Extended Abstract
Design of Measurement Noise Filters for PID Control

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1. INTRODUCTION

Control design requires a model of the process and its environment, as well as a collection of requirements such as robustness and performance. Robustness shows the sensitivity of the closed loop to process changes. Performance involves specifications with respect to load disturbance response as well as limitation of the control actions generated by measurement noise. Thus, the final design requires a compromise between the different requirements.

Most design methods focus on the attenuation of load disturbances and do not consider measurement noise. In this extended abstract the discussion will focus on trade-offs between load disturbance attenuation, robustness and reduction of control actions due to measurement noise.

2. MODELING AND FILTER DESIGN

The process \( P(s) \) is approximated with the standard FOTD system

\[
P(s) = K_p \frac{1}{1 + sT} e^{-sL},
\]

where \( K_p, L, \) and \( T \) are the static gain, the apparent time delay, and the apparent time constant. The relative time delay \( \tau = L/(L + T) \) is used to characterize process dynamics. The parameters \( K_p, L, \) and \( T \) can be determined from a step response experiment.

The PI and PID controllers have the transfer functions

\[
C_{PI}(s) = k_p + \frac{k_i}{s}, \quad C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s, \tag{2}
\]

where \( k_p, k_i, \) and \( k_d \) are the controller parameters.

Measurement noise is reduced by a second order filter with the transfer function

\[
G_f(s) = \frac{1}{1 + sT_f + s^2T_f^2/2}, \tag{3}
\]

where \( T_f \) is the filter time constant. A second order filter is used to ensure roll-off in the PID controller.

The combinations of the controllers and the filter transfer functions are

\[
C(s) = C_{PI}(s)G_f(s), \quad C(s) = C_{PID}(s)G_f(s), \tag{4}
\]

Using this representation ideal controllers can be designed for the augmented plant \( P(s)G_f(s) \).

Control performance can be characterized by the integrated absolute error

\[
IAE = \int_0^\infty |e(t)|dt, \tag{5}
\]

where \( e \) is the control error due to a unit step load disturbance. Here, it is assumed that the disturbance enters at the process input.

Robustness to process uncertainty can be captured by the maximum sensitivities \( M_s \) and \( M_t \).

It is important that the control actions generated by measurement noise are not too large. This can be observed in the transfer function from measurement noise to controller output of the closed loop system

\[
G_{un}(s) = \frac{C(s)}{1 + G_l(s)} = C(s)S(s), \tag{6}
\]

where \( G_l(s) = P(s)C(s) \) is the loop transfer function, and \( S(s) \) is the sensitivity function. In order to characterize the effects of measurement noise, the control bandwidth \( \omega_{gb} \) is considered. This quantity represents the smallest frequency where the gain of \( G_{un} \) is less than \( \beta \), where \( \beta \) is typically in the range 0.01–0.7. Considering that \( S(s) \) in (6) approaches 1 for frequencies higher than the gain crossover frequency \( \omega_{gc} \), the control bandwidth for PI and PID control can be approximated by

\[
\omega_{gb}^{PI} \approx \frac{1}{T_f} \sqrt{\frac{2k_p}{\beta}}, \quad \omega_{gb}^{PID} \approx \frac{2k_d}{\beta T_f^2}. \tag{7}
\]

The largest gain \( M_{un} \) of the transfer \( G_{un} \) is another way to characterize the effect of measurement noise

\[
M_{un} = \max_\omega |G_{un}(j\omega)|, \tag{8}
\]

Adding a filter reduces the effects of measurement noise, but it also reduces robustness and deteriorate load disturbance responses. A compromise is to choose the filter so that the impact on robustness and performance is not too large. The design suggested here is formulated as a trade-off between performance (IAE), robustness (\( M_s, M_t \)) and filtering of measurement noise (\( \omega_{gb}, M_{un} \)), where the controller parameters and the filter time constant are calculated using an iterative procedure.

The filter time constant is chosen as

\[
T_f = \frac{\alpha}{\omega_{gb}}, \tag{9}
\]

where \( \omega_{gb} \) is the gain crossover frequency. Controllers with this filter constant will be designed for different values of \( \alpha \), which is chosen as a trade-off between performance and robustness. For a given value of \( \alpha \) the design procedure is
The FOTD approximation of $P(s)$ using AMIGO [Åström and Hägglund (2005)] gives $k_p = 6.44, k_l = 17.83, and k_d = 0.24$. Table 2 shows the dependence on the filter time constant of different parameters. Figure 2 shows the response to load disturbance, the Nyquist plot of the loop transfer function $G_l$, the gain curve of the noise transfer function $G_{un}$, as well as the gain curve of $G_l$.

Table 2. Parameter dependence on the filter time constant for $P(s)$ using PID control

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\tau$</th>
<th>$L$</th>
<th>$T$</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$T_f$</th>
<th>IAE</th>
<th>$\omega_{cb}/\omega_{gc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.07</td>
<td>0.08</td>
<td>1.04</td>
<td>4.13</td>
<td>7.67</td>
<td>0</td>
<td>0.13</td>
<td>$\infty$</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.07</td>
<td>0.08</td>
<td>1.04</td>
<td>3.95</td>
<td>7.21</td>
<td>0.003</td>
<td>0.14</td>
<td>888.7</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>0.07</td>
<td>0.08</td>
<td>1.04</td>
<td>3.79</td>
<td>6.81</td>
<td>0.005</td>
<td>0.15</td>
<td>435.7</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.08</td>
<td>0.09</td>
<td>1.04</td>
<td>3.33</td>
<td>5.65</td>
<td>0.015</td>
<td>0.18</td>
<td>163.2</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.11</td>
<td>1.04</td>
<td>2.53</td>
<td>3.87</td>
<td>0.038</td>
<td>0.26</td>
<td>71.2</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.15</td>
<td>0.18</td>
<td>1.03</td>
<td>1.45</td>
<td>1.89</td>
<td>0.095</td>
<td>0.53</td>
<td>35.9</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.26</td>
<td>0.37</td>
<td>1.05</td>
<td>0.60</td>
<td>0.66</td>
<td>0.312</td>
<td>1.53</td>
<td>17.4</td>
<td></td>
</tr>
</tbody>
</table>
The iterative design is based on the FOTD model and the dynamics of the filter is accounted for by changing the apparent delay $\Delta$ and the apparent time constant $T$. Figures 3 shows that filtering has a significant effect on the magnitude of the unwanted control actions created by measurement noise. Both $M_{\text{un}}$ and $\omega_c/\omega_{gc}$ decrease rapidly with filtering. According to these results, reasonable values of $\alpha$ are in the range of 0.01 to 0.05.

4. DESIGN RULES

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For the example the filter time constant is related to the gain crossover frequency, however for design rules it is useful to relate the filter time constant to the controller parameters. The example as well as others [Romero and Hågglund and Åström (2013)] not included here for space reasons, show that the filter time constant depends on the process. Figure 4 which has been obtained using FOTD models illustrates this dependency. It shows the ratios $T_f/T_i^0$ and $T_f/T_d^0$ as a function of the relative time delay $\tau$ for different values of the design parameter $\alpha$. The parameters $T_i^0$ and $T_d^0$ are the integral time and the derivative time computed for the controller without filtering.

Simple parameter fits in Figure 4 give the following approximate rules for PI and PID control

$$T_f = 6\alpha \pi T_i^0 \text{ (PI)}$$
$$T_f = 4.5\alpha T_d^0 \text{ (PID)}$$

The rules hold for $\alpha < 0.1$. The rule for PI control is valid for all $\tau$ but the rule for PID control only holds for lag-dominated and balanced systems. Derivative action is however of little value for delay-dominated systems. A reasonable standard value is $\alpha = 0.05$.

5. SUMMARY

A drawback of feedback is that measurement noise is fed into the system, but the undesired control actions generated by the noise can be reduced using filtering. Filtering introduces additional dynamics which have to be considered in the control design. Insight into the choice of filtering has been obtained by investigating design of PI and PID controllers as a trade-off between performance and robustness.

The design problem has been solved iteratively. Process dynamics has been approximated by FOTD models and controller parameters have been determined using the AMIGO rule which give sensitivities less than 1.4. The filter has been chosen as a second order Butterworth filter which is characterized by one parameter, the filter time constant $T_f$. The iterative process starts with the nominal process model $P$. The crossover frequency $\omega_{gc}$ has been determined and the filter time constant has been chosen as $\alpha/\omega_{gc}$. A new process model has then been determined by fitting an FOTD model to $PG_f$ and the process has been repeated until convergence.

The results have shown that the control actions generated by measurement noise can be reduced significantly by filtering with only a moderate decrease of performance while maintaining robustness.

Simple design rules for choosing the filter time constant have also been developed (11).

The analysis has been made based on a particular design method AMIGO and the matching method of fitting FOTD models. It would be interesting to investigate if the design rules are similar if other methods for PID design are used.

REFERENCES