Modeling and Identification of the Restoring Force of a Marine Riser

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Abstract: The purpose of this paper is to provide a novel model to characterize the nonlinear restoring force of a marine vertical riser by using position-and-velocity-dependent polynomials. This model permits the obtaining of a specific state space representation—via the Liénard transformation—for the design of state observers that identify the structural parameters of vertical risers. The main results presented here are: (i) an approximation of the nonlinear restoring force by means of polynomials and its incorporation into a distributed parameter (DP) model, (ii) the transformation of the DP model into a Liénard system and (iii) an analysis of its observability and identifiability properties.

Keywords: Marine systems, pipelines, structural parameters, parameter identification, state observers

1. INTRODUCTION

The marine risers are underwater pipelines that perform large displacements and vibrations because of external forces. Risers adopt an important role in the extraction of petroleum from the sea (Lee (2009)) as the connection between a platform and an oil wellhead located on the seabed. Their main use is the transportation of the crude oil or sludge when a well is drilled; additionally, they can be used to safeguard the drilling column. Dynamic behavior of a riser can be modeled numerically as a harmonic oscillator with distributed parameters (masses, springs and dampers) along its structure, which is in contact with several outward forces (ocean currents, waves, platform motion) that determine its behavior over time (Niedzwiecki and Liagre (2003), Furnes (2000)).

If a structure is excited by external forces with frequencies near its natural frequency, the structure vibration is amplified rendering the entire system at risk of becoming unstable. This phenomenon is known as resonance. In mechanical structures, vibrations cause wear and can produce anomalies with undesirable outcomes. Therefore, vibration and resonance are widely studied phenomena, particularly in the reliability and assessment of building constructions Doebling (1996).

Vibration is present during the exploration and exploitation processes of petroleum in deep waters. This is caused by the force exerted by ocean currents, vortexes, the waves moving the platform and the wind on the oil extraction structures. Riser vibration induces mechanical stress, problems of fatigue and crack propagation, which require expensive inspections and repairs. Furthermore, the majority of the crude oil fields is localized in zones prone to extreme weather such as hurricanes, cyclones, polar storms, etc. Thus, the installations are always susceptible to structural damages.

To avoid economic losses and environmental damage, considering automatic on-line monitoring systems is necessary in order to estimate structural changes in the risers, i.e., structural health monitoring systems (SHMs). Several methodologies have been developed to monitor the structural conditions of the risers; one of these is the use of dynamic data—obtained by acceleration and vibration measurement techniques—which continuously update the parameters of a structural model. Among these works one finds Ghanem and Shinozuka (1995), Shinozuka and Ghanem (1995) and Doebling (1996), which are widely used—mainly in the case of land structures—to treat the parameter identification in a linear dynamical context. In the case of marine risers, however, the parameter identification problem must be tackled with nonlinear techniques, because these systems have a strong nonlinear behavior because of large displacements of the floating system caused by the environmental loads. In general, there are several factors that determine the nonlinear behavior of a riser modeled by a nonlinear restoring force: (i) the oil flowing inside the riser and its interaction with the inner walls which have a constitution that is variant according to the building materials, (ii) the interaction of the outer walls with the sea and (iii) the vibration induced vortexes (VIV).

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In this regard, many identification techniques have been developed for the identification of nonlinear systems, which can be classified according to seven categories: linearization-based methods, time-domain and frequency domain methods, modal methods, time-frequency analysis, black-box modeling and structural model updating. The reader may consult Kerschen et al. (2006) for an overview of their application on structural system identification. Another option to identify parameters of nonlinear systems (in the time domain) is the use of state observers (Busvelle and Gauthier (2002); Jiang et al. (2004); Besançon and Ticlea (2007)), which have already been employed in structural parameter identification, e.g., Lin and Betti (2004), Garrido et al. (2004), Angeles and Alvarez-Icaza (2005) and Jiménez-Fabián and Alvarez-Icaza (2010).

Various works have proposed models for the nonlinear restoring force of marine risers, e.g., force-decomposition models (Sarpkaya, 2004), single-degree-of-freedom (SDOF) models (Basu and Vickery, 1983), and wake-body coupled models. In Panneer-Selvam and Bhattacharyya (2001), the authors developed an iterative scheme for the identification of the hydrodynamic coefficients in a Morison type model and included in their analysis a nonlinear stiffness parameter (Duffing coefficient). In Bishop and Hassan (1964), the authors suggested the use of a Van der Pol oscillator to describe the time-varying forces. In Violette et al. (2007), a weak-oscillator model was developed, which is attached to each node (of the discrete solution) to simulate the hydrodynamic force in cross-flow direction.

The objective of this paper is to obtain a generalized model of the nonlinear restoring force of a marine riser using a pair of polynomials that depend on the velocity and position variables; the model then provides the following advantages: (i) the resulting model of the riser, a nonlinear polynomial oscillator, can be put into a Liénard representation to design state observers, and (ii) choosing high order polynomials to approximate nonlinearities with non-polynomial nature is possible. Finally, another contribution in this article is the analysis of observability/identifiability of the finite version of the structural DP model presented in Niedzwecki and Liagre (2003) and modified here with the inclusion of the nonlinear polynomial restoring force. The finite version in space is achieved by employing the finite difference method (FDM) and the Liénard transformation.

The paper is organized as follows. Section 2 presents the model used to simulate the behavior of a marine riser, its spatial discretization and its transformation into a Liénard system. Additionally, several expressions are shown for the modeling of the restoring force, waves and the hydrodynamic force. In Section 3, the observability/identifiability issue is discussed. Finally, results of simulation are presented in Section 4, and concluding remarks are drawn in Section 5.

2. RISER PHYSICAL MODEL

In Niedzwecki and Liagre (2003), to estimate the parameters of a marine riser, the authors proposed the following fourth order quasi-linear partial differential equation (PDE) describing the riser displacements for an external excitation $u(z, t)$:

$$EI \frac{\partial^4 u(z, t)}{\partial z^4} - T \frac{\partial^2 u(z, t)}{\partial z^2} + m \frac{\partial^2 u(z, t)}{\partial t^2} + c \frac{\partial u(z, t)}{\partial t} + p(z, t) = u(z, t) \tag{1}$$

where $(z, t) \in (0, L) \times (0, \infty)$ are the time and space coordinates respectively, $\nu(z, t)$ is the horizontal displacement of the marine riser, $m$ is the mass per unit, $c$ is the linear viscous drag coefficient, $T$ is tension, $EI$ is the bending stiffness, and the term $p(z, t)$ represents a nonlinear restoring-damping force related to the nonlinear drag force which depends on time and space. Notice that in model (1), in order to have a simplified and useful model to develop identification approaches, the physical properties as well as the tension are assumed to be uniform along the length of the riser and time invariant. Indeed, this model was used in Niedzwecki and Liagre (2003) to elaborate a frequency domain identification algorithm.

2.1 Hydrodynamic force

Hydrodynamic force acting on the risers can be calculated using a modified Morison’s equation for a cylinder in motion (Morison et al., 1950). Thus, the horizontal hydrodynamic force can be expressed as

$$u(z, t) = \rho_w C_M \frac{\pi D^2}{4} \frac{\partial d(z, t)}{\partial t} + \frac{1}{2} \rho_w C_D D (V(z, t) - d(z, t)) |V(z, t) - d(z, t)|. \tag{2}$$

Coefficients intervening in this equation are $C_M$, the inertia coefficient and $C_D$, the drag coefficient. Furthermore, $\rho_M$ is the water density, $D$ is the outer marine riser diameter and $V(z, t)$ is the current velocity, whereas $d(z, t)$ and $\partial d(z, t)/\partial t$ are the velocity and acceleration of the waves, which can be calculated by the following expressions (Wheeler (1970)):

$$d(z, t) = \sum_{j=1}^{\infty} A_j \omega_j \frac{\cosh(k_j z)}{\sinh(k_j l)} \cos(\omega_j t + \phi_j)$$
\[ \frac{\partial d(z, t)}{\partial t} = \sum_{j=1}^{\infty} A_j \omega_j \cosh(k_j z) \sinh(k_j l) \sin(\omega_j t + \phi_j) \]

where \( A_j \) is the wave component amplitude obtained based upon a particular random, \( \omega_j \) is the corresponding wave frequency and \( \phi_j \) is the random phase angle assumed to be uniformly distributed over the interval \([0, 2\pi]\), and \( k_j \) is the wave number and is related to \( \omega_j \) through the linear dispersion relation for a specified water depth \( l \): \n\[ \omega_j^2 = gk_j \tanh(k_j l). \]

2.2 Nonlinear restoring force

In Niedzwiecki and Liagre (2003), the authors suggested that the nonlinear drag force \( p(z, t) \) has polynomial types of non-linearities. In particular, they proposed a general non-linear damping-restoring term based upon the combination of classic Duffing and Van der Pol nonlinearities as follows:

\[ p(z, t) = k_3 \nu^3(z, t) + \frac{c_3}{3} \nu^3(z, t) \]  

where \( k_3 \) is the Duffing coefficient, \( c_3 \) is also the Van der Pol coefficient and notation \( \nu \) stands here for partial derivative with respect to time. This model can also be considered in the present paper as a reference in order to assess the performance of the further proposed nonlinear identification algorithm.

For this identification, a nonlinear restoring force model composed of two polynomials is presented, which depend on the position and velocity variables, with orders \( \eta \) and \( n \) respectively:

\[ p(z, t) = a_1 \nu(z, t) \nu'(z, t) + a_2 \nu^2(z, t) \nu'(z, t) + ... + a_\eta \nu^\eta(z, t) \nu'(z, t) + b_1 \nu(z, t) + b_2 \nu^2(z, t) + ... + b_n \nu^n(z, t). \]  

Thus, the complete model (1) with the force (4), i.e.,

\[ EI \frac{\partial^4 \nu(z, t)}{\partial z^4} - T \frac{\partial^2 \nu(z, t)}{\partial z^2} + m \ddot{\nu}(z, t) \]

has the form of the equation

\[ m \ddot{\nu}(z, t) + F_0(\nu) \dot{\nu}(z, t) \]

\[ + \left[ G_0(\nu) + EI \frac{\partial^4 \nu(z, t)}{\partial z^4} - T \frac{\partial^2 \nu(z, t)}{\partial z^2} \right] = u(z, t), \]

which can be seen as the generalized dynamics that govern the behavior of a second order mechanical system, with friction \( F_0(\nu) \) and forces \( G_0(\nu) + EI \frac{\partial^4 \nu(z, t)}{\partial z^4} - T \frac{\partial^2 \nu(z, t)}{\partial z^2} \) of potential functions, called Liénard system (Liénard (1928)).

2.3 State space representation

System (6) can be rewritten in the classical structure of a second order system in state space form, namely

\[ \dot{\nu}_1 = \nu_2 \]

\[ \dot{\nu}_2 = \frac{1}{m} \left[ -EI \frac{\partial^4 \nu_1}{\partial z^4} + T \frac{\partial^2 \nu_1}{\partial z^2} - F_0(\nu) \nu_2 - G_0(\nu) + u(z, t) \right], \]

where \( \nu_1 = \nu(z, t), \nu_2 = \dot{\nu}(z, t) \), and \( \dot{\nu}_2 = \ddot{\nu}(z, t) \).

Now since \( F_0 \) and \( G_0 \) are linear with respect to their parameters as \( F_0(\nu) = F_1^T(\nu) \theta \) and \( G_0(\nu) = G_1^T(\nu) \theta \), with \( \theta \) denoting the vector of parameters, the Liénard transformation can be applied as:

\[ x_1 = \nu(z, t); x_2 = \dot{\nu}(z, t) + \frac{1}{m} \int_0^t F_1^T(\sigma) \theta d\sigma, \]

obtaining as a result

\[ \dot{x}_1 = x_2 - \frac{1}{m} \int_0^t F_1^T(\sigma) d\sigma \theta \]

\[ \dot{x}_2 = -\frac{1}{m} \left[ G_1^T(x_1) \theta + EI \frac{\partial^4 x_1}{\partial z^4} - T \frac{\partial^2 x_1}{\partial z^2} - u(z, t) \right] \]

that is:

\[ \dot{x}_1 = x_2 - \frac{1}{m} \left[ c_1 x_1 + a_2 x_2^2 + a_3 x_2^3 + ... + a_n x_2^{n+1} + \frac{1}{m} \int_0^t F_1^T(\sigma) \theta d\sigma \right] \]

\[ \dot{x}_2 = -\frac{1}{m} \left[ EI \frac{\partial^4 x_1}{\partial z^4} - T \frac{\partial^2 x_1}{\partial z^2} + b_1 x_1 + b_2 x_2^2 + b_3 x_2^3 + ... + b_n x_2^n - u(z, t) \right] \]

The PDE system (9) does not have an explicit solution. Hence, using the FDM is proposed to obtain an approximation for it, yielding for each discretization section a representation

\[ \dot{x}_{1i} = x_{2i} - \frac{1}{m} \left[ c_1 x_{1i} + a_2 x_{2i}^2 + a_3 x_{2i}^3 + ... + a_n x_{2i}^{n+1} \right] \]

\[ \dot{x}_{2i} = -\frac{1}{m} \left[ EI A_i - T \bar{T}_i + b_1 x_{1i} + b_2 x_{2i}^2 + b_3 x_{2i}^3 + ... + b_n x_{2i}^n - u_i \right] \]

where \( i = 1, ..., N \) is the index of a discretization section, and \( N \) is the number of sections,

\[ \Lambda_i = \left( \frac{x_{1(i+2)} - 4x_{1(i+1)} + 6x_{1(i)} - 4x_{1(i-1)} + x_{1(i-2)}}{(\Delta z)^2} \right), \]

\[ \bar{T}_i = \left( \frac{x_{1(i+1)} - 2x_{1(i)} + x_{1(i-1)}}{(\Delta z)^2} \right), \]

and \( u_i \) is the discretized hydrodynamic force at section \( i \). Considering that displacement measurements are available over various sections, an output equation of the form:

\[ y_i = x_{1i} \]

can be appended to (10), and terms \( \Lambda_i, \bar{T}_i \) can be assumed to be functions of measurements \( y \).

Similarly to the work presented in Fortaleza (2009), in this work the boundary conditions are \( \nu(L, t) = q(t) \) (riser top), \( \nu(0, t) = 0 \) (riser bottom-end fixed) and \( (\partial\nu/\partial z)(L, t) = (\partial\nu/\partial z)(0, t) = 0 \) (rigidity condition at the fixation point)—where \( q(t) \) is the function of the platform movement—for a riser with both extremities connected to fixed supports.

3. OBSERVABILITY AND IDENTIFIABILITY

On the one hand, the identification problem is closely related to an appropriate excitation condition with the choice of an identification algorithm. There are many difficulties in generalizing these conditions for nonlinear systems; however, these have been studied in the literature obtaining
conditions for some classes of nonlinear systems (Sastry and Bodson, 1989; Hammouri and Morales, 1990; Besançon et al., 1996). On the other hand, the identification problem represents the generalization of the observation problem. Occasionally physical behaviors are unknown but can be determined by employing measurements and identification techniques. A clear example is the nonlinear force identification of a marine riser; although there are models for it (e.g. Niedzwecki and Liagre (2003)), this force is generally unknown. Therefore, there is a real necessity for using nonlinear identification techniques to approximate it.

The identification problem of an unknown function can be formulated by considering the following general form of a nonlinear system:

\[ \dot{x} = f(x, u, \varphi(\theta, x, u)) \]

\[ y = h(x, u, \varphi(\theta, x, u)) \]

where \( x \) is the state vector, \( \varphi(x, u, \theta) \) is the unknown function, \( \theta \) represents an unknown static parameter vector that characterizes the unknown function and \( y \) is the measurements’ vector.

Thus, the identification problem can be reduced to reconstruct the function \( \varphi(\cdot) \). This can be expanded, however, when the observation problem is associated with two reasons: (i) Suppose that the vector \( x(0) \) is unknown. Then, the identification problem includes a problem of observation: one must estimate both \( x \) and \( \varphi(\cdot) \). (ii) The identification topic requires an identifiability study, which is closely connected to the observability analysis, particularity when the identification techniques are based on state observers.

In Besançon et al. (2010) a state observer has been proposed for a Liénard system, because this kind of system can be put into a state-affine representation. Therefore, in the present work we propose the restoring force (4) that permits the transformation of system (5) into the Liénard system (10) through the transformation (8). In turn, the system (10) can be converted into a state-affine representation; see Besançon and Voda (2010).

Note that acceleration variables are not included in (4), because their inclusion would not allow the Liénard transformation.

Now, in order to analyze the properties of observability/identifiability of model (10)-(11) to design an observer that gives estimates of the states and the parameters, let us rewrite it as (where the section index is omitted):

\[ \dot{x} = A_0 x + \Phi(y) \theta + \Phi_o(y) \]

\[ y = C_0 x \]

and

\[ \Phi_o(y) = -\frac{1}{m} \left[ E I \Lambda - T \right] \cdot \]

Note that matrix \( \Phi(y) \) can not be constructed from the system (7), i.e. without the Liénard transformation, because the polynomial \( a_1 x_1 + a_2 x_1^2 + \ldots + a_n x_1^n \) and the coefficient \( c \) are affine to the velocity \( x_2 \), which is not a measurable state for estimation. Obviously if the velocity is measurable, there is not an observation problem, only an identification problem.

Now, let us consider the following persistent excitation condition in function of \( y(t) \):

\[ \exists T, \beta' > 0, \alpha' > 0 : \]

\[ \beta' \geq \int_t^{t+T} \Psi_y^T(\tau, t) C^T C \Psi_y(\tau, t) d\tau \geq \alpha' I, \quad \forall t \geq t_0, \]

where \( \Psi_y \) denotes the transition matrix (as in Besançon (2007)).

Proposition 1. Assume that for any initialization of system (13), the condition (15) is satisfied; the parameter vector \( \theta \) of system (13) can be then asymptotically estimated.

This result follows from a convergence condition for Kalman observers designed for state-affine systems (Hammouri and Morales (1990) and Besançon et al. (1996)).

Note that as demonstrated in Besançon et al. (2006) this condition is satisfied as soon as separate conditions for estimation of the state, on the one hand, and estimation of parameters, on the other hand, hold true. Similarly to the analysis of Besançon et al. (2010) and Besançon and Voda (2010), the condition for the estimation of the state is here obviously satisfied as well (\( A_0, C_0 \) observable), and consequently satisfying (15) reduces to satisfying a condition for the parameter estimation.

Following Besançon et al. (2006) and Zhang (2002) such a condition can be expressed as

\[ \mathbf{Y} \]

\[ \mathbf{T} \]

solution of \( \dot{\mathbf{T}} = (A_o - K C_o) \mathbf{T} + \Phi(y) \) for any \( K \) such that \( A_o - K C_o \) is stable, if satisfies

\[ \beta I \geq \int_t^{t+T} \mathbf{Y}^T(\tau) C_o^T C_o \mathbf{Y}(\tau) d\tau \geq \alpha I. \]

In that case, one can design an observer for estimation of both \( \theta \) and \( x \) for instance as proposed in Zhang (2002) as

\[ \dot{x} = A x + \Phi(y, u) \theta + (K + \Lambda \Gamma^T C^T)(y - C \dot{x}) \]

\[ \dot{\Lambda} = (A - K C) \Lambda + \Phi(y, u) \]

\[ \dot{\theta} = \Gamma \Lambda^T C(y - C \dot{x}). \]

Owing to the existence of periodic solutions in Liénard systems (see e.g. Abd-Elrady et al. (2004)), one can expect that (16) will indeed be satisfied for system (13).

4. SIMULATION RESULTS

In order to program the simulator—the dynamic behavior of the riser—based on model (1), the physical parameters
listed in Table 1 are considered. The order of the spatial
discretization is $n = 50$ sections, and the initial conditions
are assumed to be $[\nu^{(i)}, \dot{\nu}^{(i)}]^T = [0, 0]^T$. In Fig. 2, the

Table 1. Physical parameters of the riser simulator

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>[m]</td>
<td>1.4</td>
</tr>
<tr>
<td>Riser Length</td>
<td>[m]</td>
<td>875</td>
</tr>
<tr>
<td>Mass per unit length</td>
<td>[kg/m]</td>
<td>912</td>
</tr>
<tr>
<td>Linear viscous drag coeff</td>
<td>[N/s/m]</td>
<td>120</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>[N/m]</td>
<td>8000</td>
</tr>
<tr>
<td>Van der Pol coefficient</td>
<td>[N/m$^2$]</td>
<td>5000</td>
</tr>
<tr>
<td>Tension</td>
<td>[N]</td>
<td>$7 \times 10^4$</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>[Nm$^2$]</td>
<td>100</td>
</tr>
<tr>
<td>Inertia coefficient</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>-</td>
<td>1.2</td>
</tr>
<tr>
<td>Water Density</td>
<td>[Kg/m$^3$]</td>
<td>1.025</td>
</tr>
<tr>
<td>Current velocity</td>
<td>[m/s]</td>
<td>1.2</td>
</tr>
</tbody>
</table>

response of the riser when the platform oscillates periodically
can be appreciated. In this case, the movement of the
platform is given by the function $q(t) = 2 \sin 0.05t \times \sin 0.1t$
[m] In order to evaluate the proposition of some restoring

force (4) to approximate a real unknown force, an observer
with structure (17) was designed considering the following
factors: (i) the unknown ‘real force’ was modeled by (3),
(ii) the orders in the polynomials of the restoring force
were chosen $\eta = n = 2$, and (iii) given the order of the
polynomials, the coefficients to be estimated are $a_1$, $a_2$, $b_1$
and $b_2$. In Fig. 3, the parameter estimation is exposed. Note
that estimations converge to oscillating values since the
estimated coefficients are not the coefficients employed to
simulate the ‘real force’ in this test, i.e., model (5) with
(3). Nevertheless, a mean ($\bar{\cdot}$) of the sustained oscillations
can be computed once these have converged. Each mean
can be employed to parameterize the unknown restoring
force. In Fig. (4), both the estimated and ‘real’ simulated
restoring force at some section $i$ are shown. To obtain a
better approximation, the order of the polynomial should
be modified. For most structural mechanic systems, odd
polynomials are a suitable choice (e.g. third order models
without second order terms). In general, not all the terms
are significant; and a selection can be made by simply checking
the relative influence of each term by a preliminary
identification and by retaining the most important ones
(Ceravolo et al. (2013)).

Fig. 2. Response of the riser with periodic movements of
the platform.

Fig. 3. Estimation of $a_1$, $a_2$, $b_1$ and $b_2$.

Fig. 4. Estimation of the restoring force.

5. CONCLUSION

This paper introduced a nonlinear restoring model for
vertical risers based on polynomial functions. Such a restor-
ing model permits the transformation of the riser model
into a Liénard representation, which is suitable for the
conception of observers that identify structural parameters
or the riser restoring force when its physical model is not
available. Although the identification technique presented
here has been employed in a specific application, it can be
extended for being applied in other mechanical systems;
but with the caveat that a polynomial model should be
used for small nonlinearities or as a “first trial” when a
pertinent parametric model is unknown. Finally, in future
works, the use of the presented approach would be interesting
in developing schemes for the detection of structural
damages.

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