Simplified mechanistic model of the Multiple Hearth Furnace for control development*
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ABSTRACT
This paper presents the simplified mechanistic model of a Multiple Hearth Furnace (MHF), developed for process control implementation. The detailed mechanistic model of the MHF and its solving procedure are introduced. Based on the detailed model, the simplified model is developed in the nonlinear Hammerstein-Wiener form, which defines a specific type of nonlinear state space models suitable for example for Model Predictive Control (MPC) implementation. The simplified model aims to preserve the key physical-chemical phenomena taking place in the furnace and to reproduce the nonlinear dependencies between the input and output variables. Finally, the paper presents the simulation results to compare the mechanistic and the simplified models. The comparison confirms that the dynamics of the simplified model accurately follows the mechanistic model outputs.

Keywords: mechanistic model, multiple hearth furnace, calcination, model predictive control, model simplification, industrial application

1 INTRODUCTION

Furnaces, such as the rotary kilns and multiple hearth furnaces, are widely used in industry for the calcination of clay minerals, such as kaolin. However, these processes continue to provide challenges in maintaining efficient process operations. In particular, it is hard to control the final product quality, due to the difficulty in measuring the product characteristics, the solid temperature profile in the furnace, and the rates of the calcination reactions. Instead, the existing control systems mostly rely on the gas temperature measurements and traditional control implementations, such as PID. This strategy, however, does not allow achieving stable solid phase temperature profile and uniform product quality. In contrast, a Model Predictive Control (MPC), based on a model describing the physical-chemical phenomena in the furnace, would be able to stabilize the solid temperature and minimize the product quality variations.

The physical-chemical phenomena taking place in the furnaces were recently investigated in the literature. In particular, the solid phase movement in a MHF and the residence time distribution have been studied in [1]. Extensive studies have been carried out on the kaolin properties [2], [3] and the calcination reactions [4], [5]. Several mechanistic models have been proposed, describing the physical-chemical phenomena occurring during the calcination. Some of the models are in steady state, including a model for a rotary hearth [6] and a rotary kiln coke calciner [7]. More recently, a dynamic model of a MHF for the calcination of kaolin was suggested in [8], [9]. This model describes the dynamics of the MHF, including the gas phase, solid bed, central shaft, walls, rabble arms, and the cooling air. Furthermore, the model shows good accuracy compared to the plant data.

Most of the detailed mechanistic models proposed in the literature are complicated, containing tenths or even hundreds of states and equations, many of which are nonlinear. On the other hands, state space models are commonly used for model based control implementation, due to the simple matrix structure [10]. Therefore, model based control almost exclusively operate using linear models identified from process data [11]. As an example, a constrained MPC is implemented in [12] to

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enhance a lime kiln control, which results in improved product quality, reduced maintenance costs, and lower energy consumption. In [13], a ‘finite number of weights’ MPC is presented for a lime kiln, which requires a suitable weights selection when constraints are violated. While these MPCs rely on models identified from process data, there are also some MPC implementations employing mechanistic models, such as [14] and [15]. In both cases, however, the applied linearization limits the model accuracy, resulting in a model mismatch compared to the plant dynamics, which is inherently nonlinear.

This work presents a simplified mechanistic model of a Multiple Hearth Furnace for process control development. A description of the detailed MHF mechanistic model developed by Eskelinen et.al. [8] is provided. In addition, this paper proposes a detailed model of the wall temperature dynamics, aiming to improve the model accuracy. The simplified model is developed in the Hammerstein Wiener form, which defines a specific type of nonlinear state space models suitable for control development. The simplified model aims to preserve the nonlinear dependencies between the input and output variables while maintaining the information of the physical-chemical phenomena. Finally, the comparison of the mechanistic and the simplified models is carried out to confirm the accuracy of the simplified model.

This paper is organized as follows. First the process is described in Section 2, while Section 3 presents the mechanistic model. Section 4 illustrates the simplified model, derived based on the mechanistic model. Section 5 compares the dynamic response of the mechanistic model and the simplified model. The conclusion is presented in Section 6.

2 PROCESS DESCRIPTION

This paper considers a multiple hearth furnace used for kaolin calcination, having the counter-current solid and gas flows. The furnace has eight hearths, and eight burners, combusting natural gas to provide the heat necessary for the calcination reactions, are located in hearths 4 and 6. Kaolin is supplied to the first hearth located at the top of the furnace. In the calciner, the material is moved by the metal plates, called blades, which are attached to the rotating rabble arms, designed with the intention of transporting the material outwards on even-numbered hearths and inwards on odd-numbered hearths. The kaolin traversing the even numbered hearths moves outward to descend through the holes at the outside border of the hearth, while in the odd-numbered hearths kaolin falls to the next hearth through a single annulus located around the shaft carrying the rabble arms. Figure 1 illustrates the design of the calciner.

The temperature of the solid increases as it travels down through the furnace and reaches its maximum in Hearth 6. Kaolinite transforms to metakaolin in the hearths 3, 4 and 5 at the temperature between 400-700 °C. The metakaolin is released from hearth 5 at a temperature approximately 800 °C, which continues elevating in the hearth 6, where the transformation of metakaolin to the Al–Si spinel phase
occurs. Thus, the main objective of the hearth 6 is to increase the temperature in order to facilitate the absorption of aluminum into the silica phase. The control of temperature in the hearth 6 is essential to avoid overheating, which may result in the undesired formation of a more crystalline material that may generate some abrasion problems. The temperature of the solids begins to decrease in the hearths 7 and 8, and the product leaves from the hearth 8 through two discharge holes at a temperature of 750 °C.

3 DYNAMIC MODEL OF THE MHF

This section describes the mechanistic model of the MHF developed by Eskelinen et.al. [8]. The modeling equations are developed for the six parts of the MHF: the gas phase, solid bed, central shaft, walls, rabble arms, and the cooling air. The model comprises the calcination reaction kinetics, the mass and energy balances, the transport phenomena in the parts of the MHF, as well as additional equations describing the temperature dependent parameters, more details can be found in [8].

The following assumptions have been made to simplify the model development. The solid bed in the hearths is split into four or five homogenous annular volumes, depending on the rabble arm configuration. The volumes are assumed to be equal in mass content and radial direction. The mixing model, describing the solids movement in the hearths, assumes that one shaft rotation disseminates a part of the volume contents to the neighbor volumes. Thus, the solid mass distribution between the volumes of hearth $j$ can be calculated after one shaft rotation as follows:

$$m_{i+1}^j = D_j \cdot (m_i^j - R_{r,t}^j) + m_{\text{feed},t}^j$$

where $m_{\text{feed},t}^j$ and $R_{r,t}^j$ connote the feed and the mass loss in the solid phase in Hearth $j$. The mixing matrix $D_j$ is used to transform the distribution of solids $m_i^j$ in Hearth $j$ after one central shaft rotation. Specifically, the column $i$ of the matrix denotes the distribution of volume $i$ contents between the volumes of the hearth. The feed to a hearth is expressed by considering the exiting volume from the previous hearth:

$$m_{\text{feed},t}^j = (1 - \sum D_{j-1}^K)(m_t^{j-1} - R_{r,t}^{j-1})$$

where $K$ is the exiting volume of hearth $j - 1$ and $D_{j-1}^K$ is the $K^{th}$ column of matrix $D_{j-1}$. The procedure to solve the model is comprised of five steps shown in Figure 2.
The heat received by the furnace walls from the gas phase is transferred in three directions: its major part is transmitted to the solids through radiation, some thermal energy is accumulated inside the walls and a part of the heat is conducted through the walls from the furnace to the ambient, which is considered as energy losses [16]. The radiation from the walls to the solids is known to be very important. The model developed in [8] utilizes an approximated dynamics of the wall temperature. It considers the temperature of the inner walls and the temperature between the material layers. To obtain an accurate description of the dynamics of the inner wall temperature, a detailed model of the heat transfer in the wall has been developed. The wall is divided into 5 mm volumes, where the initial and the final layers represent respectively the outer and the inner wall, and the total number of layers is denoted as \( N \).

The temperature \( T_i \) of each volume \( i = 1, \ldots, N \) is considered to be uniform. An approximation to the dynamics has been made using an implicit finite difference method, as shown in Eq. (3):

\[
\frac{(T_{i+1}^t - T_i^t) C_{pi}}{\Delta t} = \frac{(T_{i+1}^{t+1} - T_i^{t+1}) k_i}{2d} + \frac{(T_{i+1}^{t+1} - T_i^{t+1}) k_i}{2d} - \frac{(T_{i+1}^{t+1} - T_i^{t+1}) k_i}{2d} \]

where \( \Delta t \) represents a time step, \( C_{pi} \) and \( k_i \) are respectively the heat capacity and the heat conduction coefficient of volume \( i \), calculated using the material properties, and \( d = 5 \text{mm} \) is the spatial step.

The heat flow \( Q_t \) through the inner surface of the wall is considered to be linearly dependent on the temperature of the inner wall, where \( \Delta \) is the coefficient of the linear part:

\[
Q_t = Q^* + \Delta T_N^t \]

The approximation of the inner wall layer temperature is obtained as follows:
After initializing the temperature of the walls $T_0^i, i = 1, ..., N$, the system of equations (3), (5) may be solved to calculate the temperature profile within the walls at each instant $t$.

4 MODEL SIMPLIFICATION

This section describes the simplified model developed based on the mechanistic model presented in Section 3. A simplification of the mechanistic model is designed, describing the dynamics and the nonlinear behavior of the system separately. In more details, the simplified model is expressed as a Hammerstein-Wiener model (HWM), decomposing the model in blocks containing the nonlinearities in static form and the linear dynamics. The linear block, enclosing the dynamics of the process, is preceded and followed by a static non-linear blocks.

The dynamics of the MHF includes the very fast component related to the gas phase, and the slower component representing the solid state. For MPC implementation, the temperature of the solid has to be described dynamically. Furthermore, as the temperature of the inner layer of the walls has a direct effect on the solid-walls heat exchange, it is also considered as a model state. The simplified model is implemented as following:

\[
x_{t+1} = ax_t + (1 - \alpha)F(u_t) \\
y_t = G(u_t, x_t)
\]

(6)

where $u_t$ is a vector containing the process inputs (kaolin feed rate, gas flows to the Hearths 4 and 6), $x_t$ is the state vector contains the temperature of the solids in each volume of the furnace and the internal wall temperature in the hearths, $y_t$ contains the gas phase temperature next to the walls in the hearths, $\alpha$ is the time constant parameter of the linear dynamic part of the modeling equations. The time constant $\alpha$ is obtained for each modeling equation by identification performed using the MATLAB® identification toolbox. $F(u_t)$ is a static nonlinear function calculating the steady state of the furnace using the process input values. In order to implement the first function $F(u_t)$, a look up table has been created by running the mechanistic model simulations with different process inputs. The obtained values are interpolated as follows:

\[
F(u_t) = \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{k=1}^{5} b_{i,j,k} h^x_i(F_K) h^y_j(F_{g4}) h^z_k(F_{g6})
\]

(7)

where $b_{i,j,k}$ are the values from the look-up table and the piecewise linear basis functions $h^x_i, h^y_j$ and $h^z_k$ have been used for the interpolation.

The second function $G(u_t, x_t)$, involved in the modeling equations (6), calculates the gas temperature profile next to the walls in the Hearths based on the current furnace state $x_t$ and the process inputs $u_t$. The function $G(u_t, x_t)$ is implemented by solving the energy balance for the gas phase derived from the mechanistic model.

5 SIMULATION RESULTS

The simulation study has been conducted to evaluate the performance of the simplified model by comparing its predictions with the results of the detailed mechanistic model. Input series have been designed for the open-loop tests to compare the dynamic response of the models. The shaft rotation...
period is used as the sampling time in the simulations, according to how it was made by the mechanistic model in [8]. The tests signals include a sequence of step and ramp changes in the input variables, including the kaolin feed rate, total gas flow ratio to the feed, and the ratio of hearth 4 gas flow to the total gas flow. In the top graph of Figure 3, the ramp changes of the kaolin feed to the process are shown. The middle graph presents the open-loop signal for the ratio of the total gas flow to the feed rate. Finally, the bottom graph illustrates the changes in the ratio of the hearth 4 gas flow to total gas flow.

The results of the simplified model are shown as dashed lines, whereas the outputs of the mechanistic model are given as solid lines in the Figures 4-6, providing the simulation results. The comparison of the gas phase temperature (Figure 4) shows excellent simplified model accuracy for all hearths, specifically in hearths 1 to 4 and 7-8. For the solid phase temperature, presented in Figure 5, the comparison also confirms the accuracy of the simplified model, with the best results similarly achieved in hearths 1 to 4 and 7-8. Finally, in the comparison of the wall temperature shown in Figure 6, the simplified model also shows satisfactory accuracy, where the best results are found in hearths 1 to 3 and 7-8.

The quantification reference for the evaluation of the simulation results shown in Figure 4, 5, and 6 is the coefficient of determination, denoted as R². This coefficient provides a simple way to discern if the simplified model is accurately reproducing the results of the detailed mechanistic model. In particular, the R² values for hearths 1 to 4 and 7-8 are above 0.8 in almost all cases, while the captured
variance statistic calculated for the results in hearths 5 and 6 is below 0.7. The reason for the elevated error of the simplified model obtained in Hearths 5 and 6 is the exothermic reaction actively ongoing in hearth 6 and complicating the temperature prediction in the hearth. In particular, rapid changes in the exothermic reaction rate in different volumes cause the more complicated temperature dynamics observed in Figure 5 near samples 1000, 4000 and 7000.

Figure 5. Comparison of the model predictions of the solid phase temperature

Figure 6. Comparison of the model predictions of the wall temperature

6 CONCLUSIONS

A simplified model of a MHF was developed in this paper, based on a mechanistic model developed previously. The simplified model is implemented in the form of Hammerstein-Wiener nonlinear dynamic model with a reduced number of dynamic equations and a state space structure. Thus, the obtained simplified model enables implementation of different MPCs, such as EMPC, in the developed simulation environment of the MHF.

The results of the simplified model are compared against the mechanistic model and are found to be good. For the gas and the solid phases, the results show higher accuracy, especially in the first four hearths and the last two hearths. The reason for the elevated simplified model error in hearths 5 and 6 can be the exothermic reaction actively ongoing in hearth 6 and complicating the temperature...
prediction in the hearth. This issue will be addressed in the future research to improve the simplified model performance.

**Literature**


