Gait Planning and Compliance Control of a Biped Robot on Stairs with Desired ZMP

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Abstract: For biped robot, friendly environment interaction makes sense. In this paper, a new method to plan the gait and control a biped robot on stairs with desired ZMP is proposed first. The desired ZMP derived from an iterative optimal algorithm not only has enough stability margin and satisfies actuator specifications, but also is available to implement and energy saving. Then, a controller with force sensing and variable impedance is proposed, which can sense and compensate the environmental disturbance well. Finally, the effectiveness of the proposed methods is confirmed by simulation examples.

Keywords: Biped robot, Gait planning, Desired ZMP, Compliance control

1. INTRODUCTION

Nowadays, biped robot that capture the mobility, autonomy and speed of living creatures has become a hot research. Some actual biped robots design and application have been mentioned in recent years, such as WABIAN-II (Ogura et al. [2006]), HRP-4 (Kaneko et al. [2009]) and Lola (Lohmeier et al. [2009]). In the biped robot field, many related issues have been studied.

The design (Pratt et al. [2008]), model (Lee et al. [2010]) and control (Hurmuzlu et al. [2006]) are the basic research fields of biped robot, and various methods have been proposed. One of the significant breakthroughs is the introduction of the zero moment point (ZMP) by Vukobratovic et al. [1972], which had been shown to provide effective, robust, and versatile locomotion for biped robots (Chevalerieau et al. [2008]). To generate or plan a proper biped gait, most studies concentrate on guaranteeing ZMP criterion from given hip trajectory. In some literature, the hip trajectory was given out by taking the biped robot as an inverted pendulum (Chen et al. [2013]). In other literature, the hip trajectory was given out by iterating to satisfy the ZMP criterion (Chevalier et al. [2008]). However, these investigations didn’t focus on dealing with the relationship among stability margin, actuator specifications and energy consumption. In this paper, we employ an iterative optimal algorithm to get a desired ZMP that synthesizes all the three aspects: stability margin, realizability and energy saving.

Under ideal conditions, the biped robot can walk well after gait planning. Yet, it is difficult to control its walking patterns (Hong et al. [2014], Shuhei et al. [2013]) and instability (Fumihiko [2013]) in the real world. Since the existence of environment interaction, the biped robot needs a controller to make adjustment to adapt the varied environments, especially a learning optimal controller (Satoshi et al. [2013]). To make biped robot walk stable, the environmental disturbance and the impact between foot and ground should be sensed and compensated. In this case, friendly environment interaction is essential for biped robot. In other words, the biped walking should be compliant. Therefore, impedance control was introduced.

Impedance control is a relatively new issue in robotic control. For biped balancing, Hyon et al. [2006] used a control approach of joint torque sensing based compliance control, which led to very robust behavior in balancing. These results on biped balancing verified the robustness properties of robot manipulators with similar controllers (Ott et al. [2008], Schaffer et al. [2007], Wimbock et al. [2006]). For environment interaction, Yang et al. [2011] proposed a novel human-like learning controller to interact with unknown environments. The controller was derived from the minimization of instability, effort, and motion error strictly. By adapting feedforward force and impedance, it compensated for the disturbance in the environment in interaction tasks. For biped robot locomotion, Park. [2001] used impedance controller to control both legs, but the damping coefficient of the impedance didn’t change smoothly. Therefore, a controller with force sensing and variable impedance is proposed based on these literature.

The paper is organized as follows. The model and walking cycle of the biped robot are described in Section 2. In Section 3, a new method to plan the gait of the biped robot with designated desired ZMP is proposed. In Section 4, the whole controller with force sensing and variable impedance is designed. The simulation results and actuator specifications are provided and discussed in Section 5. Finally, conclusions are given in Section 6.
2. MODEL OF THE BIPED ROBOT

The biped robot was considered as a multi-link model (Fig. 1) in this paper. And it was made to have a total 10 DOF for simplicity. Each leg has 5 DOF: two joints at the ankle, one joint at the knee, and two joints at the hip.

Like human, the walking of biped robot has a walking cycle (Fig. 2). Actually, each leg has a walking cycle, and there is a whole half periodic time difference ($T/2$) between the left and right leg. Generally, the walking cycle of each leg has three phases: a support phase ($SUP$), a swing phase ($SWP$), and a weight loading phase ($WLP$). As Fig. 2 shown, these three phases represents three kinds of different interactions with ground.

The proportions of these three phases in the whole walking cycle are also an important part to analyze. Since the interval of the weight loading phase $T_2$ in human locomotion is about 10% – 30%, we specify it ($T_2 = T/8$) here.

From the perspective of Human-simulated and simplifying the calculation, we make three assumptions as follows:

**Assumption 1** The torso of the biped robot is always vertical to the level ground.

**Assumption 2** The bottom of foot is always parallel to the level ground.

**Assumption 3** The biped robot can be simplified as a center of mass on the upper body.

3. GAIT PLANNING OF THE BIPED ROBOT ON STAIRS

On stairs, the biped robot should be kept balance by a proper type of ankle motion and hip motion. If both ankle trajectories and the hip trajectory are known, using kinematic constraints will derive all joint trajectories of the biped robot. Therefore, both ankle trajectories and the hip trajectory can denoted the walking pattern uniquely. In this paper, through the formulated ground constraints and the designated desired ZMP, ankle trajectories and the hip trajectory can be derived by polynomial interpolation. Fig. 3 shows the walking parameters of the biped robot.

3.1 Ankle trajectories

On the stairs terrain, the ground constraints of the ankle trajectories have 4 main factors: a) one step length ($S$); b) the feet should begin to touch the stairs with enough small or zero speed. If the speed is a little too high, it will lead to a shock; c) the feet should begin to touch the stairs with enough small or zero acceleration. If the acceleration is a little too high, it will affect the normal motion of the hip and torso; d) it is necessary to avoid the swing foot to hit the stair edges and lift the swing foot high enough to negotiate obstacles. Thus, the ankle trajectories can be generated by polynomial interpolation (1), which satisfies the second derivative continuity conditions:

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n.$$  (1)

where $n+1$ can be represented as the number of the ground constraints.

Above all, we can give out all the ground constraints in a single cycle. The ground constraints on the X axis:
Specify walking speed and step length.
Specify ankle trajectory.
Design ZMP trajectory in the stable region.
Generate hip trajectory by designated ZMP.
Compute joints’ angular velocity and angular acceleration.
Satisfy actuator specifications?
Satisfy energy saving requirements?
Decrease stability margin?
Select the hip trajectory with the largest stability margin?
Y Y N N Y N N Y

Fig. 4. Algorithm for desired ZMP.

\[
x''(T_2) = 0
x''(T) = 0
x'(T_2) = 0
x'(T) = 0
x(T_2) = -S
x(T) = S
\]

(2)

The ground constraints on the Y axis:

\[
y(t) = L_3, 0 < t < T_2
\]

\[
y(t_1) = -S/2 + L_4/2
y(t_2) = h_1
y''(T_2) = 0
y''((T + T_2)/2) = 0
y''(T) = 0
y''((T + T_2)/2) = 0
y'(T) = 0
y'((T + T_2)/2) = 0
y(T) = L_3
y'(T + T_2)/2 = H + L_3
\]

(3)

where \(h_0\) and \(h_1\) are the height margin to negotiate obstacles.

3.2 Hip trajectory

In this paper, we first design a desired ZMP trajectory, then the hip trajectory or torso motion required to achieve the ZMP trajectory can be derived. One of the advantage of this approach is that the stability margin can be very large if the desired ZMP is designed near the center of the stable region. Although, in this case, not all desired ZMP trajectories can be achieved due to the fact that hip motion is limited, and the hip acceleration may need to be large to achieve a desired ZMP trajectory, and the energy consumption increases, we can employ iteration to get a desired ZMP that synthesizes all the three aspects (Fig. 4): stability margin, realizability and energy saving. It is certain that the designated desired ZMP must satisfy the second derivative continuity conditions.

Here we give a method to get hip trajectory based on a desired ZMP. Assume the center of mass of the biped robot is \(p_c = [x_c, y_c, z_c]^T\), the ZMP is \(p = [x_p, y_p, z_p]^T\), and the ground reaction force on the supporting point is \(f\). Then the ground reaction torque around the origin is \(\tau\):

\[
\tau = p \times f + \tau_p.
\]

where \(\tau_p\) is the torque on the ZMP.

By the momentum theorem and the angular momentum theorem, we can get

\[
P = Mg + f. \quad (7)
\]

\[
L = p_c \times Mg + \tau. \quad (8)
\]

Associate (6), (7), (8), we can get

\[
\tau_p = \dot{L} - p_c \times Mg + (P - Mg) \times p. \quad (9)
\]

Make the horizontal component of \(\tau\), \(\tau_{px}\) and \(\tau_{pz}\) equal zero. Then through the ZMP principle, we can get the coordinate of ZMP:

\[
x_p = \frac{Mgx_c + z_p \dot{P}_z - \dot{L}_z}{Mg + \dot{P}_y}
\]

\[
y_p = 0
\]

\[
z_p = \frac{Mgz_c + z_p \dot{P}_z + \dot{L}_z}{Mg + \dot{P}_y}
\]

(10)

From the Assumption 3, the momentum theorem and the angular momentum theorem of the biped robot are:

\[
\begin{align*}
P &= M\ddot{p}_c = M[x_c, y_c, z_c]^T \\
L &= p_c \times M\dot{p}_c
\end{align*}
\]

(11)

Substitute (11) into (10), we can get the relationship between the ZMP trajectory and hip trajectory:

\[
x_p = x_c - \frac{y_c - y_p \ddot{y}_c}{\ddot{y}_c + g}
\]

\[
z_p = z_c - \frac{y_c - y_p \ddot{y}_c}{\ddot{y}_c + g}
\]

(12)

where \(y_c\) has the following constraints:

\[
y_c(t) = \begin{cases} H_{\text{min}}, & 0 < t < T_2 \\ H_{\text{max}} + H/2, & t = (T - T_2)/2 \\ H_{\text{min}} + H, & T < t < T + T_2 
\end{cases}
\]

(13)

where \(H_{\text{min}}\), \(H_{\text{max}}\) is respectively the lowest and highest position of center of mass \(y_c\) on the plane terrain. Meanwhile, the desired \(y_c\) should also satisfy the second derivative continuity conditions.

If the desired ZMP is designated, the center of mass trajectory can be known. From Fig. 3, we know

\[
x_{ck} = x_c + x_{ck}
y_{ck} = y_c + y_{ck}
\]

(14)

where \(y_{ck}\) can be specified to be constant, or to vary within a small fixed range. In this way, the hip trajectory \((x_{ck}, y_{ck})\) can be obtained.

So far, we have got ankle trajectories and hip trajectory. By using kinematic constraints, we can calculate the six joint angles we need to control the biped robot: \(\theta_{zh}, \theta_{zx}, \theta_{zk}, \theta_{bh}, \theta_{bk}, \theta_{bk}\).

Through the same method, we can get the last four joint angles (Fig. 5). From the Assumption 1 and Assumption 2, we know the four joint angles are equal or are opposite. Thus, \(\theta_{zc}\) is the last joint angle needed to be worked out.
The dynamics of the biped robot in Fig. 1 is formulated below:

\[ M_l \ddot{q}_l + C_l \dot{q}_l + G_l + E_l \ddot{x}_c = \tau_l + A_l F_l \] (15)

\[ M_r \ddot{q}_r + C_r \dot{q}_r + G_r + E_r \ddot{x}_c = \tau_r + A_r F_r \] (16)

\[ N_l \ddot{q}_l + N_r \ddot{q}_r + D_q \dot{q}_l + D_q \dot{q}_r + G + E \ddot{x}_c = B_l \dot{F}_l + B_r \dot{F}_r \] (17)

where \( M_j, E_j, N_j, E \in \mathbb{R}^{5 \times 5} (j = l, r) \) are the inertia-related metrics of the left and right legs and the torso, \( C_j, D_j \in \mathbb{R}^{5 \times 5} (j = l, r) \) are the centrifugal and Coriolis terms of the left and right legs, \( A_j, B_j \in \mathbb{R}^{5 \times 5} (j = l, r) \) are the Jacobian matrices of the left and right feet, \( G_j, G \in \mathbb{R}^5 (j = l, r) \) are the gravitational term of the left and right legs and the torso, \( q_j, x_c \in \mathbb{R}^5 (j = l, r) \) are the joint displacement of the left and right leg, \( F_j, \tau_j \in \mathbb{R}^5 (j = l, r) \) are the force and torque of the left and right leg, \( F_l, \tau_l \) are the force vector on the left foot, joint driving torque of the left leg and right leg, respectively.

The above equations of the biped robot include the dynamics of the center of mass (17), coupled with the dynamics of the legs (15) and (16).

Assume \( E \) is invertible, we can obtain \( \ddot{x}_c \) from (17) and substitute it in (15) and (16) to get two more familiar dynamic equations:

\[ (M_l - E_l E_l^{-1} N_l) \ddot{q}_l - E_l E_l^{-1} N_r \ddot{q}_r + (C_l - E_l E_l^{-1} D_l) \dot{q}_l + E_l E_l^{-1} D_r \dot{q}_r + G_l - E_l E_l^{-1} G \] (18)

\[ = \tau_l + (A_l - E_l E_l^{-1} B_l) \dot{F}_l - E_l E_l^{-1} B_r \dot{F}_r \]

\[ (M_r - E_r E_r^{-1} N_r) \ddot{q}_r - E_r E_r^{-1} N_l \ddot{q}_l + (C_r - E_r E_r^{-1} D_r) \dot{q}_r - E_r E_r^{-1} D_l \dot{q}_l + G_r - E_r E_r^{-1} G \] (19)

\[ = \tau_r + (A_r - E_r E_r^{-1} B_r) \dot{F}_r - E_r E_r^{-1} B_l \dot{F}_l \]

Since the periodicity of biped walking, for simplicity, the compliance control laws of the legs will be only derived in the case that the left leg is in the WLP or the SWP and the right leg is in the SUP.

4.1 Compliance Control of the Left Leg in the SWP and the WLP

As known, the foot velocity of the left leg \( \dot{x}_{lf} \in \mathbb{R}^5 \) related to the joint angular velocity of the left leg \( \ddot{q}_l \) is

\[ \dot{x}_{lf} = \dot{x}_c + J_l \ddot{q}_l \] (20)

where \( J_l \) is the Jacobian of the left foot with respect to the center of mass.

From the differentiation of (20), we have

\[ \ddot{q}_l = J_l^{-1} \left( \ddot{x}_{lf} - \dot{x}_c - \dot{J}_l \ddot{q}_l \right) \] (21)

Configure the desired impedance of the left leg as follow

\[ P_l(\ddot{x}_{lf} - \ddot{x}_{lf}) + Q_l(\dot{x}_{lf} - \dot{x}_{lf}) + K_l(x_{lf} - x_{lf}) = F - F_{ld} \] (22)

where \( P_l, Q_l, K_l \) are the desired mass, damping ratio and stiffness of the left leg, and \( K_l \) is proportional to \( F \), subscripts ‘d’ indicates the desired value. \( F \) is the resultant external force. The desired reference force \( F_{ld} \) is

\[ F_{ld} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \text{ in the WLP} \]

\[ F_{ld} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ in the SWP} \] (23)

where \( mg \) is the weight of the biped robot.

Combining (21), (22) and (23) gets the joint torque of the left leg

\[ \tau_l = M_l J_l^{-1} (\ddot{x}_{lf} - \ddot{x}_{lf}) - P_l^{-1} K_l (x_{lf} - x_{lf}) + P_l^{-1} (F - F_{ld}) - \dot{x}_c - \dot{J}_l \ddot{q}_l \] (24)

4.2 Compliance Control of the Right Leg in the SUP

Combining (15), (16) and (17) gets

\[ \ddot{E} \ddot{x}_c + \ddot{D}_l \ddot{q}_l + \ddot{D}_r \ddot{q}_r + \ddot{G} = N_r M_r^{-1} \tau_r + B_l \ddot{F}_l + B_r \ddot{F}_r \] (25)

where

\[ \ddot{E} = (N_l M_l^{-1} E_l + N_r M_r^{-1} E_r - E) \]

\[ \ddot{D}_l = (N_l M_l^{-1} C_l - D_l) \]

\[ \ddot{D}_r = (N_r M_r^{-1} C_l - D_r) \]

\[ \ddot{G} = N_l M_l^{-1} (G_l - \tau_l) + N_r M_r^{-1} G_r - G \] (26)

\[ \ddot{B}_l = (N_l M_l^{-1} A_l - B_l) \]

\[ \ddot{B}_r = (N_r M_r^{-1} A_l - B_r) \]

Configure the desired impedance of the torso as follow

\[ P_r(\ddot{x}_c - \ddot{x}_{cd}) + Q_r(\dot{x}_c - \dot{x}_{cd}) + K_r(x_c - x_{cd}) = 0 \] (27)

where \( P_r, Q_r, K_r \) are the desired mass, damping ratio and stiffness of the torso.

Combining (25) and (27) gets the joint torque of the right leg

\[ \tau_r = (N_r M_r^{-1})^{-1} \left[ \ddot{E} \ddot{x}_c - P_r^{-1} Q_r (\dot{x}_c - \dot{x}_c) - P_r^{-1} K_r (x_c - x_{cd}) \right] + \ddot{D}_l \ddot{q}_l + \ddot{D}_r \ddot{q}_r + \ddot{G} \] (28)

The whole controller is shown in Fig. 6, which includes the top level controller to track the desired ZMP and the underlying controllers with force sensing and variable impedance to track the servo objectives \( q_d \).

Actually, every DOF of biped robot is actuated by one servo system, we take the right ankle as an example. Owing to the event that the front foot begins to touch the stairs or the event that the rear foot begins to leave the stairs happening, the actuator is disturbed dramatically by it. Therefore, a force sensing mechanism is added here to sense the events and can be treated as an event trigger.
to tell the controller it is time to make adjustment, such as changing the stiffness $K$ value with respect to $F$. At the same time, variable impedance is used to compensate the periodic disturbance to achieve high tracking accuracy. The variable impedance can make actuators adapt the environments. When the disturbance event happens, the controller will study it and give out a periodic compensation to eliminate the tracking error. In short, the impedance in SWPs is the lowest, and in order to absorb impact energy in foot landings, a higher impedance is used in SUPs than that in WLPs. As for energy saving, we can lower impedance and control accuracy in SWPs appropriately.

The result of the controllers can be shown in Fig. 7 and Fig. 8. Fig. 7 shows the result of the controller without force sensing and variable impedance. For the periodic disturbing event that the front foot begins to touch the stairs or that the rear foot begins to leave the stairs cannot be known in time or foreknown, the biped walking is dramatically influenced. The tracking ZMP is very close to the edge of the stable region. In this case, the biped robot will tip over easily in unknown environments. This case is not allowed to occur and an enough large stability margin is essential. Also, under the influence of the disturbance, the tracking accuracy of ZMP is not so well in other time. Fig. 8 shows the result of the controller with force sensing and variable impedance. Compared with Fig. 7, it is easy to find the disturbance is compensated mostly by the proposed controller, and the error between the designated desired ZMP and tracking ZMP is smaller.

5. SIMULATION RESULTS

The parameters of the biped robot (Fig. 1) were set according to Table 1. Taking human walking parameters as a reference, we set the walking speed as 1.8 km/h with the step length of 0.3 m/step and the step period of 0.6 s/step.
6. CONCLUSION

In the gait planning of this paper, ankle trajectories and hip trajectory are obtained by the formulated ground constraints on stairs and the designated desired ZMP. Meanwhile, the designated desired ZMP has enough stability margin, the biped robot system is available to implement, and the actuator specification is satisfied. Also, it is energy saving. The proposed controller with force sensing and variable impedance consists of a top level controller to track the desired ZMP and ten underlying controllers to track the servo objectives $q_d$. With the controller, the robot has a good biped balancing and interact with the environment friendly. What's more, it is essential for us to focus the future work on the various walking patterns and online control.

REFERENCES


