Investigation of Critical Factors Affecting Dynamic Stability of Wind Generation Systems with Permanent Magnet Synchronous Generators

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Abstract: This paper presents an analysis to investigate the critical factors which affect the dynamic stability of wind energy conversion systems (WECSs) with permanent magnet synchronous generators (PMSGs). In this paper, at first the detail mathematical model of a WECS with a PMSG is developed and then the modal analysis technique is used to identify the critical factors. The sensitivity of these critical factors on the dynamic stability of WECSs with PMSGs, is analyzed in terms of damping ratio with changes in critical factors. The proposed approach is tested on a WECS with single PMSG for different level of wind energy penetration.

Keywords: Wind energy conversion system (WECS), permanent magnet synchronous generator (PMSG) dynamic stability, critical factors, modal analysis

1. INTRODUCTION

The integration of renewable energy sources (RESs) has been increased in recent years due to several technical, environmental, and social benefits which include the use of these sources as supplement and alternative of conventional fossil fuel generation to meet the increased energy demand, keep the environment free from fuel exhaustion. There are different types of RESs such as solar, wind, biogas, tidal, etc., and among these RESs, wind energy is the fastest growing one (Sahu et al., 2013).

Various wind turbine concepts with different types of wind generators for WECSs along with their benefits and limitations have been discussed in (Li and Chen, 2008) and it is found that the variable speed wind turbine with PMSGs is one of the latest developments in this area. However with the increasing level of wind energy penetration in power systems, grid connection and interaction issues have posed several new challenges for the development of WECSs which may significantly affect power system dynamics and operational characteristics (Bu et al., 2012; Hossain et al., 2012). The future trends of wind energy integration will strongly depend on the ability of complying the requirements of grid utilities and for this, it is essential to carry out investigations of dynamic interactions between the grid and wind farms with suitable model.

A great deal of attention has been paid to address the challenges of wind energy integration for different types of wind turbines and generators using small-signal stability analysis. The small-signal analysis of a fixed-speed wind farm is shown in (Tabesh and Iravani, 2006) to investigate the sensitivity of the system modes to system parameters and evaluate the system capability to reject electrical and mechanical disturbances. The effects of variable speed wind generation system based on doubly-fed induction generators (DFIGs) on the dynamic stability of power systems, for high penetration levels, has been discussed in (Tsourakis et al., 2009). Though the investigation of key factors for fixed-speed WECSs and variable DFIG-based WECSs are well established in literature of power systems but in an earlier stage for PMSG-based WECSs.

The small-signal stability analysis of a direct-drive PMSG used in WECS is shown in (Wu et al., 2009) to determine the control capability of proportional-integral (PI) controllers. The stability issues of a PMSG–based WECS have been discussed in (Geng and Xu, 2011) from where it can be seen that the drive train dynamics significantly affect the system stability by introducing a low-frequency oscillatory mode. But in (Geng and Xu, 2011), the dynamics model of each component is not considered in the system modeling and if the dynamics are considered some other factors could affect the stability of the system. A detailed mathematical model of the system components

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are considered in (Huang et al., 2012) where the small-signal stability analysis has been considered to show the relationship between modes and states variables and it can be seen that the traces of eigenvalue as a function of controller parameters influence the performance of the system. However, the effects of key factors on the dynamic stability of PMSG-based WECSs for high penetration levels are still uncovered.

The aim of this paper is to develop a detail model of a WECS which includes a PMSG and to identify the critical parameters and factors based on the developed model. The critical factors are investigated through modal analysis of the system. After the identification of critical issues, the dynamic stability of the PMSG-based WECS has been investigated with variations of the critical parameters. The sensitivity of each critical factor with respect to other factors and for different penetration level has also been investigated in this paper.

2. WIND GENERATION SYSTEM MODEL

The schematic diagram of a grid-connected PMSG for a direct-drive wind turbine is shown in Fig. 1 where the PMSG is connected to the grid through back-to-back full converters. In Fig. 1, the generator-side converter operates to adjust the speed of the generator with the variation of wind speed. On the other hand, the grid-side converter at the end of the DC-link capacitor connects AC grids and takes the responsibility of controlling DC voltage and thus the output AC voltage or the reactive power. To investigate the dynamic interactions of WECSs on the grid, it is essential to develop the dynamic models of the wind energy capturing mechanism of each wind turbine-generator (WTG) unit, the rotating shaft system of each WTG unit, the electrical circuitry of each induction generator, the converter, and the transmission system (Wilkie et al., 1990). The dynamical model of a WECS-based on PMSGs is developed in the following subsections.

2.1 Wind Speed Model

The wind speed modeling offers several supports for basic applications such as performance estimation of a wind generation system (i.e., prediction of the energy output) and analysis of the turbulence influence on the energy conversion and on the system dynamics. The most important application of wind speed modeling is the development of control solutions as a random process. The wind speed is considered as consisting of two elements (Leithead et al., 1991):

- a slowly varying mean wind speed of hourly averages plus a rapidly varying turbulent component which is modeled as a Rayleigh distribution

\[ p_w(v) = av \exp \left( -\frac{v^2}{2a^2} \right) \]  

(1)

where \( v \) is the hourly wind speed average and \( a \) is related to the very long time-scale mean speed;
- a rapidly varying turbulence component which is modeled by a normal distribution with mean value equal to zero and standard deviation proportional to the current value of the average wind speed. The dynamic properties of the turbulence component are given by the following Von Karman power spectrum

\[ S(\omega_r) = \frac{0.475\sigma^2 (\frac{1}{4})}{[1 + (\frac{\omega}{\beta})^2]^{5/6}} \]  

(2)

where \( \sigma \) is the turbulence intensity, \( \omega_r \) is the rotational speed of the shaft, and \( l \) is the turbulence length scale.

2.2 Wind Turbine Model

The power generated from a wind turbine \( (P_{wt}) \) is proportional to the cube of the wind speed and can be expressed as

\[ P_{wt} = \frac{1}{2} \rho A v^3 \]  

(3)

where \( \rho \) is the density of air, \( A = \pi R^2 \) is area swept by blades and \( R \) is the radius of the wind turbine blade. But a wind turbine can only extract a part of the power from the wind which is limited by the Betz limit (maximum 59%) as it assumes perfect blade design. This fraction is described as the power coefficient of the turbine \( (C_p) \) which depends on the blade pitch angle \( (\beta) \) and the tip speed ratio \( (\lambda) \). Therefore the effective mechanical power of the wind turbine extracted from the wind is

\[ P_w = \frac{1}{2} C_p (\beta, \lambda) \rho A v^3 \]  

(4)

The tip speed ratio (TSR) is the ratio between the linear blade tip speed and the wind speed which can be expressed as

\[ \lambda = \frac{\omega R}{v} \]  

(5)

Therefore, any change in the shaft speed or wind speed will cause a change in the TSR leading to power coefficient variation which in turn affects the generated power. The mechanical power of the turbine can also be expressed as

\[ P_w = \omega T_w \]  

(6)

where \( T_w \) is the aerodynamic torque of the wind turbine which can be written as

\[ T_w = \frac{1}{2} C_t (\beta, \lambda) \rho \pi R^3 v^2 \]  

(7)

where \( C_t = \frac{C_p (\beta, \lambda)}{2} \) is the torque coefficient of the wind turbine. The drive train of a PMSG consists of five parts such as rotor, low-speed shaft, gearbox, high-speed shaft and generator. Other parts of wind turbines, e.g. tower and flap bending modes can be reasonably neglected to simplify the analysis. The complexities of drive trains depend on the interest of studies. For example, when the problems such as torsional fatigue are studied, dynamics
from both sides of gearbox have to be considered. So, two-
lumped mass or more sophisticated models are required.
But when the study focuses on the interaction between
wind farms and AC grids the drive train can be treated as
one-lumped mass model for the sake of time efficiency and
acceptable precision. So the drive train can be simplified
into a motion equation as
\[
\dot{\omega}_r = \frac{1}{2H_t} (T_w - D_t\omega_r - T_g)
\]
with
\[
H_t = H_r + H_q
\]
\[
D_t = D_r + D_q
\]
where \(\omega_r\) is the generator rotor speed, \(T_g\) is the electromag-
tnetic torque applied on the generator shaft, \(H_t\) is the total
inertia, \(H_r\) is the turbine rotor inertia, \(H_q\) is the generator
rotor inertia, \(D_t\) is the total friction coefficient, \(D_r\) is the
turbine friction coefficient, and \(D_q\) is the generator friction
coefficient.

2.3 PMSG Model

The dynamic model of PMSGs can be expressed by the
equations on a reference coordinate system rotating syn-
chronously with the magnetic flux in order to define the
generator control system easily. In this frame, the q-axis is
90° ahead of the d-axis with respect to the direction of
rotation. The synchronization between the dq- and the abc-
frame is maintained by a phase locked loop (PLL). Thus
the dynamic model of the PMSG is
\[
\begin{align*}
\dot{I}_d &= -\frac{R_s}{L_d} I_d + p\omega_q \frac{L_q}{L_d} I_q + \frac{V_{td}}{L_d} \\
\dot{I}_q &= -\frac{R_s}{L_q} I_q + p\omega_d \frac{L_d}{L_q} I_d - p\omega_p \frac{\psi}{L_q} + \frac{V_{td}}{L_q}
\end{align*}
\]
where \(\psi\) is the magnetic flux, \(p\) is the number of pole pairs,
\(L_d\) & \(L_q\) are direct- and quadrature-axes inductances
of generators, \(I_d\) & \(I_q\) are direct- and quadrature-axes cur-
rents of generators, \(V_{td}\) & \(V_{tq}\) are direct- and quadrature-
axes terminal voltages of generators, and \(R_s\) is the stator
resistance. Equation (9) shows how to control the current
components by means of the applied voltage. The elec-
tromagnetic torque of the permanent magnet generator is
given by
\[
T_g = p(L_d - L_q) I_d I_q + p\psi I_q
\]
For PMSGs, the minimum loss can be achieved if it is assumed
that \(I_q = 0\) (Zhang et al., 2007) and equation (10) can be
simplified as
\[
T_g = p\psi I_q
\]
To complete the modeling of PMSG-based WECSs, it is essen-
tial to model the pulse width modulation (PWM)
converter which is shown in the following subsection.

2.4 PWM Converter Model

Though the control schemes of two converters (generator-
and grid-side converters) are different, their topologies are
similar. Therefore, the mathematical model of a voltage
source type PWM converter can be modeled in a similar
way as presented in (Mahmud et al., 2012) and can be
written as
\[
\begin{align*}
\dot{I}_d &= -\frac{R}{L} I_d' - \omega I_q' + \frac{E_d}{L} - \frac{v_{dc}}{L} K_d \\
\dot{I}_q &= \omega I_d' - \frac{R}{L} I_q' + \frac{E_q}{L} - \frac{v_{dc}}{L} K_q \\
\dot{v}_{dc} &= \frac{1}{C} I_d' K_d + \frac{1}{C} I_q' K_q - \frac{1}{C} I_L
\end{align*}
\]
where \(I_d'\) & \(I_q'\) are dq- axis AC currents, \(K_d\) & \(K_q\) are switch status functions, \(E_d\) & \(E_q\) are direct-
and quadrature-axes grid voltages, \(R\) is the resistance of
each phase, \(L\) is the filter inductance, \(C\) is the DC-link
capacity, \(v_{dc}\) is the DC-link voltage, and \(I_L\) is the DC
load current.

Equations (8), (9), and (12) represent the complete dy-
namical model of a PMSG-based WECS which can be
written in the form of following group of equations:
\[
\begin{align*}
\dot{\omega}_r &= \frac{1}{2H_t} (T_w - D_t\omega_r - n_q p\psi I_q) \\
\dot{I}_d &= -\frac{R_s}{L_d} I_d + p\omega_q \frac{L_q}{L_d} I_q + \frac{V_{td}}{L_d} \\
\dot{I}_q &= -\frac{R_s}{L_q} I_q + p\omega_d \frac{L_d}{L_q} I_d - p\omega_p \frac{\psi}{L_q} + \frac{V_{td}}{L_q} \\
\dot{v}_{dc} &= \frac{1}{C} I_d' K_d + \frac{1}{C} I_q' K_q - \frac{1}{C} I_L
\end{align*}
\]
With this complete model the dynamic stability analysis
and investigation of critical factors are discussed in the
following sections.

3. DYNAMIC STABILITY ANALYSIS

There are several factors which may affect the dynamic
behavior of a PMSG-based WECS such as turbine aero-
dynamics, pitch control or turbine mechanical control,
shaft dynamics, electrical characteristics of the genera-
tor, and electrical controls (Gautam et al., 2009). In a
WECS, turbine aerodynamics and mechanical controls are
responsible for controlling the mechanical power delivered
to the shaft. The pitch angle controller aligns the rotor
blades at a particular angle for temporary reduction of
wind turbine mechanical power when the rotor suffers from
over speed. The shaft dynamics are normally modeled as
a two-mass shaft- one mass represents rotor or turbine
blades and the second represents the generator. But in
this paper, a lumped mass shaft is considered as the
electromagnetic torque interacts with turbine rotor. The
electrical characteristics of a PMSG are different from
other synchronous generators used in conventional power
generation and these dynamics are much faster than that
of mechanical dynamics because of the huge turbine and
generator inertias.

The generator-side converter controls the active power
extracted from the wind turbine and this power can be
either maximum power for maximum power point tracking (MPPT) operation or a constant power to support the grid (Akhatov, 2003). The grid-side converter regulates the DC-link voltage as well as maintains the voltage stability. Since the focus of this paper is to identify the factors which mostly affect the dynamic stability of a PMSG-based WECS, it is essential to analyze the dynamic characteristics of the system and the analysis will be performed by using the modal analysis. To perform the modal analysis, the PMSG-based WECS as represented by (13) needs to be linearized at an equilibrium point and the linearized form of which can be written as

\[ \Delta \dot{x} = A \Delta x + B \Delta u \]  

(14)

where \( x \) represents the states of the system, \( u \) is the input of the system, \( A \) is the system matrix, and \( B \) is the input matrix. Here, the states and inputs of the PMSG-based WECS are

\[
\begin{align*}
    x &= [\omega_r \ I_d \ I_q \ I_d' \ I_q' \ v_{dc}]^T \\
    u &= [K_d \ K_q]^T
\end{align*}
\]

The dynamic performance of the PMSG-based WECS can be investigated by using modal analysis which includes the eigenvalue and participation factor analysis as discussed in the following subsections.

3.1 Eigenvalue Analysis

The nominal values of the PMSG-based WECS as shown in Fig. 1 are given in Appendix A. The dynamic characteristics of the system can be investigated from the eigenvalues of \( A \). The system matrix \( A \) can be obtained once the initial values of state variables are known which can easily be determined from load flow studies and the electrical relationships as discussed in the modeling section. When the nominal values of the system parameters are used, there exist two oscillatory modes along two monotonic modes. The monotonic modes, \(-2.225 \text{ and } -7.851\), are far away from the center of the complex s-plane as compared to the oscillatory modes (OMs): \(-0.0980 \pm 1.8186i\) and \(-0.2000 \pm 0.5596i\).

The small-signal stability problems are often associated with oscillatory problems due to the lack of sufficient damping or synchronizing torque within the system. For the first oscillatory mode (OM1) the damping ratio is 0.0538 and that of for the second oscillatory mode (OM2) is 0.3365. The damping ratio indicates that there is adequate damping within the system as normally the damping factor needs to be maintained at 5%. But these low frequency oscillations may persist for several reasons and one could be due to the mismatch in mechanical torque of the wind turbine and electrical torque of the PMSG. The underlying phenomena of these oscillations can be investigated from the participation factor analysis which is discussed in the following subsections.

3.2 Participation Factor Analysis

Participation factors specify which state variables are more significant for a given dynamic mode and the converse (Kundur, 1994). The participation matrix which combines the right and left eigenvectors as a measure of the association between the state variables and modes, denotes the eigenvalue sensitivity with respect to the diagonal elements of the state matrix.

The contribution of state variables in a PMSG-based WECS can easily be obtained from the participation factors. The state participation of oscillatory modes, OM1 and OM2, is shown in Fig. 2 from where it can be seen that the state, \( \omega_r \), has the highest contribution to the mode, OM1 and state variables \( I_d \) and \( v_{dc} \) do not have any contribution. OM1 is also affected by the states \( I_q \) and \( I_q' \) where the participation of these states is close to each other. The contribution of state \( I_d' \) is quite small. From this relationship between states and participation factors for OM1, it can be seen that the dynamics of drive train most significantly affect the dynamic stability. Also the states related to supplying active power affect the dynamic stability of the PMSG-based WECS.

On the other hand, the state \( I_q' \) has the highest contribution to the oscillatory mode, OM2 and this is related to the active power delivered into the grid rather than drive train as discussed in (Geng and Xu, 2011). Though the state \( \omega_r \) is still responsible for the oscillation in OM2 but the effects of state \( I_d' \) is considerable where that of for the remaining states, \( v_{dc}, \ I_d, \text{ and } I_q \) are negligible.

Thus Fig. 2 clearly shows that the dynamic stability of a PMSG-based WECS is not only affected by the drive train but also affected by the active power supplied by the generator and reactive power delivered by the grid-side converter. The actual factors, i.e., parameters which have paramount importance on the dynamic stability of a PMSG-based WECS are discussed in the following section.

4. INVESTIGATION OF CRITICAL FACTORS

This section is aimed to identify the critical factors based on the dynamic stability analysis as discussed in Section 3 from where it is seen that the states, \( \omega_r, I_d' \) and \( I_q' \) have the dominating affects to OMs. Therefore the coefficients of \( A \) matrix related to these state variables can be varied by changing the parameters to see the effect on dynamic stability. The similar approach is also presented in (Mahmud et al., 2010) to investigate the critical parameters of a conventional synchronous generator-based power system with dynamic loads. From (Mahmud et al., 2010), it can be seen that the open-circuit time constant and the exciter gain of synchronous generators are very sensitive to the dynamic stability.
In this paper, the friction coefficients $D_r$ and $D_s$ are varied at the first stage of identifying the critical factors as from equation (8) it is clear that the state $\omega$ is related to $D_r$ and $H_1$. If the values of $D_r$ and $D_s$ are set to zero, the oscillator mode (OM2) and other monotonic modes remain unchanged but OM1 becomes marginally stable with eigenvalues $\pm 1.8212i$. This means that there is lack of damping in OM1 and this can be increased by increasing $D_t$. Now if the values of $D_r$ and $D_s$, i.e., $D_t$ are gradually increased, there would be more damping in OM1 and the damping in OM2 is still unaffected with these changes. This scenario is shown in Fig. 3 from where it can be seen the damping ratio of OM1 is increasing with the increase in $D_t$ through a linear relationship and that of for OM2 remains constant.

From a power system point of view, the behaviors of WECSs are quite different from traditional centralized generation systems. Apart from the intermittency, most of the WECSs do not contribute to the system reserves and total inertia. But the system inertia is considered as the most important parameters for the operation of existing power systems (Mahmud et al., 2010; Nomikos and Vournas, 2005). When a frequency event occurs, the synchronous machines will inject or absorb kinetic energy into or from the grid to counteract the frequency deviation and the lower this system inertia, the more nervous the grid frequency reacts on abrupt changes in generation and load patterns (Tielens and Hertem, 2012). But wind turbines are generally equipped with back-to-back converters, as discussed in this paper, which electronically decouple the generator from the grid and thus no inertial response is delivered during a frequency event. For this reason, the impact of inertia constant on the dynamic stability is not considered in this paper. On the other hand, the increased penetration of wind energy will reduce the effective inertia constant (Gautam et al., 2009; Anderson and Fouad, 2002) and in this case, the damping of the critical mode will be reduced. The effect of increased wind energy penetration level can be seen in Fig. 4 from where it can be seen that the damping in OM1 reduces with increases in penetration level. In a similar manner if the inductance (either $L_d$ or $L_q$) and the stator resistance ($R_s$) are increased, the damping in OM1 will also be increased. But in all cases, the damping of OM2 will not be affected.

Since the damping of OM2 is unaffected for the aforementioned changes, it is essential to find out the critical factors which affect the damping of this mode. From Fig. 2 and as overshadowed earlier, it is clear that the state, $\omega_q'$, has the highest participation along with states, $\omega_d'$ and $\omega_r$. If the parameters, $R$ and $L$ related to the state, $\omega_q'$ are varied, the damping of OM2 changes. But this is not a wise way of changing the stability limits of a PMSG-based WECS as $R$ and $L$ are fixed for any system. But this could be possible solution for a newly established WECS with PMSGs. But the oscillations in OM2 can easily be reduced by controlling the power factor. With the increase in power factor, the damping in OM2 also increases and this can be seen in Fig. 5. From Fig. 5, it can be seen that there is no oscillations in OM2 when the power factor is maintained at unity and the state, $\omega_q'$ is not contributing anymore. This power factor can be controlled by controlling the grid-side converter. In this case, the damping of OM1 remains the same as in the nominal operation with highest contribution by the state, $\omega_q$, and this could be reduced by providing appropriate pitch control.

5. CONCLUSION

The effects of different factors on the dynamic stability of a PMSG-based WECS are investigated and several factors such as friction coefficients, penetration levels, power factors, etc., affect the dynamic stability of the overall system. The sensitivity of these factors with respect to the damping ratio is discussed and from where it can be concluded that friction coefficients have linear relationship with stability margin, the high penetration of wind energy reduces the stability limits of the system, and power factor control reduces the contribution of grid current in the oscillatory mode. Though the variations in friction coefficients and penetration levels affect the stability limit of a particular oscillatory mode which is significantly contributed by the drive train dynamics, but the other particular oscillatory mode remains unaffected. Therefore, it is essential to have several controllers, e.g.,

![Fig. 3. Effects of friction coefficients on dynamic stability](image1)

![Fig. 4. Effects of penetration level on dynamic stability in OM1](image2)

![Fig. 5. Effects of power factor control on dynamic stability in OM2](image3)
a pitch control to reduce the detrimental effect of drive train and converter controls for controlling power factor in order to reduce the effects of grid current. The future works will deal with the design of appropriate controllers for a PMSG-based WECS to alleviate the effects of critical factors.

REFERENCES


Appendix A. SYSTEM PARAMETERS

**Base values**

Base Power: 2 MVA Base voltage: $\frac{44}{\sqrt{3}}$ kV

**Wind turbine parameters**

Nominal wind speed: 1 pu Nominal mechanical power of the wind turbine: 1.1 pu Rotational speed of the shaft: 1 pu Nominal wind turbine inertia constant: 4.8 pu Nominal wind turbine friction coefficient: 2 pu

**PMSG parameters**

Number of pole pairs: 4 Rated speed of the generator: 1 pu Nominal generator inertia constant: 0.3 pu Nominal generator friction coefficient: 0 pu Rated generated torque: 1 pu Rated generated power: 1 pu Terminal voltage of the generator: 1 pu Inductance in $d$-frame: 0.7 pu Inductance in $q$-frame: 0.7 pu Stator resistance: 0.01 pu Filter inductance: 0.1 pu Filter Capacitance: 1 pu Line resistance: 0.02 pu Nominal power factor: 0.95

**Grid parameters**

Grid voltage: 0.99 pu