The Target Guarding Problem Revisited: Some Interesting Revelations

Raghav Harini Venkatesan∗ Nandan Kumar Sinha**

* Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai, India 600036 (e-mail: hvraghav@smail.iitm.ac.in)
** Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai, India 600036 (e-mail: nandan@ae.iitm.ac.in)

Abstract: In this paper, the target guarding problem, posed by Rufus Isaacs in his seminal textbook is revisited. Unlike the simplified version of the problem solved by Isaacs, optimal strategies are discussed for the pursuer and the evader having simple motions with different speeds. Analysis is also done for the case when the intercept occurs if the pursuer reaches within a certain distance of the evader, instead of a strict coincidence of their positions. The importance of the study is that, it is fundamental to building autonomous systems that would be used for protecting a target, for example, unmanned vehicles used in anti-poaching operations.

1. INTRODUCTION

The target guarding problem was first introduced by Isaacs [1965] in his pioneering work on differential games. It consists of a player (the evader E) trying to reach an immobile target T, and another player (the pursuer P) trying to intercept the evader before he reaches the target. The objective of the game for P is to maximize the distance between E and T at intercept, and that for E is to minimize it. Therefore, the best payoff for E would be to reach T without being intercepted.

Isaacs presented the solution to the target guarding problem, assuming equal speeds for P and E. He also assumed that interception corresponds to coincidence of the positions of P and E. When both P and E have equal speeds, capture occurs along the perpendicular bisector of their initial positions. The target to be guarded from E is an area C, which lies in the zone P can reach prior to E. The optimal strategy for either player in this case would be to head to the point closest to C on the perpendicular bisector, as shown in Fig. 1(a). If only P plays optimally, and E chooses to traverse a sub-optimal path, P traverses a curved path resulting in a better payoff for him at intercept, as shown in Fig. 1(b). Similarly, if E plays optimally, but P does not, E could well reach the target C, thus securing a better payoff for himself, as shown in Fig. 1(c).

In spite of being one of the earliest problems in differential games, the target guarding problem is one of the least studied in the literature. Unlike the problem solved by Isaacs, in reality, the players are hardly at the same speeds. Usually, the pursuer is faster than the evader. Also, intercept will occur when the pursuer reaches within a certain distance of the evader. The target might also be mobile. These variabilities to the target guarding problem have not been studied in the literature. Guarding a moving target would be the fundamental problem of interest in autonomous agents guarding mobile targets, for example, unmanned vehicles that could be used in anti-poaching operations to protect animals from poachers.

A variant of the problem of guarding a moving target was studied by Rusnak [2005] called “the lady, the bandits and the body guards” game. It consists of the bandits trying to capture the lady, and the body guards trying to intercept the bandits before they catch the lady. In this problem, the lady cooperates with the body guards and actively
tries to evade the bandits. However, in the problem of guarding a moving target, the target remains neutral in the game and does not actively cooperate with the rescuers and perform evasive action – this problem applies to, for example, in the case of guarding unsuspecting animals which get ambushed by poachers. Lau and Liu [2013] presented an online path planning algorithm using Rapidly-exploring Random Trees (RRT) for unmanned vehicles in charge of autonomous border patrol, based on Isaacs’ target guarding problem. The target considered in the study was static. The concept of RRT was used in the study because solving the differential game to obtain optimal strategies would be computationally intensive and could become infeasible for real-time systems. Therefore, the feasibility of solving the differential game would have to be considered while building real-time systems for anti-poaching operations. If found infeasible, algorithms like RRT should be explored.

2. GEOMETRY OF CAPTURE WHEN THE SPEEDS OF P AND E ARE DIFFERENT

As mentioned in the previous section, Isaacs analyzed the target guarding problem when the speeds of P and E are equal and execute simple motions. In this case, the perpendicular bisector of the line connecting their initial positions divides the playing space into regions that either of them can reach prior to the other. In other words, when both P and E travel straight towards the same point on the perpendicular bisector, they arrive at the same instant, and capture occurs. However, if the speeds of P and E are different, capture would not occur on the perpendicular bisector. To find the curve on which capture occurs, let us assume that the ratio of the maximum speeds of P and E be $p:e$. Therefore, let the maximum speed of P be $v_P = pv$, and that of E be $v_E = ev$, where $v$ is some common factor. Let $t$ be the time elapsed until capture. Both P and E will travel straight at their maximum speeds because decreasing their speed or traveling in a curved path will benefit the opponent. Let $E_0(x_c, y_c)$ be the initial position of E and $P_0(x_p, y_p)$ be that of P.

![Diagram](fig2.png)

**Fig. 2.** Game geometry when $v_P \neq v_E$ and capture is the coincidence of P and E

### 2.1 Capture Occurs when the Positions of P and E Coincide

Let us assume that the condition for capture is the coincidence of positions of P and E. Let $I(x, y)$ be the point where capture occurs, as shown in Fig. 2. Then, the distances traveled by P and E until capture would be $pt$ and $et$ respectively, and can be expressed mathematically as:

\[
(x - x_c)^2 + (y - y_c)^2 = (vt)^2 \quad (1)
\]

\[
(x - x_p)^2 + (y - y_p)^2 = (pt)^2 \quad (2)
\]

Dividing (1) by (2), we get

\[
\frac{(x - x_c)^2 + (y - y_c)^2}{(x - x_p)^2 + (y - y_p)^2} = \frac{v_t^2}{v_p^2} \quad (3)
\]

\[
\Rightarrow p^2 \left[ (x^2 - 2xx_c + x_c^2) + (y^2 - 2yy_c + y_c^2) \right] = e^2 \left[ (x^2 - 2xx_p + x_p^2) + (y^2 - 2yy_p + y_p^2) \right] \quad (4)
\]

\[
\Rightarrow \left( p^2 - e^2 \right) x^2 + \left( p^2 - e^2 \right) y^2 + 2 (x_p e^2 - x_p^2) x + 2 (y_p e^2 - y_p^2) y + p^2 (x_c^2 + y_c^2) - e^2 (x_p^2 + y_p^2) = 0 \quad (5)
\]

\[
\Rightarrow x^2 + 2 \left( x_p e^2 - x_c e^2 \right) x + y^2 + 2 \left( y_p e^2 - y_c e^2 \right) y + p^2 (x_c^2 + y_c^2) - e^2 (x_p^2 + y_p^2) = 0 \quad (6)
\]

Let $x_c = \left( \frac{xe^2 - x_p e^2}{p^2 - e^2} \right)$ and $y_c = \left( \frac{ye^2 - y_p e^2}{p^2 - e^2} \right)$. Then, (6) becomes

\[
x^2 - 2xx_c + y^2 - 2yy_c + p^2 \left( x_c^2 + y_c^2 \right) - e^2 \left( x_p^2 + y_p^2 \right) = 0 \quad (7)
\]

\[
\Rightarrow (x - x_c)^2 + (y - y_c)^2 = \frac{p^2 (x_c^2 + y_c^2) - e^2 (x_p^2 + y_p^2)}{(p^2 - e^2)} \quad (8)
\]

\[
\Rightarrow (x - x_c)^2 + (y - y_c)^2 = R^2 \quad (9)
\]

where $R = \sqrt{x_c^2 + y_c^2 - \frac{p^2 (x_c^2 + y_c^2 - e^2 (x_p^2 + y_p^2))}{(p^2 - e^2)}}$. Equation (9) is that of a circle with center $C(x_c, y_c)$ and radius $R$. Therefore, capture of E by P occurs on this circle. Also, the slope of the line connecting $C(x_c, y_c)$ and $E_0(x_p, y_p)$ is given by,

\[
y_c - y_p = \frac{y_c - y_p}{x_c - x_p} = \frac{y_c^2 - y_p^2}{x_c^2 - x_p^2} \quad (10)
\]

where $m_{E_0P_0}$ is the slope of the line $E_0P_0$. Therefore, C lies on $E_0P_0$. The capture circle looks like as shown in Fig. 3(a) and Fig. 3(b) for $v_c < v_p$ and $v_c > v_p$ respectively. It encloses $E_0$ when $v_c < v_p$ and $P_0$ when $v_c > v_p$. C always lies outside $E_0P_0$. When the speeds of P and E are equal, $(p^2 - e^2) = 0$, and therefore, (5) reduces to

\[
2 (x_p - x_c)x + 2 (y_p - y_c)y + (x_c^2 + y_c^2) - (x_p^2 + y_p^2) = 0 \quad (11)
\]

which is a straight line with slope $m_1 = \frac{y_p - y_c}{x_p - x_c}$. It can be seen that this line passes through the midpoint of $E_0P_0$, $\left( \frac{x_p + x_c}{2}, \frac{y_p + y_c}{2} \right)$, and $m_1 m_{E_0P_0} = -1$. This means that (11) is the perpendicular bisector of $E_0P_0$, which agrees with the result derived in Isaacs [1965].

### 2.2 Capture Occurs when P Reaches within a Distance $d_c$ of E

Let us now assume that the condition for capture is $P$ reaching a distance of $d_c$ from E. At capture, let
Fig. 3. Capture curve when \( v_p \neq v_c \) and capture is the coincidence of \( P \) and \( E \), and possible locations of a point target \( T \)

\[
\begin{align*}
(a) \quad v_c & < v_p \\
(b) \quad v_c & > v_p
\end{align*}
\]

Substituting the expression for \( l \) from (18) in (27), we have

\[
p^2 \left[ (x_{ei} - x_p)^2 + (y_{ei} - y_p)^2 \right] = e^2 \left[ (x_{pi} - x_p)^2 + (y_{pi} - y_p)^2 \right]
\]

Equations (14) - (16) do not simplify any further.

To gain more insight into the locus of points where capture would occur, without loss of generality, let us fix the origin of the coordinate system at \( P_0 \), and let the \( y \)-axis coincide with \( P_0E_0 \). Therefore, the coordinates of the initial positions of \( P \) and \( E \) would now be \( P_0(0,0) \) and \( E_0(0,y_0) \). Like before, the coordinates of the positions of \( P \) and \( E \) at capture would remain as \( P_1(x_{pi}, y_{pi}) \) and \( E_1(x_{ei}, y_{ei}) \). If \( E_0 \) were to travel straight vertically down from \( E_0 \), \( P \) would travel vertically up from \( P_0 \), and capture would occur when \( P \) and \( E \) have traveled distances of \( d_1 = p \left( \frac{y_0 - d_e}{p + e} \right) \) and \( d_2 = e \left( \frac{y_0 - d_e}{p + e} \right) \) respectively. This is because the triangle of Fig. 4(b) would flatten out to the line segment \( P_0E_0 \) in such a case. Therefore, if \( E \) takes any other path than vertically down, he will get to travel at least the distance \( d_2 \) before running the risk of being captured, and \( P \) will have to travel at least the distance \( d_1 \) before getting a chance to capture. Let \( e = 1 \). Therefore, in the new coordinate system, (12) - (15) become:

\[
\begin{align*}
x_{ei}^2 + (y_{ei} - y_p)^2 &= e^2l^2 \\
x_{pi}^2 + (y_{pi} - y_p)^2 &= p^2l^2 \\
x_{ei}^2 + y_{ei}^2 &= (pl + d_e)^2 \\
(x_{ei} - x_{pi})^2 + (y_{ei} - y_{pi})^2 &= d_e^2
\end{align*}
\]

Calculating \( l \) from (17) and substituting in (19), we have

\[
x_{ei}^2 + y_{ei}^2 = \left( \frac{p}{e} \sqrt{x_{ei}^2 + (y_{ei} - y_c)^2 + d_e^2} \right)
\]

As \( E_1 \) lies on the line joining \( P_1 \) and \( P_0 \), the slope of the line is given by

\[
m = \frac{y_{ei}}{x_{ei}} = \frac{y_{pi}}{x_{pi}} = \frac{y_{ei} - y_{pi}}{x_{ei} - x_{pi}}
\]

If the locus of the point \( P_1 \) is desired, subtracting (19) from (17) yields

\[
(2pl + d_e)^2 - 2y_ey_{ei} + y_{ei}^2 = e^2l^2
\]

\[
\Rightarrow y_{ei} = \frac{(pl + d_e)^2 + y_{ei}^2 - e^2l^2}{2y_e}
\]

\[
\Rightarrow x_{ei} = \frac{y_{ei}}{m} = \frac{y_{ei}}{y_{pi}}x_{pi}
\]

\[
= \frac{x_{pi}}{y_{pi}} \left( \frac{pl + d_e)^2 + y_{ei}^2 - e^2l^2}{2y_e} \right)
\]

where we have substituted the expression for \( y_{ei} \) from (24). Substituting (24) and (25) in (20), we have

\[
\left( \frac{y_{ei}}{m} - x_{pi} \right)^2 + (y_{ei} - y_{pi})^2 = d_e^2
\]

\[
\Rightarrow \left( \frac{x_{pi}}{y_{pi}} \left[ \frac{(pl + d_e)^2 + y_{ei}^2 - e^2l^2}{2y_e} \right] - x_{pi} \right)^2
\]

\[
+ \left( \frac{pl + d_e)^2 + y_{ei}^2 - e^2l^2}{2y_e} - y_{pi} \right)^2 = d_e^2
\]

Substituting the expression for \( l \) from (18) in (27), we have
\[ \left( \frac{x_{pi}}{y_{pi}} \right) \left( \frac{(\sqrt{x_{pi}^2 + y_{pi}^2} + d_e)^2 + y_c^2 - \frac{c^2}{p^2} (x_{pi}^2 + y_{pi}^2)}{2y_e} - x_{pi} \right) = \frac{1}{2} \left( \frac{x_{pi}^2 + y_{pi}^2}{A} - y_{pi} \right)^2 \\
\quad \quad + \left( \frac{(\sqrt{x_{pi}^2 + y_{pi}^2} + d_e)^2 + y_c^2 - \frac{c^2}{p^2} (x_{pi}^2 + y_{pi}^2)}{2y_e} - y_{pi} \right) = d_e^2 \]

Equation (28) is a 12th degree polynomial. When \( p = e \), it reduces to a 8th degree polynomial.

And when \( d_e = 0 \), (28) further reduces to
\[
\left( \frac{x_{pi}}{y_{pi}} \right) \left( \frac{(x_{pi}^2 + y_{pi}^2)^2 + y_c^2 - \frac{c^2}{p^2} (x_{pi}^2 + y_{pi}^2)}{2y_e} - x_{pi} \right) = \frac{1}{2} \left( \frac{x_{pi}^2 + y_{pi}^2}{A} - y_{pi} \right)^2 \\
\quad \quad + \left( \frac{(x_{pi}^2 + y_{pi}^2)^2 + y_c^2 - \frac{c^2}{p^2} (x_{pi}^2 + y_{pi}^2)}{2y_e} - y_{pi} \right) = d_e^2
\]

\[
\Rightarrow \left( \frac{x_{pi}^2}{y_{pi}^2} + 1 \right) (A - y_{pi})^2 = 0
\]

\[
\Rightarrow A - y_{pi} = 0
\]

\[
\therefore \left( 1 - \frac{e^2}{p^2} \right) (x_{pi}^2 + y_{pi}^2) + y_c^2 - 2y_e y_{pi} = 0
\]

\[
\Rightarrow \left( x_{pi}^2 + y_{pi}^2 \right) + y_c^2 - 2y_e y_{pi} = 0
\]

\[
\Rightarrow x_{pi}^2 + \left( y_{pi} - \frac{y_c}{\frac{1}{1 - \frac{e^2}{p^2}}} \right)^2 = \left( \frac{e}{p} y_c \right)^2
\]

Equation (34) is that of a circle with center at \( C \left( 0, \frac{y_c}{1 - \frac{e^2}{p^2}} \right) \) and radius \( R = \frac{e}{p} y_c \), which is the circle of (9) in the new coordinate system, where \( x_p = y_p = x_c = 0 \). When \( e < p \), the circle encloses \( E_0(0, y_c) \) and when \( e > p \), the circle encloses the origin \( P_0(0, 0) \).

In addition, when \( p = e \), \( A = \frac{y_c}{p} \) and (31) becomes
\[
\frac{y_c}{2} - y_{pi} = 0
\]

which is the perpendicular bisector of \( P_0E_0 \) in the new coordinate system.

From the above analysis, it is clear that if the criterion for capture is a proximity of \( d_e \) between \( P \) and \( E \), the locus of \( P_i \) is a nontrivial 12th degree polynomial. Even when \( v_p = v_e \), locus of \( P_i \) is not a simple curve. Only when the criterion for capture is a simple coincidence of positions, the derivation of optimal strategies is not straightforward. In many practical scenarios, the
assumption that $d_e \ll p_l$ could well be satisfactory, in
which case, $P_1$ and $E_1$ would coalesce into the capture
point $I$. Therefore, the locus of $I$ would be the capture
circle (referred to as $C_I$ henceforth) given by (9), which
will be used in the following sections.

3.1 Case $v_e < v_p$

In this case, $C_I$ encloses $E_0$. $T$ could be in two possible
locations - within $C_I$ at $T_1$ or outside $C_I$ at $T_2$ as shown
in Fig. 3(a). If $T$ is at $T_1$, $E$ would reach $T_1$ before $P$ and
the game ends with $T$ being sacrificed. However, if $T$ is at $T_2$, he could be guarded. Let $T$ be located at a position $T_0$ outside $C_I$ as shown in Fig. 5(a). $P$ strives to keep $E$ away from $T$ as possible and $E$ strives to do
the opposite. $E$ will choose to head to the point closest
to $T_0$ on $C_I$ to make sure that even if captured, he is at
the minimum possible distance from $T$ at capture. This
point $I$ is the intersection of the radial line $CT_0$ with $C_I$,
as shown in the figure. $P$ too will head to $I$ expecting $E$
to play optimally.

When the game proceeds optimally on both sides, the
dynamics of the game look like as shown in Fig. 5(b). The
locations of $P$ and $E$ at two instants during the course
of the game are shown as $P_1$, $P_2$ and $E_1$, $E_2$ respectively.
The instantaneous capture circles $C_{I1}$ and $C_{I2}$ are drawn
at these two instants and their centers are marked as
$C_1$ and $C_2$ respectively. As both $P$ and $E$ have been
playing optimally, i.e., traveling straight along $P_0I$ and
$E_0I$ respectively right from the beginning, the center $C$
of the instantaneous capture circle $C_I$ also moves along the
line $CI$. In fact, $P_0E_0 \parallel P_1E_1 \parallel P_2E_2$ because $\triangle E_0P_0I \sim
\triangle E_1P_1I \sim \triangle E_2P_2I$. And by (11), $C_1$ and $C_2$ should
lie along $P_1E_1$ and $P_2E_2$ respectively. The instantaneous
capture circle $C_{I_t}$ (the subscript $I$’s subscript $t$ being the
time instant under consideration) reduces in size as the
game proceeds, finally coalescing to the capture point $I$.
Also, all the instantaneous capture circles $C_{I_t}$’s touch each
other at $I$. The optimal strategies for $P$ and $E$ remain
the same at all instants during the game. Both of them
head towards the point of intersection of the instantaneous
radial line $CT_0$ with the instantaneous capture circle $C_I$,
which remains as $I$ if $P$ and $E$ play optimally through out
the game.

The capture point $I(x_i, y_i)$ can be found in the following
way. Let the coordinates of $T$ be $T_0(x_t, y_t)$. $I$ lies on the
line $CT_0$. Therefore, it’s slope

\[ m = \frac{y_t - y_I}{x_t - x_I} = \frac{y_t - y_I}{x_t - x_I} \] (36)
\[ \implies y_I = m(x_I - x_t) + y_t \] (37)

As $I$ also lies on $C_I$ given by (9),

\[(x_i - x_c)^2 + (y_i - y_c)^2 = R^2 \] (38)
\[ \geq 0 \] (39)

Substituting for $y_I$ from (37) in (38), we have

\[(x_i - x_c)^2 + [m(x_i - x_t) + y_t - y_c]^2 = R^2 \] (40)
\[ \implies (1 + m^2)x_i^2 + 2(mk - xc)x_i + x_c^2 + k^2 - R^2 = 0 \] (41)

where $k = y_t - y_c - mx_t$. Solving (41), we get

\[ x_i = \frac{-mk - xc \pm \sqrt{(1 + m^2)R^2 - (mx_c + k)^2}}{1 + m^2} \] (42)

Each $x_i$ has a corresponding $y_i$. Out of these two points,
the point closest to the target $T(x_i, y_i)$ is $I$.

3.2 Case $v_e > v_p$

The capture circle $C_I$ encloses $P_0$ in this case, and like
before, $T$ could be either at $T_1$ inside $C_I$ or at $T_2$ outside
as shown in Fig. 3(b). It is quite obvious that if $T$ is at $T_2$, $E$
can reach there before $P$ and $T$ is lost. However, if $T$
is at $T_1$, one might be tempted to say that $P$ can save
$T$. But it turns out that $E$ will certainly reach $T$ wherever
he is located in the plane, as shown later. Therefore, the
best $P$ can do is to reach $T$’s location and wait for $E$ to
capture him there.

Fig. 6. Game dynamics when both $P$ and $E$ play optimally

To understand the dynamics of the game when $v_e > v_p$, it
is important to understand the range of positions that $E$
can reach for a given position of $P$. Let $E$ be at $E_0(0, y_e)$
and $P$ at $P_0(0, 0)$ as shown in Fig. 6(a), in accordance
with the simplified coordinate system used in section 2.2.
$C_I$ is then given by (34), which is shown in the figure. The
tangents drawn from $E_0$ to $C_I$ meet at $M_1$ and $M_2$. Now,
the range of locations that $E$ can reach safely without any
chance of being captured by $P$ is the unshaded region in Fig. 6. This is because if $E$ tries to reach any location within the shaded region, he faces the possibility of being intercepted by $P$ on the arc $M_1M_2$. The tangents can be found in the following way: Let the slope of the tangent be $m_t$. As the tangent passes through $E_0(0, y_c)$, equation of the tangent is

$$y - y_c = m_t x$$

The point of tangency $M_1(x_{m_1}, y_{m_1})$ lies on this tangent and also on $C_T$, therefore

$$x_{m_1}^2 + (y_{m_1} - y_c)^2 = R^2$$

$$\Rightarrow x_{m_1}^2 + (y_c + m_t x_{m_1} - y_c)^2 = R^2$$

$$\Rightarrow (1 + m_t^2)x_{m_1}^2 + 2m_t(y_c - y_c)x_{m_1} + (y_c - y_c)^2 - R^2 = 0$$

Now, for (43) to be a tangent at $M_1(x_{m_1}, y_{m_1})$, (46) should have a double root. Therefore,

$$4m_t^2(y_c - y_c)^2 - 4(1 + m_t^2)[(y_c - y_c)^2 - R^2] = 0$$

$$\Rightarrow R^2m_t^2 + R^2 - (y_c - y_c)^2 = 0$$

$$\Rightarrow m_t = \pm \sqrt{\frac{(y_c - y_c)^2}{R^2} - 1} \equiv m_{t_{\pm}}$$

From (34), we have $y_c = \frac{y_0}{1 - \frac{1}{p^2}}$ and $R = -\frac{e}{p(1 - \frac{1}{p^2})}$. Therefore, (49) becomes

$$m_{t_{\pm}} = \pm \sqrt{\frac{e^2}{p^2} - 1}$$

which is independent of the instantaneous locations of $E$ and $P$. The result means that the radius of $C_T$ varies with the location $E$ in such a way that the tangents to it from $E$ always make a constant angle with the line $EP$. The points of tangency $M_1, M_2$ can be found as the intersection of the line (43) and the radial line of $C_T$ perpendicular to it, which can be written as

$$y - y_c = -\frac{1}{m_{t_{\pm}}} x$$

Solving (43) and (51), we get the points of tangency $M_1, M_2$ as $\left(\frac{m_{t_{\pm}}}{{1 - \frac{1}{p^2}}}, y_c\right)$ and $\left(\frac{m_{t_{\pm}}}{1 - \frac{1}{p^2}}, y_c\right)$ respectively, which lie on the line perpendicular to $E_0P_0$ and passing through $P_0$. Therefore, $M_1P_0 \perp E_0P_0$ and $M_1P_0 \parallel M_1M_2$, as shown in Fig. 6(a). As $m_{t_{\pm}}$ is independent of the instantaneous locations of $P$ and $E$, the line connecting the instantaneous points of tangency always is perpendicular to $EP$ and passes through $P$.

As only the relative positions of $E$ and $P$ matter, by fixing $P$ to $P_0$ and moving $E$ w.r.t him, it is easier to analyze the range of locations that $E$ can reach mathematically. Fig. 6(b) shows the inaccessible regions for $E$, depending on his location w.r.t $P_0$. If $E$ is at $E_0$, his inaccessible region is the region shaded in red. However, if $E$ is located a little closer to $P$, like at $E'$, the region inaccessible to him (indicated by the region in purple) is lesser than that at $E_0$. Therefore, as $E$ moves closer to $P$, the range of locations that he can reach without being intercepted by $P$ increases. If $E$ is located at $E''$, the inaccessible region to him (indicated by the region shaded in green) is oriented differently, opening up locations which were inaccessible to him while at $E_0$. Therefore, $E$ can reach any location in the plane by suitably positioning himself w.r.t $P$, merely by virtue of his higher speed than $P$. To save $T$, $P$'s only option then is to reach $T$'s position and wait to capture $E$ there. This would be $P$'s optimal strategy. Any effort by $P$ to intercept $E$ at a different location could result in the loss of $T$. Similarly, the optimal strategy for $E$ would be to head straight to $T$. If $P$ plays sub-optimally and happens to be on his way, he should first get arbitrarily close to $P$ and then keep traveling along one of the tangents to the instantaneous $C_T$, till the tangent points in the direction of $T$. At this point, he should leave along this tangent, thus avoiding capture and acquiring $T$.

4. CONCLUSIONS

In this paper, the target guarding problem was solved for the pursuer and evader having simple motions with different speeds. The target guarding problem is fundamental to systems that protect a target from enemies, for example, unmanned vehicles trying to protect endangered animals from poachers. Results discussed in this paper could be applied to a single drone trying to protect a static animal, under threat from a single poacher. The optimal strategy for the drone would depend on the ratio of its speed to that of the poacher. If the drone’s speed is higher than that of the poacher (which is usually the case), it would be possible for the drone to keep the poacher at bay, depending on their initial locations. However, if the poacher’s speed is more than that of the drone, an interesting result from the analysis done in this paper shows that it would not be possible for the drone to keep the poacher away from the animal if the poacher plays optimally. The best that the drone could do in such a case is to go to the animal’s location and wait there for the poacher to arrive.

To build field-fit drones that would help in anti-poaching operations, the analysis done in this paper has to be extended for mobile targets because, in reality, the animals would be moving. Also, if interception is achieved when the drone gets within a certain proximity of the poacher, calculations for optimal strategies could become intense, and it may not be possible to do such time consuming mathematical computations in real-time. Therefore, algorithms like Rapidly exploring Random Trees (RRT) will have to be explored.

REFERENCES


I. Rusnak. The Lady, the Bandits and the Body Guards - A Two Team Dynamic Game. In *Proceedings of the 16th IFAC World Congress*. 2005.