An energy perspective on modelling, supervision, and control of large-scale industrial systems: Survey and framework

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Abstract: Energy is a universal concept that can be used across physical domains to describe complex large-scale industrial systems. This brief survey and framework gives a perspective on energy as a unifying domain for system modelling, supervision, and control. Traditionally, modelling and control problems have been approached by adopting a signal-processing paradigm. However, this approach becomes problematic when considering non-linear systems. A behavioural viewpoint, which incorporates energy as basis for modelling and control, is considered a viable solution. Since energy is seen as a unifying concept, its relationship to Euler-Lagrange equations, state space representation, and Lyapunov functions is discussed. The connection between control and process supervision using passivity theory coupled with a system energy balance is also established. To show that complex industrial systems comprising multiple energy domains can be modelled by means of a single electric circuit, its application to a large-scale thermo-hydraulic system is presented. Next, a simple non-linear transmission impedance electric circuit is used to illustrate how energy can be used to not only describe a system, but also serve as basis for system optimisation. An energy-based framework is proposed whereby energy is used as a unifying domain to work in, to analyse, and to optimise large-scale industrial systems.

Keywords: Energy, modelling, supervision, control, multi-domain, large-scale, industrial systems, equivalent electric circuit.

1. INTRODUCTION

Large-scale industrial systems are characterised by a multitude of sub-systems exchanging matter and energy to accomplish a common goal. The interactions of the sub-systems can be complex and take place across different physical domains such as thermal, chemical, fluid, mechanical, and electrical, to name but a few. Most physical systems also portray non-linear behaviour of varying complexity. Global system optimisation, meeting specific performance objectives while maintaining a healthy energy profile, is therefore a complex and multi-faceted problem.

Energy is seen as a unifying concept that can be used across physical domains to characterise and describe complex large-scale industrial systems. The engineering challenge can be described as one of achieving some global plant objectives through the effective manipulation and transformation of energy. The notion to consider energy as a measure of system stability is of course the basis of Lyapunov’s second stability criterion (Shinners, 1998), where the sum of the system’s kinetic and potential energy is considered as a function, and the time derivative of the function is taken. Haddad & Nersesov (2011) proposes the global optimisation of a complex system from an energy perspective, with a unified stability analysis and control design framework for large-scale non-linear interconnected dynamic systems. The proposed framework stands on the legs of vector Lyapunov functions and passivity theory. In the analysis of large-scale non-linear industrial systems, several Lyapunov functions arise naturally from the stability properties of each individual subsystem. Furthermore, with many input, state, and output properties related to the supply, storage, transport, and dissipation of energy, extending classical dissipativity theory (Willems, 1972a, 1972b) to include storage and dissipation on the subsystem level, leads to a natural energy-based framework for large-scale industrial systems. Dissipativity refers to the system characteristic where a fraction of the energy supplied to the system is transformed into heat or losses. Passivity is a special case of dissipativity, where the energy stored in the passive systems cannot exceed the energy supplied by the external environment. It provides a fundamental framework for the analysis and design of control using a state space formalism based on system energy related considerations.

Linear or non-linear complex systems have long been modelled by means of equivalent electric circuits. This is due to the striking similarity that exists between the differential equations that describe the behaviour of physical systems in a
variety of domains, ranging from electric circuits to mechanical movement and thermodynamics. The above mentioned pattern becomes even more evident when so-called "through" and "across" variables are used (Dorf & Bishop, 2011). Examples of "through" variables (flow variables in Bond graph terminology) include current, force, torque, fluid volumetric flow rate, and heat flow rate. Examples of "across" variables (effort variables in Bond graph terminology) include voltage, velocity difference, angular velocity difference, pressure difference, and temperature difference. The energy contained in through variables is stored inductively in the form of inductors (electric circuits), springs (mechanical systems), and fluid inertia. The energy contained in across variables is stored capacitively in the form of capacitors (electric circuits), mass (mechanical systems), fluid capacitance, and thermal capacitance. Lastly, energy is also dissipated in a similar manner across various domains by means of resistors (electric circuits), dampers (mechanical systems), and thermal resistance. In addition, equivalent electric circuits typically represent lumped parameter models of the examined system. Combined with the use of either positive or negative feedback, it is therefore possible to approximate most systems (in a particular domain) by means of an equivalent electric circuit.

The notion of using energy as basis for global optimisation is extended in this work to the more general concept of applying energy for the purpose of system representation. The idea stems from two apparently unrelated works. The first derives a generic procedure for state space model extraction of large-scale thermo-hydraulic systems, whereby the transparency of the system components is retained in the final model (Uren & van Schoor, 2013a, 2013b). In the state space model, each state represents stored energy associated with that particular state, and therefore, an important link can be made between energy as basis for modelling and control. In the second work, a methodology is devised to extract enthalpy and entropy fault signatures of a large-scale thermo-hydraulic system for the purpose of fault detection and diagnosis (FDD) (du Rand & van Schoor, 2012a, 2012b). A connection is thus established between energy and process supervision, and visualising the condition of the system using energy signatures.

Note that in this work, terminology in the field of process supervision and FDD are adopted according to the IFAC SAFEPROCESS Technical Committee (Isermann & Ballé, 1997). Supervision constitutes a continuous task of determining the condition of the process (monitoring) whereby system anomalies are detected, diagnosed, and corrected. The diagnosis task comprises fault isolation (type, location, and time) and fault identification (magnitude and time-variant behaviour). Also, an energy signature does not signify a simple “best-fit" line between specific calculated indices (Belussi & Danza, 2012; Yu & Chan, 2005), but aims to optimally depict system knowledge based on energy indicators.

This paper gives an energy perspective on modelling, supervision, and control of large-scale industrial systems in the form of a brief survey and proposed framework. The paper aims to establish an apparent relation between the tasks mentioned using energy as the unifying domain. The paper starts out in Section 2 with a historical development of energy as the basis for system modelling, supervision, and control. Section 3 presents the relevance of energy as a unifying domain for the tasks mentioned. A perspective on realistic systems is then given in Section 4, portraying equivalent electric circuits as representative of complex multi-domain industrial systems. Next, a non-linear electric circuit case study is examined in Section 5, demonstrating how energy can form the basis for global system optimisation. Section 6 describes an envisaged energy-based framework using energy formalisms as a unifying concept for system modelling, supervision, and control. Some final thoughts and concluding remarks are presented in Section 7.

2. HISTORICAL DEVELOPMENT

Considering the general history of control theory, it is interesting to notice that the control system design process has been traditionally approached from a signal theoretic perspective (R Ortega, van der Schaft, Mareels, & Maschke, 2001). This perspective can be traced back to the 1930s with the development of the first feedback amplifiers. In this paradigm, the sub-systems of the control system are viewed as signal processing devices that transform input signals into output signals. The design specifications are based on minimising error signals and reducing the effect of disturbance signals in the presence of model uncertainties. Mathematically this translates to the assumption that the disturbance signals and unmodelled dynamics are norm bounded. This means that the control performance is determined by the size of the operator gains that map the various input signals to the output signals. The mathematical framework that supported the modelling, analysis, and synthesis of control systems was based on input-output ideas with Fourier and Laplace transforms being the dominant mathematical tools. This paradigm worked particularly well in the case of linear time-invariant control systems since filtering using frequency-domain considerations can be implemented successfully.

During the 1960s and 1970s, this paradigm took a new direction due to the introduction of the state-space formalism. In a mathematical framework, mapping between the inputs and outputs of the control system are based on the transformation of the internal state of the control system. Moreover, in this approach, the mathematical tools turned towards ordinary differential equations. The celebrated concepts of controllability, observability, and optimality were introduced that led to powerful controller design techniques.

From the 1990’s research is focused towards the development of a paradigm that allows for the treatment of a more generalised class of systems, the goal typically being to consider non-linear systems represented by

$$\dot{x} = f(x,u),$$  
$$y = h(x,u).$$

This notion was inspired by (Willems, 1991, 2007) and is generally referred to as the behavioural approach towards
systems modelling and control. In this paradigm a mathematical model is viewed as a subset of a universum of possible descriptions of reality. That is, before a mathematical model is derived of the real system, all outcomes in the universum are in principle possible. After a mathematical model is accepted as a convenient description of reality, only then a certain subset of outcomes is possible. This subset is called the behaviour of the mathematical model. Proceeding from this perspective, one arrives at the notion of a dynamical system as simply a subset of time-trajectories. This paradigm is not as restrictive as the input/output point of view. In fact, most physical systems do not have a preferred signal flow direction, and it is important to let the mathematical structures reflect this. The behavioural paradigm therefore starts from a mathematical model obtained from first principles resulting in a set of differential and algebraic equations. Among the vector of time trajectories satisfying these equations are components that are available for interconnection. The process of controller design reduces to defining an additional set of equations for these interconnection variables to impose a desired behaviour on the system. This paradigm naturally supports the fundamental concept of energy conservation. Therefore, complex dynamic systems consisting of sub-systems and controllers are viewed as energy-transforming devices that interconnect via power conserving connections to achieve not only a desired response, but also an optimal system response.

Considering the modelling and control of large-scale systems, the same kind of restrictions surface as with the signal theoretic paradigm. According to (Haddad & Nersesov, 2011), the behavioural paradigm is a much more natural fit. It follows that energy-based modelling arises naturally in large-scale dynamic systems. Fig. 1 illustrates the two paradigms regarding the modelling and control of large-scale dynamic systems. Considering the behavioural paradigm, three modelling approaches can be followed using energy concepts (Janschek, 2011):

- Energy-based modelling employing scalar energy functions using either Euler-Lagrange or Hamiltonian formalisms.
- Multi-port modelling employing component-based system models with power-conserving rules utilising e.g. Kirchhoff networks or Bond/linear graph approaches.
- A combination of the Hamiltonian and port-based modelling formalism called Port-Hamiltonian modelling.

The port-Hamiltonian formalism is especially of great importance regarding the modelling of complex, large-scale systems due to the following advantages (Duindam, Macchelli, Stramigioli, & Bruyninckx, 2009):

- It is highly scalable, and therefore naturally allows the modelling of very large interconnected multi-physics systems.
- Due to a strong differential geometric base, it has the ability to incorporate non-linearities while retaining underlying conservation laws.

- It has the ability of treating both finite-dimensional and infinite-dimensional components.

![Fig. 1. Paradigms for modelling and control of large-scale dynamic systems.](image)

The port-Hamiltonian formalism is able to match the “old” framework of port-based network modelling of multi-domain physical systems with the “new” framework of geometrical dynamic systems and control theory. This allows for the systematic approach of modelling, analysis, condition monitoring/fault detection and control, via

- separation of the interconnection structure of the system from the constitutive relations of its components;
- enforcing power conservative interconnections by means of Dirac structures;
- analysing the system making use of the interconnection structure and component constitutive relations;
- the achievement of control by means of Casimir function generation, energy shaping, energy routing and port and impedance control.

From a geometric perspective, the Dirac structure lies central in describing port-Hamiltonian systems. The Dirac structure has a strong link with bond graphs, especially 0- and 1-junctions are prime examples of the general concept of a Dirac structure. Generalised flow and effort vectors \( \mathbf{e}, \mathbf{f} \) are elements of abstract finite-dimensional linear spaces \( \mathcal{F} \) and \( \mathcal{E} \) respectively. The effort space is defined as the dual space of \( \mathcal{F} \), that is \( \mathcal{E} = \mathcal{F}^* \). The total space of flow and effort variables is \( \mathcal{F} \times \mathcal{E} \) and is generally called the space of port variables.
On this total space of port variables, the power is defined by

\[ P = \langle e | f \rangle \quad (f, e) \in \mathcal{F} \times \mathcal{F}, \]  

(3)

where \( \langle e | f \rangle \) denotes the dual product, that is, the linear functional \( e \in \mathcal{F}^* \) acting on \( f \in \mathcal{F} \). A Dirac structure on \( \mathcal{F} \times \mathcal{F} \) is then a subspace \( \mathcal{D} \subseteq \mathcal{F} \times \mathcal{F} \) such that

- \( \langle e | f \rangle = 0 \), for all \( (e, f) \in \mathcal{D} \),
- \( \dim \mathcal{D} = \dim \mathcal{F} \).

This illustrates the notion that port-Hamiltonian system descriptions share common ground with geometric nonlinear control theory and geometric mechanics.

Traditional and advanced methods for process supervision and FDD are well documented in the literature (Bokor & Szabó, 2009; Das, Maiti, & Banerjee, 2012; Hwang, Kim, Kim, & Seah, 2010; R Isermann & Ballé, 1997; Rolf Isermann, 1984; Qin, 2012; Venkatasubramanian, Rengaswamy, Kavuri, & Yin, 2003a, 2003b, 2003c). However, in the last few decades, FDD based on energy formalisms (i.e. not energy based signal transformations) did not develop to any great extent. The most notable contributions relate to an energy balance or conservation principle, which offers great possibilities due to its clear physical meaning and easy implementation (Wei Chen, 2011). The energy can be representative of the true physical system energy, or an abstract energy function defined via Lyapunov theory.

Model-based fault detection based on energy balance calculations takes its origins from chemical process control in the 1970s (Gertler, 1998 that refers to previous works of Himmelblau, 1978; Vaclavek, 1974). Berton & Hodouin (2003) introduced a conservation model obtained via linear and bilinear state equations describing mass and energy balances. Interactions between different conservation laws, i.e. mass and energy, are evaluated using single bilinear FDD residual vectors. (Sunde & Berg, 2003) successfully achieved the notion of fault detection by way of plant-wide mass and energy balances for a 3,300 MW; boiling water reactor turbine cycle. The balance equations were implemented as constraints to a minimisation problem. (Theilliol, Noura, Sauter, & Hamelin, 2006) exploits the energy balance of a SISO closed-loop system to generate residuals of the energies involved without an input-output model. Very often, such input-output models are almost impossible to obtain for complex large-scale industrial systems. In this case, the energy balance offers an intuitive way to perform FDD. Energy indices are used by (Tinaut, Melgar, Laget, & Dominguez, 2007) to investigate the interchange of energy between different engine components for the purpose of fault detection. An energy model corresponding to the change in total kinetic energy of the moving parts facilitates a transparent and straightforward FDD approach.

Subsequent energy supported FDD works follow from passivity theory. (Yang, Cocquempot, & Jiang, 2008) constructs a global passivity energy relation by an inequality that comprises system states, inputs and outputs. FDD is realised by monitoring this inequality for “fault” energy. Since implicit FDD is only required for part of faults under the passivity framework, certain conditions on the system structure can be relaxed for implicit fault diagnosis (Gertler, 1998). (W Chen, Ding, Khan, & Abid, 2010) extends the passivity framework to a more inclusive energy based framework taking into account the system’s dissipative properties. Therefore, unlike an energy inequality, an energy balance is achieved which offers optimal FDD. For unmeasurable system states, FDD is accomplished by an optimal approximation of the energy balance based on system inputs and outputs. In (Wei Chen, 2011), the energy balance based FDD is further developed to accommodate passive non-linear systems. FDD design procedures are established for two classes of passive non-linear systems namely input-affine and Lagrangian systems.

3. ENERGY AS A UNIFYING CONCEPT

3.1 Prelude to energy as a universal concept

Energy is a universal concept in systems and processes found in all domains, and therefore, also in multi-domain systems. In this work, the term system implies a closed environment that represents energy exchanges internal to the system, but also to and from the system. A simple example of a multi-domain system demonstrating this concept is a permanent magnet DC motor, in which electrical energy is converted to mechanical energy and vice versa. Specifically, the armature current is converted to mechanical torque, which results in acceleration of the motor’s rotor. Conversely, the mechanical angular speed of the motor results in a back electromotive force. This inherently feeds back to the electrical subsystem of the motor, thereby controlling the magnitude of the rotor current. Consequently, the amount of mechanical torque produced is controlled. Another important point to be made is that the general theory of systems assumes that the examined system is linear, and if not, that the system can be linearised in the operating region of interest. An important salient feature of energy is that it applies equally to both linear and non-linear systems. Therefore, by considering a system from the viewpoint of energy, the very limiting requirement of the system to be linear proves to be superfluous and can thus be disposed of.

3.2 Euler-Lagrange equations and energy

The Euler-Lagrange equations used for deriving the differential equations that models a given problem forms part of the subject called Calculus of Variations. The basic problem herein is to infer a function \( x(t) \) (i.e. not a variable’s value) that minimises a specified definite integral. The integrand of the latter is a function of the original function as well as certain derivatives thereof. In its simplest form, the problem is that of determining a once-differentiable scalar function \( x(t) \) of a single independent variable \( t \), for which the integral

\[ I[x(t)] = \int_{t_0}^{t_f} F(t, x(t), \dot{x}(t)) \, dt \]  

(4)
is minimised. The function $F$ in (4) is called the Lagrange function.

It follows readily (Hildebrand, 2012) that the function $x(t)$ that minimises the integral (4) satisfies the Euler-Lagrange equation

$$\frac{d}{dt} \left( \frac{\partial F}{\partial \dot{x}_i} \right) - \frac{\partial F}{\partial x_i} = 0. \tag{5}$$

Conversely, (5) is a necessary but not sufficient condition for a function $x(t)$ to minimise the integral (4). It is possible to generalise the above to problems involving $n$ dependent and $m$ independent variables. The particular case of interest is that of modelling a system using an ordinary differential equation of order $n$ with $t$ as the independent variable. In this case, the set of Euler-Lagrange equations (Hildebrand, 2012) in (5) needs to be solved.

$$\frac{d}{dt} \left( \frac{\partial F}{\partial \dot{x}_i} \right) - \frac{\partial F}{\partial x_i} = 0, \quad \text{for } i = 1, \ldots, n \tag{6}$$

For our purpose, the Lagrange function is selected to be the energy difference $F(t, x, \dot{x}) = T(x, \dot{x}) - V(x)$. Here, $T$ and $V$ represent the total kinetic and potential energy of the system respectively, and are expressed in terms of the variable set $\{x_1, \ldots, x_n\}$ called generalised coordinates, and their derivatives $\{\dot{x}_1, \ldots, \dot{x}_n\}$. Taking this set of generalised coordinates to be the state variables then links the Euler-Lagrange equations to state space modelling. Therefore, by applying this method for the purpose of system modelling, a state space representation of the system can be obtained. Refer to (Jeltsema & Scherpen, 2009) for a more comprehensive discussion of Euler-Lagrange equations.

### 3.3 Relationship between state space representation and energy

Just as energy is a universal concept across various domains, it comes as no surprise that the state space representation is also a universal concept, allowing a system’s dynamical behaviour to be expressed as a set of first order ordinary differential equations (refer to equation (7) below).

For an arbitrary multi-domain system, a particular state space representation, called the standard form, can be obtained by choosing the output of each energy storage element to be a state variable of the system (Shinners, 1998). Here, the output of a storage element is the dependent variable associated with the storage element as dictated by the given system’s configuration (e.g. the current passing through a voltage-fed inductor). For this particular state-variable assignment, the time average of a state-variable squared is easily shown to be proportional to the energy stored in the associated energy storage element. This confirms our suspicion that there exists a deep connection between energy flow in a system, and the state space representation of the system. Moreover, in more elaborate paradigms (e.g. as encountered in optimal control), even generalisations of power and energy namely cross-power and cross-energy are considered. Cross-power terms are those represented by the product of two different state-variables.

For an arbitrary $n$th order driven system with $m$ outputs, the general form of the state space representation is given by the state equation

$$\dot{x}(t) = f(x(t), t), \quad f : \mathbb{R}^{n+1} \to \mathbb{R}^n \tag{7}$$

together with the output equation

$$y(t) = h(x(t), t), \quad h : \mathbb{R}^{n+1} \to \mathbb{R}^m \tag{8}$$

for all $t \geq t_0$ with $x(t_0) = x^0$.

If the functions $f$ and $h$ are independent of time, then the system is said to be autonomous (i.e. not driven). A large class of non-linear driven systems can be represented in the form

$$\dot{x}(t) = g(x(t)) + Bu(t), \quad y(t) = Cx(t) \tag{9}$$

where $u(t)$ represents the input vector of the system. In (Hrusak, Stork, & Mayer, 2009) it is shown that for the case where the function $g$ is of the form $g(x) = Ax(x)$, the instantaneous value of the output power $P(t)$ and the corresponding average energy $E(t)$ of the state, up to time $t$, are related by

$$\frac{dE}{dt} = -P(t) = -\|y(t)\|^2. \tag{10}$$

Then, for the zero input case, the energy present in the system at time instant $t_0$ is

$$E(t_0) = \int_0^{t_0} \|y(t)\|^2 \, dt. \tag{11}$$

A state space representation with the $A$ matrix of the form

$$A = \begin{bmatrix}
-a_{11} & a_{12} & 0 & 0 & 0 \\
-a_{21} & -a_{22} & a_{3} & 0 & 0 \\
0 & a_{5} & -a_{13} & \ddots & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & a_{n-2} & 0 \\
0 & 0 & 0 & -a_{n-1} & -a_{n-1,n-1} & a_{n} \\
0 & 0 & 0 & 0 & -a_{n} & -a_{nn}
\end{bmatrix} \tag{12}$$

exists and is termed physically correct (Hrusak et al., 2009). By (Hrusak et al., 2009), a large class of non-linear systems, all for which $g(x) = Ax(x)$, can be represented by a state space representation for which (12) holds. For the non-linear case, components of $A$ in (12) depend on $x$.

In (Hrusak et al., 2009) and references therein, it is shown that a necessary and sufficient condition for dissipativity of this class of non-linear systems is that $a_1 > 0$, while a necessary and sufficient condition for conservativity is that $a_1 = 0$. A necessary but not sufficient condition for asymptotic stability is that $a_1 > 0$. Refer to (Hrusak et al., 2009) for more detail. In (Mayer, Hrusak, & Stork, 2013), this energy state space approach is applied to gain insight into the mechanisms.
responsible for chaotic behaviour of two non-linear coupled oscillators.

3.4 Lyapunov functions and how they relate to energy

Another subtle connection is worthwhile to emphasise. In the theory of autonomous ordinary differential equations (ODEs), Lyapunov functions are scalar functions that enable stability analyses of an equilibrium point of the ODE. For a system that can be represented by the state space representation (7), a once-differentiable function \( V : D \times \mathbb{R} \to \mathbb{R} \), with \( D \subset \mathbb{R}^n \) a neighbourhood of the state space’s origin, is called a Lyapunov test or candidate function if both \( \dot{V}(x,t) > 0 \) for \( x \neq 0 \), and \( V(0,t) = 0 \) for all \( t \geq t_0 \) (Jordan & Smith, 2007). A Lyapunov candidate function that has the property of a non-positive orbital derivative, i.e.

\[
\dot{V}(x,t) = \nabla V \cdot f(x,t) + \frac{\partial V}{\partial t} (x,t) \leq 0, \quad \forall t \geq t_0,
\]

is called a Lyapunov function for the system (7). The existence of a Lyapunov function then guarantees that the state space’s origin is stable. By coordinate translation, considering a signal that comprises the difference between the state vector of a system and a non-zero equilibrium point of the same system, the power of this signal is an indication of how far the signal travels from the equilibrium point and hence, is a Lyapunov candidate function for the equilibrium point. In (Stork, Hrusak, & Mayer, 2005), this approach is referred to as the energy-metric approach. For a certain class of systems, this choice of candidate function does comprise a Lyapunov function for equilibrium points. By non-linear warping of the power as a function of the state variable \( x \), Lyapunov functions for a larger class of non-linear systems may be obtained (Guckenheimer & Holmes, 1997). Clearly, for a large class of systems, Lyapunov functions for studying stability are intimately connected with the concept of power and hence to energy, relative to an equilibrium point in state space.

Although not directly related to this paper’s focus, as a final note on Lyapunov functions, the reader is referred to the development presented in (Malisoff & Mazenc, 2009). As presented herein, Lyapunov functions are used to design controllers to satisfy specific stability requirements for the closed-loop system.

3.5. Passivity and the energy balance

It is noteworthy to show that passivity theory and the energy balance can be used as a unifying energy paradigm to perform both system control and FDD. Passivity theory is an established approach for stability analysis and control of non-linear systems (Brogliato, Lozano, Maschke, & Egeland, 2006; R Ortega et al., 2001). Consider a system with states \( x \in \mathbb{R}^n \), inputs \( u \in \mathbb{R}^n \), and outputs \( y \in \mathbb{R}^m \). The mapping \( u \mapsto y \) is called passive if there exists a state function \( H(x) \), bounded from below, and a non-negative function \( d(t) \geq 0 \) such that

\[
\frac{H[x(t)] - H[x(0)]}{\text{stored energy}} + \frac{d(t)}{\text{dissipated energy}} = \int_0^t u'(s)y(s)\,ds. \tag{14}
\]

A control action \( u(t) = \beta(x(t)) + v(t) \) may be applied such that the closed-loop system is again a passive system, with energy function \( H_d(x(t)) \), with respect to \( y \mapsto y') \), and such that \( H_d(x(t)) \) has a global minimum at a desired point \( x'(t) \). If a function \( \beta(x(t)) \) can be found for some function \( H_d(x(t)) \) such that

\[
-\int_0^t \beta'(x(s))y(s)\,ds = H_d(x(t)), \tag{15}
\]

then the closed loop system will also be passive, with input \( v(t) \) and an energy function

\[
H_d(x(t)) = H(x(t)) + H_d(x(t)). \tag{16}
\]

This methodology of assigning an energy function with a minimum at the desired values is generally referred to as energy shaping. In some cases the natural dissipation term may be replaced by some function \( d(t) \geq 0 \). This is called damping injection (Romeo Ortega, Schaft, Maschke, & Escobar, 2002).

The notion of passivity based control adopting the energy balance is extended to process supervision as previously discussed (W Chen et al., 2010). The system in (13) is dissipative with respect to the supply rate \( S(u,v) = y'Mu \) and storage function \( V(x) = x'Px/2 \) if

\[
PA + A'P \leq 0, \tag{17}
\]

and

\[
PB = C'TM. \tag{18}
\]

Assuming \( x(0) = 0 \), the energy balance is

\[
\frac{1}{2} \int_0^t x'(t)'Px(t)\,dt - \frac{1}{2} \int_0^t x'(t)'(PA + A'P)x(t)\,dt \geq \int_0^t y'Mudt. \tag{19}
\]

For the faulty case, (19) becomes an inequality. The energy balance for faults can therefore be expressed as

\[
E_f = \frac{1}{2} \int_t^T x'_f(T)'Px_f(T)\,dT - \frac{1}{2} \int_0^t x'_f(T)(PA + A'P)x_f(T)\,dT - \int_0^t y'Mudt. \tag{20}
\]

Fault detection is realised for component, actuator, and sensor faults by (20), (21), and (22) respectively.

\[
E_f = \int_0^t x'_f P\Delta x\,dt + \int_0^t x'_f P\Delta Budt \tag{21}
\]
\[ E_f = \int_0^T x_i^T P B f_i dt \]  
\[ E_f = -\int_0^T f_i^T M u dt \]  

In the equations, \( \Delta A \) and \( \Delta B \) are component faults, \( f_i \) is an actuator fault, and \( f_c \) is a sensor fault. To perform fault isolation, the type of energy change is first identified, i.e. stored or dissipative. Next, the fault location is established by writing (20) as a summation of energy-storing and energy-dissipating components, and checking a hypothesis involving the system states. For input-affine and Lagrangian non-linear systems, the energy balances can be written as (Wei Chen, 2011)

\[ V \left[ x(t) \right] - \int_0^T \frac{\partial V}{\partial x} f(x) dt = \int_0^T y^T u dt, \]  

and

\[ \left[ V(t) - V(0) \right] + \int_0^T \frac{\partial R}{\partial q} \frac{\partial q}{\partial x} dt = \int_0^T q^T u dt. \]  

4. A PERSPECTIVE ON REALISTIC SYSTEMS

Equivalent electric circuits is a suitable choice to describe large-scale industrial systems using energy:

- Energy and power are easily calculated in electric circuits.
- Equivalent electric circuits can be used to model systems in a variety of domains. In fact, in the years predating digital computers, differential equations could be solved by means of analogue computers (Howe, 2005). In an analogue computer, operational amplifier circuits are used to model phenomena and solve the resulting differential equations.
- In general, there is no restriction that the components in an equivalent electric circuit have to be linear. For example, hysteresis can be modelled by means of non-linear inductors. Similar, amplifiers with saturation can be represented by voltage or current sources combined with both normal or Zener diodes.

An example of a complex large-scale industrial system is the power conversion unit (PCU) of the pebble bed modular reactor (PBMR) concept (van Nickerk, Pritchard, van Schoor, & van Wyk, 2006). The PBMR PCU entails a three-shaft closed Brayton cycle and is depicted in Fig. 2. In addition to piping and valves, the PCU consists of a pebble bed nuclear reactor, high- and low-pressure turbines (HPT and LPT), a power turbine (PT), recuperator, pre-cooler, low-pressure compressor (LPC), intercooler, and a high-pressure compressor (HPC).

The PCU can be simplified by assuming that the heat exchangers are all actively controlled, resulting in constant outlet temperatures (Uren & van Schoor, 2013b). After simplification, the conceptual model of the PCU is given in Fig. 3. In this figure, the helium injection and extraction components are modelled by two externally controlled mass flow sources. The equivalent electric circuit for the hydraulic domain is shown in Fig. 4. In this model, the turbines and compressors are modelled by non-linear frequency dependent current sources, while pipe elements are modelled using RLC networks. A number of RC networks, voltage dividers, and amplifiers can be used to model each individual turbine in more detail (see block diagram model of a generic turbine (Dynamic models for fossil fueled steam units in power system studies, 1991)).
Note that the equivalent circuit shown in Fig. 4 is limited to a single domain. In the case of the PBMR, four domains, i.e. electrical, mechanical, hydraulic, and thermal, are tightly knit together into a single system. It is difficult to describe energy flows in such a system if the various domains are modelled by means of separate equivalent circuits. However, equivalent circuits representing different domains can be coupled by means of generalised transformers. Similar to the operating principle of an electrical transformer, a generalised transformer can be used to model the coupling between various energy domains (Cheng, Wang, & Arnold, 2007). In terms of electric circuit components, a generalised transformer can be constructed from two current controlled voltage sources. As an example, Fig. 5 shows the coupling between the mechanical and electrical domain in a magnetic energy harvesting device (Cheng et al., 2007). A single equivalent electric circuit can be obtained from the model by reflecting the impedances of one domain over to the other side (Cheng et al., 2007).

![Fig. 5. Equivalent circuits coupled by a generalised transformer.](image)

**5. ELECTRIC CIRCUIT CASE STUDY**

In the previous section, a connection was made between realistic systems and equivalent circuits. Therefore, to illustrate the application of energy as unifying concept for system optimisation, consider the simple non-linear electric circuit in Fig. 6. Non-linearities are introduced to realistically represent actual industrial systems. The circuit takes the form of a typical transmission system with a source $V_s$, a transmission impedance represented by non-linear resistances $R_{s1}$ and $R_{s2}$ associated with an inductance $L_{s1}$ and capacitance $C_{s2}$ respectively, and a load $R_L$.

The state space formulation of the circuit is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{L_{s1}} & -\frac{1}{L_{s1}} \\ \frac{1}{C_{s2}} & -\frac{1}{C_{s2}} \left( \frac{1}{R_{s2}} + \frac{1}{R_L} \right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{s1}} \\ 0 \end{bmatrix} V_s$$

with the state variables $x_1$ and $x_2$ denoting the current through $L_{s1}$ and the voltage across $C_{s2}$ respectively.

![Fig. 6. Simple electric circuit.](image)

In accordance with (Seshu & Reed, 1961), the power of the individual components $p_i$ adds up to zero given by

$$\sum_{i=1}^{N} p_i = 0.$$  \hspace{1cm} (27)

In this case, the total number of components $N$ equals six. The components can be grouped into power sources, energy storage elements, dissipative components, and load components. Equation (27) therefore translates to

$$\sum_{i=1}^{2} p_i = \sum_{i=1}^{4} p_{si} + \sum_{i=1}^{2} p_{di} = 0. \hspace{1cm} (28)$$

with $p_{si}$, $p_{di}$, and $p_{di}$ representing the sources, storage, dissipation, and load power components respectively. $P_i$, $P_{si}$, $P_{di}$, and $P_L$ represent the total power associated with the respective component groups.

An optimal operating point from a power loss perspective will be at maximum power efficiency. The power efficiency can be written in terms of $P_s$ and $P_L$ as follows

$$\eta_P = \frac{P_L}{P_s}.$$  \hspace{1cm} (29)

Given that $R_{s1}$ and $R_{s2}$ are non-linear functions of $x_1$ and $x_2$ respectively, the total dissipation losses is

$$P_s = \sum_{i=1}^{2} p_{si} = x_1^2 R_{s1}(x_i) + \frac{x_2^2}{R_{s2}(x_2)}.$$  \hspace{1cm} (30)

with the load power

$$P_L = \frac{x_2^2}{R_L}.$$  \hspace{1cm} (31)

Substituting (29) and (30) into (28) therefore results in

$$\eta_P = \frac{x_2^2}{x_2^2 + x_1^2 R_{s1}(x_1) + \frac{x_2^2}{R_{s2}(x_2)}}.$$  \hspace{1cm} (32)

For this case study, $R_{s1}$ and $R_{s2}$ are chosen to be non-linear functions of $x_1$ and $x_2$, as portrayed in Figs. 7 and 8 respectively. Fig. 9 shows the mesh diagram of power efficiency as a function of $x_1$ and $x_2$.

The theoretic maximum efficiency operating point $(x_{1m}, x_{2m})$ is given by

$$x_{1m}, x_{2m} = \arg \max \eta_P(x_1, x_2).$$  \hspace{1cm} (33)

This maximum point will be one of the critical points of $\eta_P$, determined from the partial derivatives of $\eta_P$

$$\eta_P(x_1, x_2) = \frac{\partial}{\partial x_1} \eta_P(x_1, x_2) = 0$$

$$\eta_P(x_1, x_2) = \frac{\partial}{\partial x_2} \eta_P(x_1, x_2) = 0$$

where $\eta_P(x_1, x_2)$ is the efficiency as a function of $x_1$ and $x_2$. This maximum point is the point at which the total dissipation losses are minimised, resulting in maximum power efficiency.
and

\[ \eta_p(x_1, x_2) = \frac{\partial}{\partial x_2} \eta_p(x_1, x_2) = 0. \]  

(35)

Applying (34) and (35) to (32) results in

\[ 2R_{x1} + x_1 \frac{d}{dx_1} R_{x1} = 0 \]  

(36)

and

\[ 2x_2 R_{x2} + \frac{x_2^2}{R_{x2}^2} \frac{d}{dx_2} R_{x2} = 0. \]  

(37)

Solving the expressions in (36) and (37) will give the maxima and minima of \( \eta_p \), including the point \((x_{1m}, x_{2m})\). The simulation results in Fig. 9 produced a maximum power efficiency point of (2.98, 100.55), corresponding with an efficiency of 98.74%.

The concept of reachability (Ohta, Maeda, & Kodama, 1984) now becomes important to determine whether the point of maximum efficiency is reachable within finite time using a specific input. The concept of least norm input for reachability (Boyd, Ghaoul, Feron, & Balakrishnan, 1994) can be used to realise the desired state transition with the least amount of energy input.

In the context of global system optimisation through modelling, supervision, and control (the latter two are not shown), this case study demonstrates how an equivalent electric circuit can serve to represent an energy model with associated state space that is suitable for dissipativity analyses. The simple circuit representation of Fig. 6 can be extended to a more general system representation constituting sources (current and voltage), energy storage elements (inductive and capacitive), dissipative elements, and loads of which some elements can also be non-linear functions of the system states. The development of generic topological representations of such electric circuits is a topic for continued research, and is therefore not included in this paper.

6. AN ENERGY-BASED FRAMEWORK

The fact that energy is a universal concept that holds across different domains is the reason for us to focus on the use thereof for large-scale industrial system analyses. It is envisioned that unified energy formalisms can be established to facilitate modelling, supervision, and control of these energy systems. Fig. 10 depicts the method implied by the vision. The system block represents any large-scale industrial system such as a power or petrochemical plant. In these systems, the principles described will not only apply on a global systems level, but also on sub-system or even component level.

Reviewing Fig. 10, the examined system is firstly transformed to an equivalent electric circuit. The process of abstraction to obtain an energy signature implies obtaining a representation of the system based on an energy formalism. This can for instance denote the system’s energy distribution in terms of energy supplied, stored, transported, and dissipated. The energy formalism is then used for feature extraction to describe a reference energy signature. This transformation will entail finding a global optimum in terms of some global system objectives, encapsulated in an optimal energy distribution profile. It is envisaged that such an optimal energy distribution will be associated with an optimal set of system states. Comparing an actual energy signature with the reference case will serve to evaluate and supervise
system performance. The process of compiling an actual energy signature may include some measurement inference or estimation of system variables. Analogous to normal feedback control, the comparison of energy signatures yields a set of residuals. If this energy mismatch violates certain bounds, a fault is detected. The diagnosis task then establishes the fault location and type based on the energy fault signature. A control action finally aims to restore or optimise system performance (may also include system maintenance or conditioning).

The link between complex multi-domain large-scale industrial systems and equivalent electric circuits provide a unified representation of multi-domain systems. The framework proposed, forms a direct analogy with conventional control theory, using energy as a basis for global system performance optimisation. The greatest challenge in terms of the proposed framework is the abstraction of usable energy formalisms and the subsequent transformation to energy signatures. This warrants further research to explore the viability of the energy-based framework.

REFERENCES


