y_s$) taking into account the variations of the process gain as follows:

$$G_p(e^{-\beta\Omega}) = (1 + \delta_k)\sum_{i=0}^{n} y_I(k)e^{-\beta\Omega}$$

(23)

whereas the impulse response tends to zero at instant $k$. Assuming a filter $G_f(q^{-1})$ with static gain $K_f$ to damp the oscillations of the estimated disturbance signal, the sufficient stability condition of the closed-loop system can be introduced in the frequency domain for $|q|^{-1} = |e^{-\beta\Omega}| = 1$ as:

$$K_f < \frac{1 + k_0q^{-d}G_m(p^{-1})}{1 + \sum_{i=1}^{n} [k_i(1-q^{-d}) - k_i]G_m(q^{-1})}$$

(24)

The stabilizing gain in (24) can be higher than the gain in (22) whereas for an aperiodic filter (first-order filter usually) the absolute value $|G_f(q^{-1})| \leq 1 \forall |q|^{-1} = 1$. This is expected because the filter is stabilizing the control more than without filter.

The resulted stabilizing gain conditions (22) and (24) are applied for the simulated processes. The designed gains are presented in Table 2.

Table 2: Designed stabilizing filter’s gain

<table>
<thead>
<tr>
<th>Process</th>
<th>Zero-order $K_0$</th>
<th>First-order $K_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delayed process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-order</td>
<td>0.9566</td>
<td>1.0776</td>
</tr>
<tr>
<td>Third-order</td>
<td>0.2449</td>
<td>0.2667</td>
</tr>
</tbody>
</table>
Fig. 6 shows PFC with estimated disturbance fed-forward using a gain $K_{f0}$ (without filter, $T_f = 0\, s$) (see Table 2). The plots in Fig. 6a and Fig. 6b show that the designed stabilizing gain $K_{f0}$ results in oscillating signals with very small damping, which shows that $K_{f0}$ is very close to the boundary stabilizing gain and it is smaller than the critical value. Applying a smaller filter’s gain damps the oscillations, and applying a higher filter’s gain moves the process control toward instability.

Fig. 6c shows PFC with estimated disturbance feed-forward of the delayed third-order process assuming nominal third-order model with dead time. The gain $K_f = 1.5K_{f0} \approx 0.366$, which leads to instability in Fig. 6b, is applied in Fig. 6c. Also higher values are applied to show that with nominal model the stabilizing gain can be increased whereas it is stable for $K_f = 0.75$ and unstable for $K_f = 1$.

Fig. 7 shows the simulation of PFC with estimated disturbance feed-forward using first-order stabilizing filter with gain $K_{f1}$ (see Table 2) and time constant $T_f = 3\, s$. The plot shows that the designed stabilizing gain $K_{f1}$ results in oscillating signals with very small damping, which demonstrates that $K_{f1}$ is smaller than and very close to the
limit value of the stabilizing gain. Applying a smaller filter’s gain damps the oscillations, and applying a higher filter’s gain moves the process control toward instability.

6. CASE STUDY

Mixing and tempering of liquids is a fundamental operation in the chemical industry. Therefore, this process has been chosen as a case study to illustrate the new design method. A fluid is fed into the mixer. This liquid is stirred and tempered. The temperature is controlled via a heating jacket which is supplied with water. The schema of the mixer is shown in Fig. 8.

![Fig. 8: Schema of the mixer](image)

The following symbols and parameters are used:

- **θ_M_in**: inlet temperature of fluid in mixer
- **θ_M_out**: outlet temperature of fluid in mixer
- **F_M_in** and **F_M_out**: inlet and outlet flow of fluid in mixer
- **V_M**: volume of mixer
- **c_p,M**: heat capacity of fluid in mixer
- **ρ_M**: density of fluid in mixer
- **θ_M**: temperature of fluid in mixer
- **J_in**: water inlet temperature into jacket
- **J_out**: water outlet temperature from jacket
- **F_J_in** and **F_J_out**: water inlet and outlet flow in jacket
- **V_J**: volume of jacket
- **c_p,J**: heat capacity of water
- **ρ_J**: density of water
- **A_J**: reactor heat transfer surface.
- **U**: thermal transmittance
- **UA**: heat transfer area
- **K**: thermal transmittance

The jacket is described by (25).

\[
V_J \cdot c_p,J \frac{d\theta_J(t)}{dt} = UA \left[ \theta_M(t) - \theta_J(t) \right] + \rho_J \cdot c_p,J \cdot F_J \left[ \theta_{J,in}(t) - \theta_J(t) \right]
\]

From (25) we obtain the following differential equation

\[
T_J \frac{d\theta_J(t)}{dt} + \theta_J(t) = K_{J,M} \cdot \theta_M(t) + K_{J,J,in} \cdot \theta_{J,in}(t)
\]

This equation reflects the following parameters of the jacket:

\[
T_J = \frac{V_J \cdot \rho_J \cdot c_p,J}{(UA + \rho_J \cdot c_p,J \cdot F_J)} = 351 \text{ s} = 5.85 \text{ min} \quad \text{time constant}
\]

\[
K_{J,M} = \frac{UA}{(UA + \rho_J \cdot c_p,J \cdot F_J)} = 0.0252 \quad \text{gain of input } \theta_M
\]

\[
K_{J,J,in} = -\frac{\rho_J \cdot c_p,J \cdot F_J}{(UA + \rho_J \cdot c_p,J \cdot F_J)} = 0.9748 \quad \text{gain of input } \theta_{J,in}
\]

The mixer is described by (26).

\[
V_M \cdot \rho_M \cdot c_p,M \frac{d\theta_M(t)}{dt} = UA \left[ \theta_J(t) - \theta_M(t) \right] + \rho_M \cdot c_p,M \cdot F_M \left[ \theta_{M,in}(t) - \theta_M(t) \right]
\]

By transforming of (26) we get the following differential equation:

\[
T_M \frac{d\theta_M(t)}{dt} + \theta_M(t) = K_{M,J} \cdot \theta_J(t) + K_{M,M,in} \cdot \theta_{M,in}(t)
\]

This equation reflects the following parameters of the mixer:

\[
T_M = \frac{V_M \cdot \rho_M \cdot c_p,M}{(UA + \rho_M \cdot c_p,M \cdot F_M)} = 307 \text{ s} = 5.1 \text{ min} \quad \text{time constant}
\]

\[
K_{M,J} = \frac{UA}{(UA + \rho_M \cdot c_p,M \cdot F_M)} = 0.1469 \quad \text{gain of input } \theta_J
\]

\[
K_{M,M,in} = -\frac{\rho_M \cdot c_p,M \cdot F_M}{(UA + \rho_M \cdot c_p,M \cdot F_M)} = 0.8531 \quad \text{gain of input } \theta_{M,in}
\]

The process model is shown in Fig. 9 with the controller.

![Fig. 9: Control schema of the mixer model](image)

The process was simulated by sampling time of \( \Delta t = 1 \text{ min} \). First-order model with static gain \( K_n = 0.1437 \) and time constant \( T_n = 160 / 3 \text{ min} \) was assumed in PFC, the tuning parameters were: \( T_e = 160 \text{ min} \) and \( n_p = 10 \). Estimated disturbance feed-forward control was applied with PFC estimator parameters: \( T_e = 30 \text{ min} \) and \( n_p = 10 \).

Set point level \( \theta_{M,r} : \) 40°C→41°C was applied at \( t = 50 \text{ min} \) and after 500 min a disturbance signal step \( \theta_{M,in}: \) 40°C→41°C °C was used in the inlet fluid temperature to the mixer.

Fig. 10 shows simulations of the temperature control without and with estimated disturbance feed-forward control using a stabilizing gain (that means no first-order filter). Fig. 10a shows the simulations using a designed critical stabilizing gain \( K_f = K_{f,0} = 0.976 \); this plot shows good disturbance rejection with decreasing oscillations. Smaller gain \( K_f = 0.75 K_{f,0} \) is simulated in Fig. 9b; the plot shows good disturbance rejection with smaller oscillating signals. Higher
gain $K_f = 1.025 K_f_0$ is simulated in Fig. 10c, the plot shows very strongly oscillating signals.

Fig. 10a: Stabilizing gain $K_f = K_f_0$

Fig. 10b: Stabilizing gain $K_f = 0.75 K_f_0$

Fig. 10c: Stabilizing gain $K_f = 1.025 K_f_0$

Fig. 11 shows simulations of the mixer temperature control without and with estimated disturbance feed-forward control using a first-order stabilizing filter with $T_f = 10\text{ s}$.

Fig. 11a: Stabilizing first-order filter with $K_f = K_f_1$

Fig. 11b: Stabilizing first-order filter with $K_f = 1.5 K_f_1$

7. CONCLUSION

PFC is a simple predictive algorithm which can be implemented easily mainly for SISO processes. The disturbance estimator using PFC algorithm was recommended by Dr. Richalet to avoid an additional implementation of another disturbance estimator algorithm. The advantage of this disturbance observer is that the recommended algorithm uses the same tools like the already implemented PFC.

The algorithm was simulated for delayed first- and higher-order processes using first-order model with dead time. Good estimation of the external and structural (gain variation) disturbances were shown even in the case that the third-order process was approximated by a first-order model.
Pulse-transfer function of the manipulated variable with estimated disturbance feed-forward through a gain

\[ u(k) = \frac{k_0(y_k - k_0q^{-d\delta}G_p(q^{-1})dist(k))}{1 + k_0q^{-d\delta}G_p(q^{-1}) + \sum_{i=1}^{n}[k_i(1-q^{-d\delta}) - k_i]G_{m_i}(q^{-1})} \]

\[ K_f \{1 + \sum_{i=1}^{n}[k_i(1-q^{-d\delta}) - k_i]G_{m_i}(q^{-1})\}dist(k-1) \]

(A1)

Pulse-transfer function of the disturbance is:

\[ \hat{dist}(k) = u(k) - u(k - d_m - n_{\mu}) \]

\[ \hat{dist}(k) = \frac{k_{\mu}y(k) - q^{-d\delta + \mu}u(k)}{1 + \sum_{i=1}^{n}(k_{\mu} - k_{\mu})G_{m_i}(q^{-1})} \]

\[ \hat{dist}(k) = k_{\mu}q^{-d\delta}G_p(q^{-1})[u(k) + \hat{dist}(k)] - q^{-d\delta + \mu}u(k) \]

\[ \hat{dist}(k) = k_{\mu}q^{-d\delta}G_p(q^{-1}) \]

\[ 1 + \sum_{i=1}^{n}(k_{\mu} - k_{\mu})G_{m_i}(q^{-1}) \]

\[ q^{-d\delta}u(k) \]

A new design method was illustrated for estimating and compensating temperature disturbances occurring with chemical reactions. The authors thank for Dr. J. Richalet for initiating this research work and for his advices.

REFERENCES


APPENDIX

Pulse-transfer function of the manipulated variable with estimated disturbance fed-forward through a gain \( K_f \) is:

\[ u(k) = k_0(y_k - y_m(k) + y_m(k - d_m)) \]

\[ + \sum_{i=1}^{n} k_i y_{m_i}(k) - K_f \hat{dist}(k-1) \]

\[ u(k) = k_0(y_k - k_0q^{-d\delta}G_p(q^{-1})[u(k) + \hat{dist}(k)] \]

\[ - k_0(1-q^{-d\delta})G_{m_i}(q^{-1})[u(k) + K_f \hat{dist}(k-1)] \]

\[ + \sum_{i=1}^{n} k_i G_{m_i}(q^{-1})[u(k) + K_f \hat{dist}(k-1)] - K_f \hat{dist}(k-1) \]

Substituting (A3) into (A1) results in:

\[ u(k) = \frac{k_0y_k - k_0q^{-d\delta}G_p(q^{-1})dist(k)}{1 + k_0q^{-d\delta}G_p(q^{-1}) + \sum_{i=1}^{n}[k_i(1-q^{-d\delta}) - k_i]G_{m_i}(q^{-1})} \]

\[ K_f \{1 + \sum_{i=1}^{n}[k_i(1-q^{-d\delta}) - k_i]G_{m_i}(q^{-1})\} \]

\[ 1 + \sum_{i=1}^{n}(k_{\mu} - k_{\mu})G_{m_i}(q^{-1}) \]

\[ q^{-d\delta}u(k) \]
Characteristic equation of the closed-loop control system is:

\[ K_f \left[ 1 + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1}) \right] \\
+ 1 + k_o q^{-s_o} G_p(q^{-1}) + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1}) \\
q^{-s_o} - \frac{k_0 G_p(q^{-1})}{1 + \sum_{i=1}^{n} (k_{0e} - k_{0w}) G_m(iq^{-1})} q^{-s_o} u(k) = 0 \]

\[ K_f \left[ 1 + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1}) \right] \\
1 + k_o q^{-s_o} G_p(q^{-1}) + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1}) \\
q^{-s_o} - \frac{k_0 G_p(q^{-1})}{1 + \sum_{i=1}^{n} (k_{0e} - k_{0w}) G_m(iq^{-1})} q^{-s_o} = 0 \]

\[ 1 = \frac{K_f \left[ 1 + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1}) \right]}{1 + k_o q^{-s_o} G_p(q^{-1}) + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1})} \\
q^{-s_o} - \frac{k_0 G_p(q^{-1})}{1 + \sum_{i=1}^{n} (k_{0e} - k_{0w}) G_m(iq^{-1})} q^{-s_o} = 0 \]

\[ K_f = \frac{q^{-s_o} [1 + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1})] - k_{0e} G_p(q^{-1})}{q^{-s_o} [1 + \sum_{i=1}^{n} (k_{0e} - k_{0w}) G_m(iq^{-1})] - k_{0e} G_p(q^{-1})} \]

The sufficient condition of the stability in the frequency domain \( |q^{-1}| = 1 \) is:

\[ K_f > \frac{1 + k_o q^{-s_o} G_p(q^{-1}) + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1})}{1 + \sum_{i=1}^{n} [k_i (1-q^{-s_i}) - k_r] G_m(iq^{-1})} \\
q^{-s_o} [1 + \sum_{i=1}^{n} (k_{0e} - k_{0w}) G_m(iq^{-1})] - k_{0e} G_p(q^{-1}) \]

Assuming a stabilizing filter with gain \( K_f \), the sufficient condition of the stability in the frequency domain is:

\[ G_f(q^{-1}) \leq 1 \]

First-order filter (with unity gain) as an example has an absolute value of the pulse-transfer function as:

\[ \frac{1 + a_f}{1 - a_f} \leq 1 \quad \forall |q^{-1}| = 1 \]

From (A7) we can see that the upper limit of the stabilizing gain in (A6) is higher or equals the upper limit of the stabilizing gain in (A5)

\[ |G_f(q^{-1})| \leq 1 \]

\[ |G_f(q^{-1})| \leq 1 \]

(A4)

(A5)