A Simple Strategy for Optimal Operation of Heat Exchanger Networks

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The objective of this work is to propose a systematic procedure to find a control structure for optimal operation of heat exchanger networks. Optimal operation in this context requires that 1) all controlled temperatures are kept at their targets and 2) utility cost is minimized. The degrees of freedom of heat exchanger networks are analyzed and used to identify if the operation is structurally feasible and if the utility cost can be optimized. The LP problem formulation for optimal operation of HENs implies that optimal operation is always at active constraints in terms of target temperatures and zero or maximum heat transfer (such as fully open or fully closed bypass valves). The information from an offline optimization is used to identify the set of nominally active constraints, and then split-range and selective controls are used to track the active constraints during operation for optimality.

Keywords Active constraint control; Heat exchanger networks; Optimal operation; Split-range control

1. Introduction

Heat exchanger networks (HENs) are used to internally transfer heat within the process, in which hot streams are cooled by cold streams which need to be heated and vice versa. One result is that utility consumption is reduced. However, to achieve the reduction in practice one needs a good operation strategy and this may be a challenging task. Several strategies were proposed to solve the operation of HENs. Marselle et al. (1982) proposed a method based on graph theory to suggest a control structure and developed a control policy to adjust flow distributions in the HEN to meet target temperatures with minimum utility usage. Calandranis and Stephanopoulos (1988) used the structural characteristics of HENs to identify routes to allocate loads to available sinks and developed an expert controller to select the best route. Another method based on structural information using a sign matrix was proposed by Glemmestad et al. (1996). The work based on online and periodic optimizations for the operation of HENs were studied by Aguilera and Marchetti (1998), Glemmestad et al. (1999) and González et al. (2006).

In practice, to achieve optimal operation of HENs, one needs to consider the plant as a whole. However, in order to simplify the problem, we here consider the special case of a detached HEN, where the interaction with the overall process is given by specifying target temperatures for some of the outlet streams of the HEN. Within this context optimal operation of HENs requires (1) all controlled temperatures are kept at their targets and (2) utility cost is minimized under the variation of operating conditions.

The objective of this work is to propose a systematic procedure to find a control structure for optimal operation of HENs. The assumptions in this work are 1) each process exchanger has a bypass, 2) only a single bypass is used and 3) a stream split is not used as a manipulated variable. The remaining part of this article is divided into six sections. In the second section, degrees of freedom of HENs

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are analyzed and used to identify if the operation is structurally feasible and if the utility cost can be optimized. Then the LP problem formulation for optimal operation of HENs is proposed and used to prove that the operation is an active constraint control problem. In the fourth section, the sign of directional effect of manipulations to controlled variables is defined and used to identify split-range signals. In the fifth section, a systematic procedure to find a control structure for the optimal operation of HENs is proposed. Then a HEN case study is considered in the sixth section. The last section is the conclusions.

2. Degree of freedom of HEN

The steady-state manipulated variables (or degrees of freedom, DOF) of a HEN are usually effective heat exchange (duty of each heat exchanger, \( Q_i \)) which may be indirectly changed by the use of process exchanger bypasses, utility flowrates and stream splits. Thus, the total number of degrees of freedom \( N_{DOF,\text{total}} \) is:

\[
N_{DOF,\text{total}} = N_{\text{units}} + N_{\text{splits}}
\]

(1)

Where \( N_{\text{units}} \) is the number of process exchangers and utility exchangers and \( N_{\text{splits}} \) is the number of splitters in a HEN.

Traditionally, when considering operation of HENs, it is assumed that \( N_t \) target temperatures (equality constraints) have to be satisfied. Marselle et al. (1982) defined the number of remaining degrees of freedom \( N_{DOF} \) by

\[
N_{DOF} = N_{DOF,\text{total}} - N_t
\]

(2)

\( N_{DOF,\geq 0} \) is a necessary condition for the operation of HENs to be feasible and utility cost optimizable (Theorem 4 in Marselle et al. (1982)). However, \( N_{DOF} \) calculated using equation (2) is not sufficient to identify if all target temperatures can be independently controlled and there are some remaining DOF that can be used for utility cost optimization. For example a HEN with some loops which has no utility exchanger may have \( N_{DOF,\geq 0} \) but is not controllable. A more precise definition of degrees of freedom with respect to utility cost optimization (DOF\(_U\)) which can be used to check if the operation of HENs is structurally feasible and possible to perform utility cost optimization was given by Glemmestad and Gundersen (1998). The equation is shown below:

\[
N_{DOF,U} = R + N_U - N_t
\]

(3)

where \( N_{DOF,U} \) is the number of remaining DOF\(_U\), \( R \) is the dimensional space spanned by the manipulations in the inner HEN to the outer HEN (see the calculation method in Glemmestad and Gundersen (1998)) and \( N_U \) is the number of utility types.

Glemmestad and Gundersen (1998) identified three cases:

1. \( N_{DOF,U}<0 \): The operation of the HEN is not feasible because all target temperatures cannot be controlled independently using the available manipulations.
2. \( N_{DOF,U}=0 \): The operation of the HEN is structurally feasible because all target temperatures can be controlled independently using the available manipulations. However, there is no degree of freedom available for utility cost optimization.
3. \( N_{DOF,U}>0 \): The operation of the HEN is structurally feasible because all target temperatures can be controlled independently using the available manipulations and there are some degrees of freedom for utility cost optimization.

The second and third cases \( (N_{DOF,U,\geq 0}) \) are sufficient to identify that each target temperature can be controlled independently using the available manipulations while the third case \( (N_{DOF,U,>0}) \) is sufficient to identify the availability of degrees of freedom for the utility cost optimization.
3. LP problem formulation for optimal operation of HENs

Aguilera and Marchetti (1998) focused that if stream splits are not used as manipulations and only single bypasses are used in HENs, then optimal operation of HENs can be formulated as a linear programming (LP) problem. In this section, the LP problem formulation for the optimal operation of HENs is developed as shown below:

\[ \min c^T x \]  
Subject to:
\[ Ax \leq b \]  
\[ A_{eq} x = b_{eq} \]

The elements of the variable vector \( x \) consist of the inlet and outlet temperatures of hot side (\( T_{\text{hot,in}}^i \) and \( T_{\text{hot,out}}^i \)) and cold side (\( T_{\text{cold,in}}^i \) and \( T_{\text{cold,out}}^i \)) and the duty of each exchanger (\( Q_{\text{process exchanger}}^i \), \( Q_{\text{cold utility exchanger}}^i \) and \( Q_{\text{hot utility exchangers}}^i \)). All elements of the cost vector \( c \) are zero except the elements related to the duty of utility exchangers. The equality constraints are obtained using process models and connecting conditions (with supply temperatures \( T_{s_i}^i \), target temperatures \( T_{t_i}^i \), and internal connections) while the inequality constraints are obtained using the lower and upper bounds of the duty of each exchanger. The LP problem formulation for optimal operation of HENs is shown in equation 5a-5m:

**Objective function:**
\[ \sum_i c_{ci}Q_{ci} + \sum_j c_{cj}Q_{cj} \quad i \in CU \text{, } j \in HU \]  

subject to

**Equality constraints:**

a) Process models

for process exchanger \( i \):
\[ Q_{ci} - CP_i^{\text{cold}}(T_{\text{cold,out}}^i - T_{\text{cold,in}}^i) = 0 \quad i \in PHX \]  
\[ Q_{ci} - CP_i^{\text{hot}}(T_{\text{hot,in}}^i - T_{\text{hot,out}}^i) = 0 \quad i \in PHX \]

for cooler \( i \):
\[ Q_{ci} - CP_i^{\text{hot}}(T_{\text{hot,in}}^i - T_{\text{hot,out}}^i) = 0 \quad i \in CU \]

for heater \( i \):
\[ Q_{hi} - CP_i^{\text{cold}}(T_{\text{cold,out}}^i - T_{\text{cold,in}}^i) = 0 \quad i \in HU \]

b) Connecting equations

supply connection:
\[ T_{\text{hot,in}}^i = T_{s_i}^i \quad i \in HXHS \]  
\[ T_{\text{cold,in}}^i = T_{s_i}^i \quad i \in HXCS \]

target connection:
\[ T_{\text{hot,out}}^i = T_{t_i}^i \quad i \in HXHT \cup \text{CUT} \]  
\[ T_{\text{cold,out}}^i = T_{t_i}^i \quad i \in HXCT \cup \text{HUT} \]

internal connection:
\[ T_{\text{hot,out}}^i - T_{\text{hot,in}}^j = 0 \quad i \in HXHO \text{, } j \in HXHI \]  
\[ T_{\text{cold,out}}^i - T_{\text{cold,in}}^j = 0 \quad i \in HXCO \text{, } j \in HXCI \]
Inequality constraints:

lower bound:
\[-Q_i \leq 0 \quad i \in PHX \cup CU \cup HU \]  \hspace{1cm} (5l)

upper bound (limited by thermal efficiency):
\[Q_i \leq P_{h,i} CP_{i}^{hot} (T_{i}^{hot,in} - T_{i}^{cold,in}) \quad i \in PHX \cup CU \cup HU \]  \hspace{1cm} (5m)

where

- PHX: set of all process-process heat exchangers
- CU: set of cold utility exchangers
- HU: set of hot utility exchangers
- HXHT: subset of PHX with hot side outlet is a controlled target
- HXCT: subset of PHX with cold side outlet is a controlled target
- CUT: subset of CU with outlet is a controlled target
- HUT: subset of HU with outlet is a controlled target
- HXHO: subset of PHX with hot side outlet entering a hot side inlet of the adjacent exchanger
- HXCO: subset of PHX with cold side outlet entering a cold side inlet of the adjacent exchanger
- HXHI: subset of PHX with hot side inlet coming from a hot side outlet of the adjacent exchanger
- HXCI: subset of PHX with cold side inlet coming from a cold side outlet of the adjacent exchanger
- HXHS: subset of PHX with hot side inlet directly coming from a hot supply
- HXCS: subset of PHX with cold side inlet directly coming from a cold supply

\[P_{h,i}: \text{thermal efficiency of exchanger } i, \quad P_{h,i} = \frac{NTU_{h,i} (1 - e^{NTU_{c,i} - NTU_{h,i}})}{NTU_{h,i} - NTU_{c,i} e^{NTU_{c,i} - NTU_{h,i}}} \]

\[NTU_{h,i} = \frac{(UA)_i}{CP_{i}^{hot}}, \quad NTU_{c,i} = \frac{(UA)_i}{CP_{i}^{cold}} \]

\[CP_{i}^{cold} \quad \text{and} \quad CP_{i}^{hot}: \text{heat capacity flowrate of cold and hot streams (kW/°C)} \]

\[(UA)_i: \text{product of heat transfer coefficient and heat transfer area of exchanger } i \text{ (kW/°C)} \]

As shown in the formulation, one process exchanger generates five variables (inlet and outlet temperatures of hot and cold side, and heat duty, see equation 5b-5c) while one utility exchanger generates three variables (inlet and outlet temperatures and heat duty, see equation 5d-5e). Therefore, for a HEN containing \(N_{hx}\) process exchangers, \(N_{cu}\) coolers and \(N_{hu}\) heaters, the number of variables (\(N_{var}\)) can be written:

\[N_{var} = 5N_{hx} + 3N_{cu} + 3N_{hu} \]  \hspace{1cm} (6)

Considering to the number of the equality constraints, one process exchanger generates two equality constraints (removed heat on hot side and received heat on cold side, see equation 5b-5c) while one utility exchanger generates one equality constraint (removed heat to a cooler or received heat from a heater, see equation 5d-5e). The number of connecting equations come from the number of supply specification (\(N_{s}\) see equation 5f-5g), the number of target specification (\(N_{t}\) see equation 5h-5i) and the number of internal variable connection between the adjacent heat exchangers (\(N_{int,connect}\) equation 5j-5k). Therefore, the number of equality constraints (\(N_{eq}\)) can be written:

\[N_{eq} = 2N_{hx} + N_{cu} + N_{hu} + N_{s} + N_{t} + N_{int,connect} \]  \hspace{1cm} (7)
For the number of the inequality constraints, each process exchanger and utility exchanger generate two inequality constraints (see equation 51-5m) and hence the number of inequality constraints ($N_{ineq}$) can be written:

$$N_{ineq} = 2(N_{in} + N_{cu} + N_{hs})$$  \hspace{1cm} (8)

With the formulation problem in equation 5a-5m, it is clear that optimal operation of HENs is a LP problem. An important property of a LP problem is that one optimal solution is always in a “corner” that implies it is optimal to use all degrees of freedoms to satisfy active constraints.

Example: A trivial HEN

The HEN in Figure 1 contains one process exchanger and two utility types ($N_{u}$=2) and has two controlled outlet temperatures ($N_{t}$=2). The dimensional space spanned by the manipulations in the inner HEN to the outer HEN ($R$) is equal to 1 (see the calculation method in Glommestad and Gundersen, 1998). Using equation (3), we have $N_{DOF,U}=1+2-2=1$. This implies there is one remaining degree of freedom for utility cost optimization. The following explanation is used to explain the meaning of $N_{DOF,U}$.

Figure 1 A trivial HEN

The required amount of heat for cooling hot stream ($Q_{h}^{tot}$) from the inlet temperature 190 °C to the desired outlet temperature 100 °C is the summation of the amount of heat exchange at exchanger 1 ($Q_{1}$) and cooler ($Q_{c}$) as shown in equation (9). The required amount of heat for heating cold stream ($Q_{c}^{tot}$) from the inlet temperature 40 °C to the desired outlet temperature 80 °C is the summation of amount of heat exchange at exchanger 1 and heater ($Q_{h}$) as shown in equation (10).

$$Q_{1} + Q_{c} = Q_{h}^{tot} = 20(190-100) = 1800 \text{ kW}$$  \hspace{1cm} (9)

$$Q_{1} + Q_{h} = Q_{c}^{tot} = 50(80-40) = 2000 \text{ kW}$$  \hspace{1cm} (10)

Subtracting equation (10) from (9),

$$Q_{h} - Q_{c} = 200 \text{ kW}$$  \hspace{1cm} (11)

Equation (11) implies $Q_{h}$ is 200 kW more than $Q_{c}$. To minimize utility cost, $Q_{c}$ should be as low as possible. If the bypass of exchanger 1 ($u_{hi}$) is fully closed, we have $Q_{1} = 1827$ kW which is more than the required $Q_{h}^{tot}$. Therefore, $Q_{c}$ is set to zero to minimize utility cost while $u_{hi}$ is set at the value to have $Q_{1}=Q_{h}^{tot}=1800$ kW. In this case $Q_{h}$ and $u_{hi}$ are used for regulatory control while $Q_{c}$ is served as the remaining DOF$_{U}$ ($N_{DOF,U}=1$) at a constraint (zero utility load) for optimality.

The network is considered again with increasing the inlet temperature of hot stream to 200 °C. The required amount of heat for cooling hot stream and heating cold stream are shown in equation (12) and (13) respectively.

$$Q_{1} + Q_{c} = Q_{h}^{tot} = 20(200-100) = 2000 \text{ kW}$$  \hspace{1cm} (12)

$$Q_{1} + Q_{h} = Q_{c}^{tot} = 50(80-40) = 2000 \text{ kW}$$  \hspace{1cm} (13)
Subtracting equation (13) from (12),

\[ Q_h - Q_c = 0, \text{ or } Q_h = Q_c \tag{14} \]

Equation (14) implies \( Q_h \) is equal to \( Q_c \). However, if the bypass of exchanger 1 is fully closed \((u_{b1}=0)\), we have \( Q_1\text{=}1949 \text{ kW} \) which is less than the required \( Q_{H\text{tot}} \) (or \( Q_{C\text{tot}} \)). Therefore, the possible lowest amount of \( Q_c \) is 51 kW to have \( Q_{C\text{tot}}\text{=}2000 \text{ kW} \) while \( u_{b1} \) is set to zero to maximize heat integration (or minimize utility cost). In this case, \( Q_h \) and \( Q_c \) are used for regulatory control while \( u_{b1} \) is served as the remaining DOF \( U(N_{DOF,U}\text{=}1) \) at a constraint (fully closed bypass) for optimality.

Again, if the inlet temperature of hot stream increases to 210 °C, we have

\[ Q_1 + Q_c = Q_{H\text{tot}} = 20(210-100) = 2200 \text{ kW} \tag{15} \]
\[ Q_1 + Q_h = Q_{C\text{tot}} = 50(80-40) = 2000 \text{ kW} \tag{16} \]

Subtracting equation (15) from (16),

\[ Q_c - Q_h = 200 \text{ kW} \tag{17} \]

Equation (17) implies \( Q_c \) is 200 kW more than \( Q_h \). To minimize utility cost, \( Q_h \) should be as low as possible. If the bypass \( u_{b1} \) is fully closed, we have \( Q_1\text{=}2071 \text{ kW} \) which is more than the required \( Q_{C\text{tot}} \). Therefore, \( Q_h \) is set to zero to minimize utility cost while \( u_{b1} \) is set at the value to have \( Q_1\text{=}2000 \text{ kW} \). In this case \( Q_c \) and \( u_{b1} \) are used for regulatory control while \( Q_h \) is served as the remaining DOF \( U(N_{DOF,U}\text{=}1) \) at a constraint (zero utility load) for optimality.

The example shows the cases that \( Q_c, Q_1(u_{b1}) \) and \( Q_h \) perform the remaining DOF \( U \) at their constraints for optimality. The result supports that optimal operation of HENs lies at constraints.

4. Split-range control for optimal operation of HENs

Split-range control is a well-known constraint control technique used for a system which has excess manipulations (number of manipulations is more than number of controlled variables). With split-range control when a manipulation is saturated (at a constraint), another manipulation is activated. Assuming two manipulations \( (MV_1 \text{ and } MV_2) \) are combined as a split-range control to control a controlled variable \( (CV) \). If we define the directional effect of a manipulation to a controlled variable by a sign element:

\[
[-] = \text{ increasing MV decreases CV (or decreasing MV increases CV)} \\
[+] = \text{ increasing MV increases CV (or decreasing MV decreases CV)} \\
[\pm] = \text{ increasing (or decreasing) MV may increase or decrease CV} \\
[0] = \text{ increasing (or decreasing) MV have no effect to CV}
\]

and the multiplication of sign elements:

\[
[-],[+] = [-] \\
[+],[-] = [+] \\
[-],[-] = [-] \\
[+],[0] = [+] \\
[0],[0] = [\pm],[0] = [0] \\
[\pm],[+] = [\pm],[+] = [\pm]
\]

then the relationship between the directional effect of manipulations to controlled variables and split-range signal can be shown in Table I.
Table I The relationship between the directional effect of MV to CV and split-range signal

- **MV<sub>1</sub> and MV<sub>2</sub> have opposite directional effect to CV**
  (Multiplication result of directional effect of MV<sub>1</sub> and MV<sub>2</sub> to CV is [-])

**Type I** Lower constraint protection

This split-range combination happens when two manipulations switch to their lower constraints. (such as after fully closing a bypass, a utility load is used).

**Type II** Upper constraint protection

This split-range combination happens when two manipulations switch to their upper constraints. (such as after fully opening a bypass and a utility load is decreased).

- **MV<sub>1</sub> and MV<sub>2</sub> have same directional effect to CV**
  (Multiplication result of directional effect of MV<sub>1</sub> and MV<sub>2</sub> to CV is [+])

**Type III** Lower constraint protection

This split-range combination happens when two manipulations switch to their constraints by MV<sub>1</sub> at lower constraint and MV<sub>2</sub> at upper constraint.

**Type IV** Upper constraint protection

This split-range combination happens when two manipulations switch to their constraints by MV<sub>1</sub> at upper constraint and MV<sub>2</sub> at lower constraint.

- **MV<sub>2</sub> cannot affect to the paired CV of MV<sub>1</sub>**
  (Multiplication result of directional effect of MV<sub>1</sub> and MV<sub>2</sub> to CV is [0])

No split-range combination is needed.

- **The directional effect of MV<sub>2</sub> to the paired CV of MV<sub>1</sub> is unclear**
  (Multiplication result of directional effect of MV<sub>1</sub> and MV<sub>2</sub> to CV is [±])

Split-range signal is unclear.

*Note* MV<sub>1</sub> (—)=the input used under “normal” conditions (i.e. not active constraint under normal conditions) , MV<sub>2</sub> (•••)=the input unused under “normal” conditions (i.e. active constraint under normal conditions), and CV=controlled variable.
5. Control structure design for optimal operation of HENs

As described in the previous section, the LP problem formulation implies optimal operation of HENs is an active constraint control problem. Therefore, after some degrees of freedom are used for regulatory control, it is optimal to use all free degrees of freedom to satisfy active constraints such as fully close of bypasses or utility loads. In more specific implication, after some DOF are used for regulatory control, it is optimal to keep $N_{DOF,U}$ DOF at constraints for optimality. A method to handle an active constraint control was proposed by Arkun and Stephanopoulos (1980). However, their method requires an online optimization strategy. In most cases, an active constraints control problem can be handled using a split-range control strategy. In this section, a procedure for a control structure design based on using split-range and selective controls for optimal operation of HENs is proposed. The obtained control structure is decentralized and easy to understand by plant operators. The procedure is shown below:

1. Calculate $N_{DOF,U}$ by using equation (3).
   1.1 For $N_{DOF,U} < 0$, the operation of the HEN is not feasible. The HEN should be redesigned.
   1.2 For $N_{DOF,U} = 0$, the operation of the HEN is structurally feasible but the utility cost cannot be optimized because there is no DOF available. The utility cost is fixed by equality constraints (target temperatures).
   1.3 For $N_{DOF,U} > 0$, the operation of the HEN is structurally feasible and the utility cost can be optimized because there are some DOF for utility cost optimization. The active constraints should be rightly controlled for optimality.

   Go further to step 2 if $N_{DOF,U} > 0$.

2. Formulate a utility cost optimization problem of the HEN by using equation 5a-5m and solve the problem for the nominal case.
   2.1 Nominally inactive manipulations (nominally inactive constraints) are used as primary manipulations for system pairings. If the number of nominally inactive manipulations is less than the number of controlled variables, some nominally active manipulations (nominally active constraints) are selected as additional primary manipulations for system pairings.
   2.2 Free nominally active manipulations are used as secondary manipulations to protect primary manipulations from saturation.

3. Find the directional effect of each primary manipulation to the paired controlled-variable and the directional effect of each secondary manipulation to all controlled variables, and then use Table 1 to generate split-range signal to see which secondary manipulation can be used to protect which primary manipulation from saturation at which constraint (lower or upper constraints).
   3.1 If a secondary manipulation can protect only one primary manipulation, a split-range control is used.
   3.2 If a secondary manipulation can protect more than one primary manipulation, a selective control is required.

4. Use the information in step 3 for the control structure design.

The proposed procedure cannot guarantee the optimality when the directional effect of some secondary manipulations is unclear and there is possibly more than one secondary manipulation to protect a primary manipulation from saturation at a constraint (such as two secondary manipulations can protect a primary manipulation from saturation at the lower constraint). The first case may happen when there are some loops in the HEN which causes opposite effects from manipulations to controlled variables while the second case may happen when there are several secondary manipulations and active constraints change very often. In the case that the proposed procedure cannot guarantee the optimality, the additional information from an offline optimization with expected perturbations is suggested for helping the control structure design.
6. Case study of HENs

The HEN shown in Figure 2 comes from the work of Glømestad et al. (1999). The HEN contains two process exchangers and two utility types ($N_U = 2$). There are three target temperatures ($N_T = 3$) which are the outlet temperatures of stream H1 ($T_{H1}^{\text{out}}$), stream C1 ($T_{C1}^{\text{out}}$) and stream C2 ($T_{C2}^{\text{out}}$). The manipulations are bypasses of exchanger 1 and 2 ($u_{b1}$ and $u_{b2}$) and utility loads of cooler and heater ($Q_c$ and $Q_h$). A control structure for optimal operation of this HEN can be obtained by the following steps:

![Figure 2 A HEN case study (Glømestad et al., 1999)](image)

Step 1: The dimensional space spanned by the manipulation in the inner HEN to the outer HEN ($R$) is equal to 2. By using equation (3), we have $N_{DQ,CL} = 2 + 2 - 3 = 1$ which implies the operation of this HEN is structurally feasible and there is one remaining degree of freedom for utility cost optimization.

Step 2: Formulate a LP problem for optimal operation of this HEN by using equation 5a-5m and solve the problem. The optimal values of manipulations are:

<table>
<thead>
<tr>
<th>$Q_c$ (kW)</th>
<th>$Q_h$ (kW)</th>
<th>$u_{b1}$</th>
<th>$u_{b2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>81</td>
<td>0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The result shows there are three nominally inactive manipulations ($Q_c$, $Q_h$ and $u_{b2}$) which should be used as primary manipulations. For direct control effect, the pairing should be using $Q_c$ to control $T_{H1}^{\text{out}}$ and $Q_h$ to control $T_{C1}^{\text{out}}$ and $u_{b2}$ to control $T_{C2}^{\text{out}}$. Therefore, $u_{b1}$ is the available secondary manipulation.

Step 3: Considering to the directional effect of primary manipulations to controlled variables, increasing $Q_c$ decreases $T_{H1}^{\text{out}}$, increasing $Q_h$ increases $T_{C1}^{\text{out}}$ and increasing $u_{b2}$ decreases $T_{C2}^{\text{out}}$. For the directional effect of secondary manipulations to controlled variables, increasing $u_{b1}$ increases $T_{H1}^{\text{out}}$ and $T_{C2}^{\text{out}}$ while decreases $T_{C1}^{\text{out}}$. These results are concluded in Table II. By using the information from Table I and II, the split-range signal can be shown in Table III.

<table>
<thead>
<tr>
<th>Controlled variable</th>
<th>Primary MV</th>
<th>Secondary MV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{H1}^{\text{out}}$</td>
<td>$Q_c$</td>
<td>$u_{b2}$</td>
</tr>
<tr>
<td>$T_{C1}^{\text{out}}$</td>
<td>$Q_h$</td>
<td>$+$</td>
</tr>
<tr>
<td>$T_{C2}^{\text{out}}$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Step 4: The result from step 3 shows $u_{b2}$ can be used to protect all primary manipulations from saturation at the lower constraint using a split-range manner. However, because $u_{b1}$ can protect more than one primary manipulation, a selective control is additionally required. The obtained control structure is shown in Figure 3.
**Optimal operation of HENs**

<table>
<thead>
<tr>
<th>Primary MV</th>
<th>$u_{b1}$ multiplication of sign</th>
<th>Split-range signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_c$</td>
<td>$-$</td>
<td></td>
</tr>
<tr>
<td>$Q_h$</td>
<td>$-$</td>
<td></td>
</tr>
<tr>
<td>$u_{b2}$</td>
<td>$-$</td>
<td></td>
</tr>
</tbody>
</table>

Table III The split-range signal of manipulations

The port 1 of the split-range temperature control (SR-TC) block represents the signal to the primary manipulation while the port 2 represents the signal to the secondary manipulation. The high signal selective control (HSS) block is used to select the signal from the port 2 of three SR-TC blocks when a primary manipulation is saturated.

The obtained control structure was tested by performing dynamic simulation of the HEN on Aspen Dynamics v12.1. The network flowsheet was firstly created by Aspen Plus, and then was exported to Aspen Dynamics in Flow Driven mode. The disturbances are the decrease of inlet temperature of hot stream H1 to 185 °C at time 5 seconds, and the increase to 195 °C at time 15 seconds as shown in Figure 4a. The response of controlled temperatures and utility consumption are shown in Figure 4b and 4c respectively. Figure 4d shows the effect of disturbances to the bypasses of exchanger 1 and 2. At the time 5 seconds when the inlet temperature of hot stream H1 decreases to 185 °C, the heat duty of exchanger 2 is not high enough to keep the outlet temperature of stream C2 at the target even after fully closing the bypass $u_{b2}$ ($u_{b2}$ is saturated at the lower constraint). Therefore, the control structure switches to use the bypass $u_{b1}$ to control the outlet temperature $T_{C1}^{out}$ instead. Then the bypass $u_{b1}$ is partially opened to increase the duty of exchanger 2. In this perturbation, the bypass $u_{b2}$ is served as the remaining DOF$_U$ ($N_{DOF,U}=1$) at the constraint for optimality. At the time 15 seconds when the inlet temperature of hot stream H1 increases to 195 °C, the heat duty of exchanger 2 is over the requirement. Therefore, the bypass $u_{b1}$ is gradually closed to decrease the excess duty of exchanger 2. Nevertheless after fully closing the bypass $u_{b1}$ ($u_{b1}$ is saturated at the lower constraint), the duty of exchanger 2 is still over the requirement. As a result, the control structure switches back to use the bypass $u_{b2}$ to control the outlet temperature $T_{C1}^{out}$. Then the bypass $u_{b2}$ is partially opened to decrease the duty of exchanger 2. In this perturbation, the bypass $u_{b1}$ is served as the remaining DOF$_U$ ($N_{DOF,U}=1$) at the constraint for optimality. The optimality is confirmed by Figure 4c which shows the control structure can bring $Q_c$ and $Q_h$ to the optimal line.
7. Conclusions

The LP problem formulation for optimal operation of HENs implies the operation is an active constraint control problem. By tracking the right active constraints, the optimality and feasibility are obtained. A systematic procedure to suggest a control structure for optimal operation of HENs is proposed. An offline optimization in the nominal case is used to identify the set of primary and secondary manipulations. The sign of directional effect of manipulations to controlled variables is used to suggest the required split-range and selective controls for tracking the active constraints during the operation.

The proposed procedure suggests only one optimal control structure. For some HENs the optimal solution is not unique, there may be more than one optimal control structure. Hence, additional considerations (such as dynamic consideration) are needed to select the best control structure. This is the topic for ongoing research.

8. Acknowledgement

Financial support from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program (Grant No. PHD/0145/2547) and the support from the department of chemical engineering, Norwegian University of Science and Technology, Trondheim, Norway during a visit of Mr. Veerayut Lersbamrungsuk are gratefully acknowledged.

9. Reference

Optimal operation of HENs


