EPAIA: Design, modelling and control of a novel electro-pneumatic adaptable impedance actuator

Berno J.E. Misgeld, Joergen Stille, Anake Pomprapa, Steffen Leonhardt

Chair for Medical Information Technology, Helmholtz-Institute for Biomedical Engineering, RWTH Aachen University, Aachen, Germany (e-mail: misgeld@hia.rwth-aachen.de).

Abstract: In this contribution we present the novel electro-pneumatic adaptable impedance actuator (EPAIA) consisting of a brushless DC-motor and a rotational pneumatic actuator, applied in series to the gear train. The pneumatic actuator is employed as a pneumatic torsional spring with a continuously adaptable stiffness, controlled by air inflow and pressure control. Thus, a rotary variable stiffness actuator is obtained, for which the stiffness can be expressed as a smooth nonlinear function of chamber pressures. The mechanical design furthermore utilises a planetary and a bevel gear reduction, to provide a torque amplification ratio necessary for human assistive knee joint actuation. Besides large and precise torque generation, the actuator provides a smoothly adjustable output impedance, backdrivability and a low cost, yet quality sensitive torque sensor. Following a detailed description of the mechanical design of the actuator, a dynamical model is derived using the Lagrange formalism. We propose a robust $H_2$-torque-controller design procedure for the linearised actuator model, for which a constraint for passivity of the load transfer function is employed. The resulting controller is tested in simulations using the nonlinear validated model.

Keywords: rehabilitation robotics, Euler-Lagrange modelling, electro-mechanical, pneumatic, control

1. INTRODUCTION

Human and animal locomotion has to be stable in face of an uncertain environment undergoing unknown disturbances. The human and animal musculo-skeletal system is thus well equipped with the ability to modulate mechanical impedance by antagonistic co-activation of muscles and nonlinear sinews, attached to the joint. The mechanical impedance, in this context, is the static and dynamic relationship between muscle-tendon complex (MTC) and velocity due to imposed stretch. Its important role in general locomotion and interaction with variable environments was investigated in the 1980’s (Hogan (1984)) and still is an active research topic. Early investigations on muscular impedance working on a joint during human locomotion was for example reported by Cavagna et al. (1977). In addition to the compliant behaviour, necessary when interacting with the environment, the MTC is able to store and release energy in cyclic or explosive movements (Misgeld et al. (2012)). These properties are highly desirable in rehabilitation robotics applications, where actuator and trajectory control has to guarantee stable interaction with variable degree of support or compliant behaviour of the robotic manipulator. Force or torque generating industrial manipulators are classically designed as highly stiff systems and are suited for tasks like high bandwidth positioning involving limited contact to the environment. In contrast to that, a new form of actuators that comprise an additional compliant element was introduced in the 1990’s (Pratt and Williamson (1995); Pratt et al. (1995)). First applied to hydraulic and electric actuators, the use of an elastic element (spring), in series with actuator and load, was motivated by a number of reasons. Among the most prominent reasons to use a compliant element in an otherwise stiff actuator is safety, which includes a stable human robot interaction and the minimisation of high energies due to end-effector collisions, Zinn et al. (2003). Other reasons are for example the energy storage in cyclic and explosive movements (Grebenstein et al. (2011)) and a cheap torque sensor, assuming the stiffness of the compliant element is known and the rotation angle difference of the spring can be measured. Compliant actuators have found their way into many application areas, like for example walking robots, robots that consider safe human-robot interaction and exoskeletons for rehabilitation and movement augmentation. Different compliant actuators have been introduced in the last two decades, but one of the main drawbacks, besides the additional mechanical element, is the limited bandwidth imposed by a fixed elastic element. This limitation is partially lifted by an active impedance control (or virtual stiffness control), where a virtual stiffness is set by a controller, that is closed around the impedance control-loop. However, the stiffness value cannot be increased above the stiffness value of the compliant element, without providing energy to the compliant element (Vallery et al. (2007)). Therefore, most recent approaches consider offline or online adjustable compliant elements (Ham et al. (2009)). However, drawbacks are the need for a second actuator and an often complicated mechanical set-up leading to heavy designs. Depending on the technology used, the compliant element may not be continuously adaptable under varying load conditions and may show adverse effects like backlash and hysteresis.

In this contribution, and in contrast to previous approaches, we present the design of a new continuously adjustable stiffness actuator, the so called Electro-Pneumatic Adaptable Impedance Actuator (EPAIA). EPAIA is designed to be applied, but not limited, to a rehabilitation robotics scenario. In order to guar-
The electro-pneumatic actuator is shown in Fig. 1 and consists of a brushless direct current (BLDC)-motor with encoder and planetary gear box (d), an electronic power and control unit, a pneumatic torsional spring (c), a proportional valve, two pressure sensors, a second encoder (b), a bevel gear box (a) and a housing. In order to be used in movement assistance, the actuator has to guarantee a certain maximum torque and angular velocity, leading to a maximum (nominal) motor power. For our case, providing assistance to a knee joint, the maximum power is derived from kinematic data (Kong et al. (2012)). The data is obtained for a healthy person of 70 kg showing an average power of 70-80 W, needed for sufficient support. Including a safety margin of factor 2, covering transmission power reduction due to non-ideal mechanical elements, an electrical motor of 150 W is chosen (EC 45, Maxon Motor AG, Sachseln, Switzerland) at the motor was selected with a reduction ratio of 26:1. The resulting maximum torque of the BLDC motor, disregarding any efficiency reduction of the gears, is 174 mNm. However, with a maximum stall torque of 1656 mNm the maximum load torque over a short period of time can be 667.5 mNm. The pneumatic torsional spring is chosen considering maximum torque, minimum constructed space and the resulting stiffness range, which depends on chamber pressure and volume. As a rotary pneumatic actuator, a relatively small actuator was selected (M/60284/TI, Norgen GmbH, Alpen, Germany), which is able to deliver 9.5 Nm at 6 bar chamber pressure. Table A.1 in Appendix A lists all important parameters for the pneumatic rotary spring. Since for a higher stiffness of the pneumatic spring a small chamber volume is of paramount importance, blocking valves were implemented in the first prototype (153467 HE-2-QS-6, Festo, Esslingen am

2. MECHANICAL DESIGN OF EPAIA

Three exemplary curves at low medium and high chamber pressure show the possible range of stiffness values. Since the pneumatic pressure supply tubes of the rotational pneumatic spring are mechanically guided via the housing, sufficient space for uncoiling has to be provided for the knee rotation angle range times gear reduction ratio. Thus, to obtain the required torque, the planetary gearbox (GP 52, Maxon Motor AG, Sachseln, Switzerland) at the motor was selected with a reduction ratio of 26:1. The resulting maximum torque of the BLDC motor, disregarding any efficiency reduction of the gears, is 174 mNm. However, with a maximum stall torque of 1656 mNm the maximum load torque over a short period of time can be 667.5 mNm. The pneumatic torsional spring is chosen considering maximum torque, minimum constructed space and the resulting stiffness range, which depends on chamber pressure and volume. As a rotary pneumatic actuator, a relatively small actuator was selected (M/60284/TI, Norgen GmbH, Alpen, Germany), which is able to deliver 9.5 Nm at 6 bar chamber pressure. Table A.1 in Appendix A lists all important parameters for the pneumatic rotary spring. Since for a higher stiffness of the pneumatic spring a small chamber volume is of paramount importance, blocking valves were implemented in the first prototype (153467 HE-2-QS-6, Festo, Esslingen am

Fig. 1. EPAIA dimensions and components: (a) Bevel gear, (b) load encoder, (c) pneumatic spring, (d) Maxon motor EC45 with planetary gear and encoder.

Fig. 2. Stiffness of the pneumatic torsional spring, evaluated at different initial chamber operating pressures and over pressure range.
3. SYSTEM MODELLING

The actuator system is modelled by using the Lagrange formalism. Consider the Euler-Lagrange equations

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i = \frac{\partial W}{\partial q_i}, \]

for which the number of external forces is \( \tau_i \), the external work is \( W \). The number of generalised coordinates \( q_i \) are expressed by the column vector

\[ q^T = [q_1 \ q_2 \ \cdots \ q_i \ \cdots q_M] \]

for the \( M \) degrees-of-freedom system. In (1), \( L \) is the Lagrange-function, which is a scalar function defined as the difference between kinetic \( T(q_i, \dot{q}_i) \) and potential \( V(q_i) \) energy

\[ L = T(q_i, \dot{q}_i) - V(q_i). \]

(3)

In the following section, the Lagrange-function is set-up for each subsystem, with a subsequent derivation of the Euler-Lagrange equations. For the sake of clarity and to avoid confusion with electrical charge, we will use the original physical variables instead of \( q \).

Generalised coordinates for the electrical motor are the electrical charge \( q_M \) and the angle of the mechanical shaft \( \phi_M \) (with derivatives \( I_M \) and \( \omega_M \), respectively). The kinetic energy of the system consists of energy, stored in the armature coil and the rotation of the mechanical shaft

\[ T = \frac{J_M}{2} \omega_M^2 + \frac{L}{2} I_M^2 \]

(4)

with \( J_M \) the rotor inertia, \( L \) the armature inductivity, \( \omega_M \) the revolution speed and \( I_M \) the BLDC-motor current.

The potential energy of the system consists of energy, stored in the pneumatic spring

\[ V = \frac{c_{PS}(\phi, \omega)}{2} \phi^2 \]

(5)

with \( c_{PS} \) the stiffness of the spring, \( \omega \) is the basic chamber pressure and \( \phi \) the angle of deflection. The external forces acting on the electrical system can be stated as

\[ \partial W_e = U_M \partial q_M - R_M \partial q_M - K_{emf} \partial q_M \]

(6)

in which \( U_M \) is the external motor supply voltage, \( R_M \) is the armature resistance, \( q_M \) is electrical charge and \( K_{emf} \) is the back-electromotive force (EMF). The external forces acting on the mechanical system are given by

\[ \partial W_m = -M_{R1} \phi_M \partial \phi_M - \frac{N_1}{N_2} d_{PS}(\phi) \left( \frac{N_1}{N_2} \phi_M - \phi_L \right) \partial \phi_M \]

\[ + K_{M, emf} I_M \partial \omega_M \]

with \( M_{R1} \) the rotor viscous friction, \( \phi = \frac{N_2}{N_1} \phi_M - \phi_L \) is the spring deflection angle, \( \phi_L \) is the load angle, \( d_{PS} \) is the damping of the spring and \( M_{PS} \) is the moment over the pneumatic spring. The differential equations for the BLDC-motor follow as

\[ \frac{d}{dt} I_M = -\frac{R}{L} I_M - \frac{K_{emf}}{L} \omega_M + \frac{1}{L} U_M \]

\[ \frac{d}{dt} \omega_M = \frac{K_{M, emf}}{J_M} \frac{M_{R1}(\omega_M)}{J_M} \left( \frac{N_1}{N_2} \right)^2 d_{PS}(\phi) \frac{\phi}{J_M} \]

\[ - \frac{N_1}{N_2} \frac{c_{PS}(\phi, \omega)}{J_M} \frac{\phi}{J_M} \omega_M \]

\[ \frac{d}{dt} \phi = \frac{N_1}{N_2} \frac{\omega_M - \omega_L}{J_M} \]

in which \( M_{R1}(\omega_M) \) is nonlinear friction and \( c(\phi, \omega) \) is the stiffness of the torsional pneumatic spring. We assume an isothermal process for the torsional pneumatic spring with \( p_{a1} V_{a1} = p_{a2} V_{a2} \), denoting the system change under constant temperature. The moment acting on pneumatic impeller is given by

\[ M(p_c) = \int_{r_1}^{r_2} p_c l_i r dr \]

\[ = p_c l_i \frac{1}{2} (r_2^2 - r_1^2) = p_c k_{volume} \]

(9)

(10)
where \( p_c \) is the pressure of a single chamber, \( l_i \) is the length of the pneumatic impeller, and \( r_1 \) and \( r_2 \) are the start and end of the impeller, as seen from torsional pneumatic spring centre, respectively. The volume of the chamber \( V_c \) decreases or increases with the impeller deflection \( \phi \) as

\[
V_c = V_c \frac{\theta_p + \phi}{\theta_p} + V_{\text{tub}},
\]

(11)

in which \( V_c \) denotes the total volume and \( \theta_p \) denotes the maximum deflection angle. Combining equations (10) and (11) leads to the following equation

\[
M_c \left( \phi, p_c \right) = \frac{p_c V_c + V_{\text{tub}}}{V_c \frac{\theta_p + \phi}{\theta_p} + V_{\text{tub}}} - k_{\text{volume}}.
\]

(12)

The moment thus depends on the basic chamber pressure \( p_b \) and an angle deflection \( \phi \). The torsional pneumatic spring consists of four chambers of which two in each case are connected and two are arranged such, that they are opposing each other. On a nonzero deflection angle, the air in the two chambers is compressed while at the same time an expansion of the air in the other two chambers takes place. The following expression for the moment is obtained

\[
M_{PS}(\phi, p_b) = 2M_{n1} - 2M_{n2} = 2p_b (V_c + V_{\text{tub}}) k_{\text{volume}} \frac{-2 \theta_p V_c}{V_c \frac{\theta_p + \phi}{\theta_p} + V_{\text{tub}}} \phi^2.
\]

(13)

and the stiffness of the spring

\[
c_{PS}(\phi, p_b) = 4p_b (V_c + V_{\text{tub}}) k_{\text{volume}} \frac{- \theta_p V_c}{V_c \frac{\theta_p + \phi}{\theta_p} + V_{\text{tub}}} \phi^3.
\]

(14)

Defining the model outputs as

\[
\begin{align*}
\omega_c & = \text{sign}(\omega_d) + M_{f1}\omega_d, \\
d_{PS} & = M_{fPS}*\text{sign}(\Delta \omega) + M_{fPS1}*\Delta \omega.
\end{align*}
\]

(15)

(16)

The friction in the motor and damping term of pneumatic spring can be modelled by

\[
M_{R1}(\omega_d) = M_{fM0}*\text{sign}(\omega_d) + M_{fM1}*\omega_d
\]

and

\[
d_{PS}(\Delta \omega) = M_{fPS0}*\text{sign}(\Delta \omega) + M_{fPS1}*\Delta \omega.
\]

4. CONTROLLER DESIGN

As the first step of the design procedure, a motor current controller is designed using the linear quadratic regulator approach, minimising the cost function

\[
J(u) = \int_0^\infty \left[ x^T Q x + u_{\text{in}} R u_{\text{in}} \right] dt,
\]

(18)

with respect to the linearised plant (17) and with weightings \( Q = Q^T \geq 0 \), defined here as diagonal state-weighting matrix

\[
Q = \text{diag}(q_1, q_2, 0, 0)
\]

and the control input weighting \( R \in \mathbb{R}_+ \). The current controlled state space model, represented by matrices \( (A_1, B_1, C_1, D_1) \), is subsequently brought to transfer function matrix representation \( G(s) = C(sI - A_1)^{-1}B_1 + D_1 \). High frequency dynamics of the transfer function

\[
G(s) = \frac{1}{(s + \alpha_1)(s + \alpha_2 s + \alpha_3)},
\]

(19)

\[
\begin{bmatrix}
\beta_{11}s & \beta_{12}(s + \alpha_1)(s + \alpha_2) \\
\beta_{21} & \beta_{22}(s + \alpha_1)(s + \alpha_2)(s + \eta)
\end{bmatrix},
\]

corresponding to the pole of the current dynamics \( p_1 = -\alpha_1 = 5.723 \times 10^3 \text{ rad/s} \) are neglected. In (19), system parameters \( \alpha_2, \alpha_3 \) and \( \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22} \) are obtained from state-space conversion and have to be corrected for the numerator constants to \( \tilde{\beta}_{11} = \frac{p_1}{\alpha_1} \text{ and } \tilde{\beta}_{21} = \frac{p_1}{\alpha_1} \). The system of reduced order and input dimension \( G_{\text{red},1}(s) : u_{\text{in},1}(s) \mapsto [u_{\text{in}}(s) M_{PS}(s)]^T \) is given by

\[
G_{\text{red},1}(s) = \frac{1}{s^2 + \alpha_2 s + \alpha_3},
\]

(20)

where \( u_{\text{in},1} \) is the new control input variable of the current controlled system. We propose a post-compensator for (20), which is based on the idea to place the compensator induced transmission zero such, that the phase of the plant near crossover frequency is increased, obtaining a squared down system. Let the Smith and the Smith-McMillan form for the system (20) be given by

\[
U(s)S(s)V(s) = \begin{bmatrix} \frac{1}{\beta_{11}} & 0 \\ \frac{1}{\beta_{21}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{\beta}_{21} = \left[ \begin{array}{c} \tilde{\beta}_{11} \\ \frac{1}{\beta_{21}} \end{array} \right].
\]

(21)

We define post-compensator by introducing the row vector \( C_c(s) = [a] \), in which \( a \) is a proportionnal-gain applied to the motor speed and the property of the open-loop torque function over the pneumatic spring \( M_{PS} \) should not be changed by \( C_c(s) \) in the low frequency range. By using (21), the set of transmission zeroes can be obtained by solving

\[
\det(C_c(s) \begin{bmatrix} \tilde{\beta}_{11} \\ \frac{1}{\beta_{21}} \end{bmatrix}) = 0,
\]

for which we obtain

\[
z_t = \frac{\frac{1}{\beta_{21}}}{\frac{1}{\beta_{11}} - a}.
\]

The transmission zero is placed such that phase and bandwidth of the open-loop squared down system are increased for torque interaction control. Towards this goal, we choose a zero position \( z_t = 1.9518 \alpha_{PS} \), with \( \alpha_{PS} \) the crossover frequency of the open-loop transfer function from \( u_{\text{in},1}(s) \) to \( M_{PS}(s) \). The post-compensator parameter results in \( a = 0.038 \text{Hz} \), which leads to an increase of 10.5 deg. in phase and 0.5 dB in magnitude at the crossover frequency \( \alpha_{PS} \).

The reduced-order squared down plant \( G_{\text{red}}(s)C_c(s) \) is subsequently extended by weighting functions to shape sensitivity
Fig. 4. Augmented plant with post-compensator and weighting functions (impedance function \(Z(s) = \frac{z_3(s)}{w_2(s)}\)).

and complementary sensitivity functions. A block diagram, depicting the structure of the generalised plant is, shown in Fig. 4. Note that the plant \(G_{red}(s)\) includes the second input \(\omega_2(s)\), which is used as an external input \(w_2(s)\) in order to minimise the load impedance function \(Z(s) = \frac{M_{red}(s)}{w_2(s)}\). The two additional complex weighting functions are \(W_1(s)\) and \(W_2(s)\). To limit the control effort, \(W_2(s)\) is introduced as a lead-lag complex weighting to the control input \(u(s) = c_{u,1}(s)\) as

\[
W_2(s) = \frac{k_2(z_2s + 1)}{p_2s + 1},
\]

(22)
corresponding to the closed-loop transfer function \(R(s) = K(s)(1 + GK(s))^{-1}\), with the static gain \(k_2\), the zero inverse \(z_2\) and the pole inverse \(p_2\). The parameters were determined according to desired sensor noise attenuation and are listed in Tab. A.3 in Appendix A. Tracking performance of the closed-loop plant is specified for the sensitivity transfer function, penalising the control system error \(e(s)\) by \(W_1(s) = W_{11}(s)W_{12}(s)\) with

\[
W_{11}(s) = \frac{k_{11}(z_{11}s + 1)}{p_{11}s + 1},
\]

(23)
Here, \(W_{11}(s)\) is designed to yield integral action with a tolerable steady-state error (low frequency range), a certain crossover frequency and minimisation of peak sensitivity gain \(M_r = \|S(s)\|_{\infty}\), for frequencies larger than the 3dB bandwidth of the sensitivity transfer function. An additional phase advance for frequencies, higher than the crossover frequency of the open-loop compensated plant \(\omega_2\) is introduced by the first-order lead-lag weighting

\[
W_{12}(s) = \frac{z_{12}s + 1}{p_{12}s + 1},
\]

(24)
in which \(z_{12}\) and \(p_{12}\) correspond to the desired lead-lag weighting and \(p_{12,HP}\) to an additional pole introduced at high frequencies to guarantee the generalised plant matrix \(D_{11} = 0\). The generalised plant is thus given by the interconnection of the transfer functions as

\[
P(s) = \begin{bmatrix} W_1(s) & 0 & -W_1(s)C_1(s)G_{red,21}(s) \\ 0 & 0 & 0 \\ G_{red,11}(s) & 0 & -C_1(s)G_{red,21}(s) \end{bmatrix}
\]

(25)
for which the linear time-invariant state-space model with \(D_{11} = 0\) and \(D_{22} = 0\) is obtained by

\[
\begin{align*}
\frac{d}{dt}x &= A_x + B_1w + B_2u \\
z &= C_1x + D_{12}u \\
y &= C_2x + D_{21}w.
\end{align*}
\]

(26)
Standard controllability and observability conditions for (26) are thus obtained by a minimal realisation and \(D_{12}^T D_{12} > 0\) and \(D_{21}^T D_{21} > 0\). Since orthogonality assumptions are not given for (26), \(D_{12}^T C_1 \neq 0\) and \(D_{21}B_1^T \neq 0\), a loop-shifting procedure is applied to (26) to result in the transformed generalised plant

\[
P = \begin{bmatrix} A & B_1 & B_2 \\ \tilde{C}_1 & 0 & 0 \\ \tilde{C}_2 & 0 & 1 \end{bmatrix} = N_iP\tilde{N}_i,
\]

(27)
where the transformation matrices \(N_i = \text{diag}(I,S_w,S_u)\) and \(N_i = \text{diag}(I,S_0,S_0)\) are obtained from a singular value decomposition. The \(H_2\)-control problem is extended with a positive-real constraint on the controller. For the closed-loop transfer function, given by the lower linear fractional transformation \(H_{zw}(s) = \mathcal{F}_2(P(s),C(s))\), the minimisation

\[
\min \left\| H_{zw}(s) \right\|^2_2: C(s) \in \mathcal{P}_C
\]

(28)
leads to a synthesised controller \(C(s) \in \mathcal{P}_C\) that belongs to the family of positive-real transfer functions. Towards that goal, we use the approach of (Germol and Gapski (1997)) which is rearranged for the given problem and applied to the transformed generalised plant of (27). The consecutive steps are as follows. Solve the continuous-time algebraic Riccati equation, associated with observer feedback of the transformed plant (27)

\[
Y_2(A - \tilde{B}_2\tilde{D}_{21}^T\tilde{C}_2)^T + (A - \tilde{B}_2\tilde{D}_{21}^T\tilde{C}_2)Y_2
\]

\[
- Y_2\tilde{C}_2\tilde{C}_2^Ty_2 + \tilde{B}_1\tilde{E}_2\tilde{B}_2^T = 0,
\]

(29)
for which a stabilising solution is obtained by \(L = L_2 = \tilde{Y}_2\tilde{C}_2 + \tilde{B}_1\tilde{D}_{21}\) for the given controller structure

\[
\frac{d}{dt}x = A_x + \tilde{B}_1\tilde{u} + L_2(y - \tilde{C}_2x),
\]

\[
u = -A_kx,
\]

(30)
with \(A_k = A - \tilde{B}_2K\) and transfer function matrix

\[
\tilde{C}_K = K(sI - [A_k - L_2\tilde{C}_2])^{-1}L_2.
\]

(31)
As a second step, we restate the \(H_2\)-SPR problem in terms of its system of LMI's. For that, we define the fixed matrices

\[
\tilde{A} = A - \tilde{B}_2\tilde{D}_{21}\tilde{C}_1, A_2 := \tilde{A} - L_2\tilde{C}_2, \text{ and } Q_2 := \tilde{B}_2\tilde{L}_2 + L_2\tilde{B}_2.
\]

Then the linear matrix inequality constraint corresponding to the SPR-condition is given by

\[
WA_2^T + A_2W - Q_2 \leq -\varepsilon I < 0
\]

(32)
with matrix decision variable \(W = W^T > 0\) and \(\varepsilon > 0\), an arbitrary small parameter. To obtain the controller gain matrix
5. RESULTS

The continuous time controller was discretised at $T_c = 0.0025s$ and implemented with a high-gain anti-windup feedback strategy (Hyde (1995)) to prevent excessive overshoot in cases of system input current limitation. Nonlinear model and controller were implemented in MATLAB/Simulink. For measurements at different pneumatic torsional spring stiffness values, both chambers of the pneumatic spring were initialised at chamber pressures, ranging from 1 to 7 bar. A number of tests for reference tracking, disturbance rejection and load impedance measurements were conducted with the closed-loop impedance controller and at varying load conditions. Fig. 5 shows the worst-case example of a step response at fixed load and with varying pneumatic chamber pre-pressures of 1, 4 and 7 bar. As a reference value a unit step was applied to the impedance controller, corresponding in commanded input voltage and motor current. It can be easily seen that although the motor current is not in saturation and the system response is sufficiently fast for active knee support, with a rise time $t_{90}$ of 0.25s for the stiff pre-setting. The controller was tested in extensive simulations and over varying operating points of varying amplitude. Similar results like those shown in Fig. 5 were obtained in step like reference setpoint changes with larger torque amplitude, resulting in a nonlinear spring characteristic. However, note that varying the varying spring constant was not explicitly addressed in the controller design.

In addition to extensive reference tracking and disturbance rejection tests, the load impedance function $Z(s) = \frac{M_{ps}(s)}{u(s)}$ was analysed in closed-loop condition using the nonlinear plant. Fig. 6 shows the bode plot of the linearised feedback-controlled system. The range of possible load impedance variation for different pneumatic spring constants is given in Fig. 6. In addition, the positive-real property of $Z(s)$ is satisfied for all possible pneumatic spring stiffness values.

6. DISCUSSION AND OUTLOOK

In this contribution, the modelling and the design of an impedance controller was proposed for the novel type of an electro-pneumatic adaptable impedance actuator EPAIA. Using a detailed linearised version of a nonlinear model, obtained by the Lagrange formalism, a norm optimal $H_2$-controller was proposed to include an additional constraint leading to a strictly positive real controller design. The controller design procedure was rearranged to be better suited for loop-shaping approaches leading to a convex optimisation problem and was applied to the generalised plant, transformed by a similarity transformation to relax orthogonality assumptions. The controller design procedure included a squaring down approach with a resulting...
Appendix A. MODEL AND DESIGN PARAMETERS

Table A.1. Parameters of the pneumatic torsional spring.

<table>
<thead>
<tr>
<th>Description</th>
<th>Param. Value Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter + tubes &amp; sensors</td>
<td>m</td>
</tr>
<tr>
<td>Spring constant</td>
<td>N/m rad</td>
</tr>
<tr>
<td>Spring friction</td>
<td>N/m rad</td>
</tr>
</tbody>
</table>

Table A.2. Mechanical parameters of EPAIA.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value Unit</th>
<th>Param.</th>
<th>Value Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>m</td>
<td>L</td>
<td>N/m rad</td>
</tr>
<tr>
<td>K_{mf}</td>
<td>V/m rad</td>
<td>I_{max}</td>
<td>A</td>
</tr>
<tr>
<td>J_M</td>
<td>cm³</td>
<td>U_N</td>
<td>V</td>
</tr>
<tr>
<td>N_1</td>
<td></td>
<td>N_2</td>
<td></td>
</tr>
<tr>
<td>V_c</td>
<td>m³</td>
<td>k_{volute}</td>
<td>m³</td>
</tr>
<tr>
<td>α_1</td>
<td></td>
<td>α_2</td>
<td></td>
</tr>
<tr>
<td>α_3</td>
<td></td>
<td>β_11</td>
<td>10⁶</td>
</tr>
<tr>
<td>β_12</td>
<td></td>
<td>β_21</td>
<td>10³</td>
</tr>
<tr>
<td>β_12</td>
<td></td>
<td>β_21</td>
<td>10³</td>
</tr>
<tr>
<td>α_4</td>
<td></td>
<td>γ</td>
<td>75.25</td>
</tr>
</tbody>
</table>

Table A.3. CONTROLLER WEIGHTINGS AND PARAMETERS.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value Unit</th>
<th>Param.</th>
<th>Value Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_p</td>
<td></td>
<td>z_p</td>
<td>1.25</td>
</tr>
<tr>
<td>p_z</td>
<td>10⁻²</td>
<td>k_z</td>
<td>1000</td>
</tr>
<tr>
<td>z_{i1}</td>
<td>2.94·10⁻²</td>
<td>r_{i1}</td>
<td>2.94·10⁻²</td>
</tr>
<tr>
<td>z_{i2}</td>
<td>9.55·10⁻³</td>
<td>r_{i2}</td>
<td>1.11·10⁻²</td>
</tr>
<tr>
<td>P_{2,4,6}</td>
<td>1·10⁻³</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REFERENCES


