Trajectory planning for AGVs in automated container terminals using avoidance constraints: a case study

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Abstract: Automated guided vehicles (AGVs) are used to transport containers between the quayside and the stacking area in automated container terminals. The behavior of AGVs becomes complex when the trajectories of AGVs need to be scheduled with interacting machines, while satisfying collision avoidance constraints. This paper proposes a new two-level energy-aware approach for generating the trajectories of AGVs in automated container terminals. The higher-level controller decides an energy-efficient schedule based on the minimal-time calculation of all machines. The higher-level controller solves optimal control problems to determine collision-free trajectories of individual AGVs. This obtained control problem of an AGV is then formulated as a mixed integer linear programming problem. Simulation results illustrate the potential of the proposed approach in a case study.

Keywords: Automated guided vehicles; trajectory planning; collision avoidance; optimal control.

1. INTRODUCTION

The performance of container terminals needs to be improved to adapt the expected significant growth of global freight transport, in which over 60% of deep-sea cargo is transported via containers (Stahlbock and Voß [2007]). An automated container terminal aims to achieve a high cost-efficiency of a transport process by controlling automated equipment. In an automated container terminal, automated guided vehicles (AGVs) transport containers between the quayside and the stacking, interacting with quay cranes (QCs) and automated stacking cranes (ASCs), respectively. Compared with QCs and ASCs, AGVs have more complex behavior due to its two dimension trajectory and two-side interactions. Therefore, research on the operation of AGVs has received much attention (see Stahlbock and Voß [2007]).

As one main topic of the operational control of AGVs in container terminals, trajectory planning has been investigated, which determines the actual trajectory of vehicle when multiple AGVs are employed for transporting containers (e.g., Duinkerken et al. [2006]), Duinkerken et al. [2006], Marinica et al. [2012] and Bähr et al. [2013] have investigated flexible algorithms for complex behavior of AGVs. However, these approaches only emphasize feasible trajectories without considering the link to the scheduling of interacting machines. For trajectory planning of AGVs, collision avoidance must be taken into account on the one hand, while vehicles need to make contact in order to exchange containers on the other hand. The lack of this link will not result in the optimal performance of the container handling system. In addition, there is a lack of collision avoidance trajectory planning taking into account energy efficiency.

This paper provides an approach for determining the collision-free trajectories of AGVs integrating with the scheduling of interacting machines. This paper in this way extends the work of Xin et al. [2014]. Optimal control is proposed to achieve generate the trajectories of each AGV taking into account collision avoidance. The control problem is then formulated as a mixed integer linear programming (MILP) problem, which can be solved by state-of-the-art solvers.

This paper is organized as follows. Section 2 describes the dynamical model of AGVs. Section 3 proposes an optimal controller taking into account both static obstacles and possible collision of AGVs. Section 4 illustrates the proposed trajectory planning approach in a simulation study. Section 5 concludes this paper and provides directions for future research.

2. MODEL OF AGVS

2.1 Dynamical model

In container terminals, AGVs are employed to transport containers between the quayside area and the stacking area. We assume that there are n AGVs. For simplicity, the dynamics of the AGVs are assumed to be identical. A
point-mass model is used to approximate the dynamical behavior of an AGV in two-dimensional space. This simplified model is used to avoid intractable computation and then simplify the interaction between trajectory planning of AGVs and the scheduling of all interacting machines. The model is described as follows:

$$\begin{bmatrix} r_i(k+1) \\ v_i(k+1) \end{bmatrix} = \begin{bmatrix} I_2 & \Delta t I_2 \\ O_2 & I_2 \end{bmatrix} \begin{bmatrix} r_i(k) \\ v_i(k) \end{bmatrix} + \begin{bmatrix} 0.5(\Delta t)^2 I_2 \\ \Delta t I_2 \end{bmatrix} u_i(k),$$

(1)

where AGV $i$ has a position $r_i(k) = [r_i^x(k) \ r_i^y(k)]^T$ and a velocity $v_i(k) = [v_i^x(k) \ v_i^y(k)]^T$. Each AGV is assumed to respond to control actions $u_i(k) = [u_i^x(k) \ u_i^y(k)]^T$. $\Delta t$ is the time step size.

An approximation for maximal velocity and acceleration constraints by polygons is used. Fig. 1 illustrates the approximation of velocities. The exact constraint is the circle, described as $v_i^x(k)^2 + v_i^y(k)^2 \leq v_{\text{max}}^2$. This nonlinear constraint could result in time-consuming computations. To avoid this, the constraints on velocities and accelerations are approximated by polygons using linear equalities. The velocity and action constraints are given as follows [Richards and How, 2002]: $\forall i \in [1, \ldots, n]$, $\forall m \in [1, \ldots, M]$

$$u_i^x(k) \sin\left(\frac{2\pi m}{M}\right) + u_i^y(k) \cos\left(\frac{2\pi m}{M}\right) \leq u_{\text{max}}$$

(2)

$$v_i^x(k) \sin\left(\frac{2\pi m}{M}\right) + v_i^y(k) \cos\left(\frac{2\pi m}{M}\right) \leq v_{\text{max}},$$

(3)

where $u_{\text{max}}$ and $v_{\text{max}}$ are limits on the acceleration and velocity, respectively. The constraints on the speed and the control variables are approximated with as $M = 10$ as in [Richards and How, 2002].

![Fig. 1. The approximation of the velocity limit of AGVs.](image)

### 2.2 Collision avoidance

For trajectory planning of the AGVs, we consider two types of collisions. The first one is the collision with a static obstacle, e.g., the neighborhood of the quayside and stacking area. For instance, there are two tracks of a stacking crane on one side of the stack where containers are handled. For security reasons, AGVs cannot approach the area of these tracks too closely. The other possibility for collision is the moving collision of two different AGVs when they are transporting containers.

Moving obstacles In the case when multiple AGVs are transporting containers to different destinations, collisions between vehicles need to be considered. At each time step every pair of AGVs $i_1$ and $i_2$ must be a minimal distance.

![Fig. 2. Two static obstacle areas near the stacking area.](image)

![Fig. 3. The illustration of the static obstacle area.](image)

**Static obstacles** The static obstacles considered in this paper are the areas of the tracks used by stacking cranes. These static obstacle areas are illustrated in Fig. 2. Such a static obstacle is represented by a rectangular area as illustrated in Fig. 3 (see [Schouwenaars et al., 2001]). The rectangular area can be described by the lower left corner $(s_{\text{low},x}, s_{\text{low},y})$ and the upper right corner $(s_{\text{high},x}, s_{\text{high},y})$. To avoid the static obstacle, the position of AGV $i$ must be outside of this rectangular area at all times. This collision avoidance mechanism can be described with the following constraints:

$$r_i^x(k) \leq s_{\text{low},x} - d$$

or $r_i^x(k) \geq s_{\text{high},x} + d$

or $r_i^y(k) \leq s_{\text{low},y} - d$

or $r_i^y(k) \geq s_{\text{high},y} + d,$

(4)

where $d$ is a safety distance for the zone of an AGV.

By introducing binary variables, (4) can be rewritten for a standard optimization problem formulation:

$$r_i^x(k) \leq s_{\text{low},x} - d + R b_{m,1}$$

$$r_i^x(k) \geq s_{\text{high},x} + d - R b_{m,2}$$

$$r_i^y(k) \leq s_{\text{low},y} - d + R b_{m,3}$$

$$r_i^y(k) \geq s_{\text{high},y} + d - R b_{m,4}$$

(5)

$$\sum_{\tau=1}^{4} b_{m,\tau} \leq 3, \forall i \in [1, \ldots, n],$$

(6)

where $R$ is a large and positive number and $b_{m,\tau} \in \{0, 1\}$ $(\tau = \{1, 2, 3, 4\})$. Equations (5) and (6) ensure that at least one of the equalities in (4) is satisfied, which guarantees the AGV is outside of the static obstacle area.
apart from each other in terms of \((x, y)\) coordinate. Considering \(d\) is the safety distance of two AGVs. Then constraints can be described as follows: for \(\forall i_1, i_2 \in \{1, ..., n\}, i_1 \neq i_2\)
\[\|r_{i_1}^x(k) - r_{i_2}^x(k)\| \geq 2d \text{ or } \|r_{i_1}^y(k) - r_{i_2}^y(k)\| \geq 2d.\] (7)
Constraint (7) is rewritten using binary variables in order to obtain the standard optimization formulation as follows: for \(\forall k\),
\[r_{i_1}^x(k) \leq r_{i_2}^x(k) - 2d + Rb_{i_1,i_2}^1(k)\]
\[r_{i_1}^y(k) \geq r_{i_2}^y(k) + 2d - Rb_{i_1,i_2}^2(k)\]
\[r_{i_1}^x(k) \leq r_{i_2}^x(k) - 2d + Rb_{i_1,i_2}^3(k)\]
\[r_{i_1}^y(k) \geq r_{i_2}^y(k) + 2d - Rb_{i_1,i_2}^4(k)\]
\[\sum_{\tau = 1}^4 b_{i_1,i_2}^\tau(k) \leq 3,\] (9)
where \(R\) is a large and positive number and \(b_{i_1,i_2}^\tau \in \{0, 1\}\) (\(\tau = \{1, 2, 3, 4\}\)). Equation (8) and (9) ensure (7) is satisfied.

3. CONTROLLER DESIGN

To coordinate the scheduling of interacting machines and the trajectory planning of machines in container terminals, a hierarchical control structure is proposed in [Xin et al., 2014], illustrated in Fig. 4. The control structure aims at achieving energy-efficient scheduling by combining the scheduling of equipment and time-dependent dynamics of all machines. The control architecture consists of three levels: the supervisory controller, the stage controllers and the controllers of equipment. The supervisory controller aims at determining the energy-efficient scheduling. The stage controller is then used to assign a container to specific equipment within a given time window. The controller of individual machines will determine its trajectory by means of minimizing energy consumption within the given time window. In this paper, we focus on the AGVs since AGVs may collide with one another.

The complete procedure of the hierarchical control structure for the AGV operations is as follows:

(1) the stage controller sends the minimal time \(s_1^j\) to transport container \(j\) from the quayside to the stack and the minimal time \(s_2^j\) to return from the stack to the quayside after transporting container \(j\);
(2) the supervisory controller computes the schedule for all jobs and sends the time window \([t_{s1}^{jx}, t_{s1}^{jy}]\) and \([t_{s2}^{jx}, t_{s2}^{jy}]\) of container \(j\) to the stage controller. \(t_{s1}^{jx}\) and \(t_{s1}^{jy}\) are the starting time and the ending time of transporting container \(j\) from the quayside to the stack. \(t_{s2}^{jx}\) and \(t_{s2}^{jy}\) are the starting time and the ending time from the stack to the quayside after transporting container \(j\);
(3) the stage controller assigns the time window to a specific AGV \(i\) as \([t_{s1}^{ix}, t_{s1}^{iy}], t_{s2}^{ix}, t_{s2}^{iy}\) \((i = 1, 2, ..., n)\). \(t_{s1}^{ix}\) and \(t_{s1}^{iy}\) are the starting time and the ending time of AGV \(i\) from the quayside to the stack for transporting container \(j\). \(t_{s2}^{ix}\) and \(t_{s2}^{iy}\) are the starting time and the ending time from the stack to the quayside of AGV \(i\) after transporting container \(j\);
(4) the controller of equipment receives the time window and computes the trajectories that the equipment should follow (taking into account energy minimization). The trajectory of the AGV with the earlier schedule is planned with a higher priority.

The details of the supervisory controller can be found in Xin et al. [2014]. Below, the details of the minimal-time calculation of AGVs will be introduced. Then the minimal-energy controller of AGVs for determining the trajectory of all AGVs will be proposed.

3.1 Minimal-time calculation

In this calculation problem, the AGV is required to reach the target \(r_f = [r_x^f, r_y^f]^T\) as fast as possible. Here we use a numerical approach to calculate the minimal time required by an AGV to transport container \(j\) from origin \(r_0\) to destination \(r_f\) subject to its dynamics and the static obstacle avoidance. Suppose \(T\) is the length of the given time window. Within a given interval \([0, ..., T-1]\), the AGV reaches \(r_f\) only at a certain moment, which is guaranteed by a binary variable at time \(k\). This constraint can be presented as follows: for \(\forall k \in [1, ..., T-1]\),
\[r_x^f(k) - r_x^f(0) \leq R(1 - b(k))\]
\[r_y^f(k) - r_y^f(0) \geq -R(1 - b(k))\]
\[r_x^f(k) - r_x^f(0) \leq R(1 - b(k))\]
\[r_y^f(k) - r_y^f(0) \geq -R(1 - b(k))\]
\[\sum_{k=1}^{T-1} b(k) = 1, \forall k \in [1, ..., T-1]\] (11)
where \(b(k) \in \{0, 1\}\) is a binary variable, \(R\) is a large and positive number to guarantee the constraints in (10) are active only when \(b(k) = 1\). Equations (10) and (11) force the position \(r(k)\) of an AGV to reach the target \(r_f\) with condition \(b(k) = 1\).

If we define \(t(k)\) as the elapsed time at time \(k\) \(t(k) = k\) since \(k = 0\), then \(t(k)b(k)\) is the finishing time when \(b(k) = 1\). Therefore, the minimal time for transporting container \(j\) can be obtained by minimizing the sum of finishing times according to different \(r_f\) as follows:
\[\min_{u,b} \sum_{k=1}^{T-1} t(k)b(k) = 1,\] (12)
subject to (1)-(5), (10) and (11),
where \(u = [u(0), u(1), ..., u(T-1)]^T\) denotes continuous decision variables and \(b = [b(0), b(1), ..., b(T-1)]^T\) denotes binary decision variables in optimization problem (6), respectively. The value of the objective function (11) is the minimal time needed for transporting container \(j\), this information is needed for \(s_1^j\) and \(s_2^j\).

3.2 Energy-efficient controller

Under a schedule that maximizes the freedom of AGVs, some AGVs do not aim for the minimal-time of transporting a container unnecessarily due to interaction of different types of machines. Instead, these AGVs can reduce energy consumption within the wide time window given by the higher-level controller. Therefore, the objective function of
AGVs is formulated as follows:

$$\min_{\mathbf{u}, \mathbf{b}} \sum_{k=0}^{T_{ij}-1} (|u_i^x(k)| + |u_i^y(k)|)$$

subject to (1)-(9).

The controller of an AGV will determine the trajectory of the vehicle based on the given schedule. The formulated optimization problem (14) is a mixed integer linear programming problem, which can be solved by state-of-the-art solvers (e.g., SCIP in Currie and Wilson [2012]).

### 4. CASE STUDY

To illustrate the performance of the proposed controller for trajectory planning of AGVs, we choose part of a benchmark system Xin et al. [2014] as a case study.

In this benchmark system, a container vessel, three QCs and six stacks are considered, shown in Fig. 5. One stacking crane is installed per stack. The features of this benchmark system are given as follows:

- The distance between the furtherest container and the interchange point of the QC is 100 m;
- The quayside transport area is 150 m × 200 m;
- Each stack has a length of 36 TEU, a width of 10 TEU and a max height of 6 TEU for capacity;
- The maximum speed of an AGV is assumed as $v_{\text{max}} = 6 \text{ m/s}$.

![Fig. 4. The hierarchical control structure proposed for a container terminal.](image)

![Fig. 5. The layout of the container terminal benchmarking system.](image)

- The maximum acceleration of an AGV is assumed to be $a_{\text{max}} = 1 \text{ m/s}^2$;
- The weight of an empty AGV and a TEU are 15t and 15t, respectively;
- Each piece of equipment can only transport one TEU container at a time;
- Each AGV can move autonomously (i.e., it is free ranging);

### 4.1 Scenario

We choose 1 QC, 2 AGVs and 3 ASCs (in stack 1-3) as a setup to test the trajectory planning of the AGVs. In this scenario, eight containers arriving at the same time are transported from the quayside to the three stacks. Each container has a different storage location in a stack. The time windows for each vehicle associated with a transported container, as provided by the schedule from the supervisory controller, are given in Table 1 and Table 2. The inbound move refers to the move of AGV from the quayside to the stacking area, while the outbound move is the opposite against the inbound move.

Table 1 and Table 2 present the time windows of two AGVs. In Table 1 and Table 2, the arrival stack and the arrival direction of each AGV are given. Also, the
Table 1. The time table of AGV 1

<table>
<thead>
<tr>
<th>stack</th>
<th>direction</th>
<th>departure</th>
<th>arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>inbound</td>
<td>48s</td>
<td>83s</td>
</tr>
<tr>
<td>1</td>
<td>outbound</td>
<td>83s</td>
<td>150s</td>
</tr>
<tr>
<td>2</td>
<td>inbound</td>
<td>150s</td>
<td>180s</td>
</tr>
<tr>
<td>2</td>
<td>outbound</td>
<td>180s</td>
<td>258s</td>
</tr>
<tr>
<td>1</td>
<td>inbound</td>
<td>258s</td>
<td>293s</td>
</tr>
<tr>
<td>1</td>
<td>outbound</td>
<td>293s</td>
<td>364s</td>
</tr>
<tr>
<td>1</td>
<td>inbound</td>
<td>364s</td>
<td>399s</td>
</tr>
<tr>
<td>1</td>
<td>outbound</td>
<td>399s</td>
<td>477s</td>
</tr>
</tbody>
</table>

Table 2. The time table of AGV 2

<table>
<thead>
<tr>
<th>stack</th>
<th>direction</th>
<th>departure</th>
<th>arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>inbound</td>
<td>100s</td>
<td>125s</td>
</tr>
<tr>
<td>3</td>
<td>outbound</td>
<td>125s</td>
<td>204s</td>
</tr>
<tr>
<td>3</td>
<td>inbound</td>
<td>204s</td>
<td>229s</td>
</tr>
<tr>
<td>3</td>
<td>outbound</td>
<td>229s</td>
<td>314s</td>
</tr>
<tr>
<td>3</td>
<td>inbound</td>
<td>314s</td>
<td>330s</td>
</tr>
<tr>
<td>3</td>
<td>outbound</td>
<td>330s</td>
<td>412s</td>
</tr>
<tr>
<td>2</td>
<td>inbound</td>
<td>412s</td>
<td>442s</td>
</tr>
<tr>
<td>2</td>
<td>outbound</td>
<td>442s</td>
<td>477s</td>
</tr>
</tbody>
</table>

departure time and the arrival time of vehicle associated with different destinations are listed.

4.2 Results

The collision-free trajectories of the AGVs are shown in Fig. 6. The relative distance of the AGVs over time is presented in Fig. 7 to show the collision avoidance. The velocity and acceleration profiles of the AGVs (the absolute velocity and the absolute acceleration) are given in Fig. 8 and Fig. 9. Table 3 and Table 4 present the energy cost of both two AGVs.

It can be seen from Fig. 8 and Fig. 9 that the absolute velocity profiles of AGVs are different when it comes to the inbound move and the outbound move. For an inbound move, the AGVs are operating at the high speed from the quayside to stacking area to guarantee the high handling capability. On the contrary, for an outbound move the AGVs are operating at the low velocity for energy efficiency when AGVs return from the stacking area to the quayside. In particular, there is more fluctuation in the acceleration and deceleration of the AGVs in an outbound move than in an inbound move. In an inbound move the AGV has to decelerate fully when it passes by the static obstacle. In an outbound move the AGV first accelerates and maintain a low speed. Then it accelerates to change the direction for preventing from the static obstacle. It can be seen from Fig. 8 and Fig. 9 that the absolute velocity profiles of AGVs are different when it comes to the inbound move and the outbound move. For an inbound move, the AGVs are operating at the high speed from the quayside to stacking area to guarantee the high handling capability. On the contrary, for an outbound move the AGVs are operating at the low velocity for energy efficiency when AGVs return from the stacking area to the quayside. In particular, there is more fluctuation in the acceleration and deceleration of the AGVs in an outbound move than in an inbound move. In an inbound move the AGV has to decelerate fully when it passes by the static obstacle. In an outbound move the AGV first accelerates and maintain a low speed. Then it accelerates to change the direction for preventing from the static obstacle. Table 3 and Table 4 present the energy cost of both two AGVs when they are operated between the quayside and the stacking area. As seen in Table 3 and Table 4, the energy used for the outbound move is significantly more than in an inbound move. In an inbound move the AGV has to decelerate fully when it passes by the static obstacle. In an outbound move the AGV first accelerates and maintain a low speed. Then it accelerates to change the direction for preventing from the static obstacle. Table 3 and Table 4 present the energy cost of both two AGVs when they are operated between the quayside and the stacking area. As seen in Table 3 and Table 4, the energy used for the outbound move is significantly more
Table 3. The energy usage of AGV 1.

<table>
<thead>
<tr>
<th>stack number</th>
<th>inbound move</th>
<th>outbound move</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.3</td>
<td>7.6</td>
</tr>
<tr>
<td>2</td>
<td>16.5</td>
<td>5.4</td>
</tr>
<tr>
<td>1</td>
<td>16.3</td>
<td>7.2</td>
</tr>
<tr>
<td>1</td>
<td>16.3</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 4. The energy usage of AGV 2.

<table>
<thead>
<tr>
<th>stack number</th>
<th>inbound move</th>
<th>outbound move</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>16.9</td>
<td>4.3</td>
</tr>
<tr>
<td>3</td>
<td>16.9</td>
<td>4.0</td>
</tr>
<tr>
<td>3</td>
<td>16.9</td>
<td>4.6</td>
</tr>
<tr>
<td>2</td>
<td>16.5</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Table 5. The time table of AGV 1.

<table>
<thead>
<tr>
<th>AGV number</th>
<th>stack</th>
<th>direction</th>
<th>departure</th>
<th>arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>inbound</td>
<td>0s</td>
<td>35s</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>outbound</td>
<td>35s</td>
<td>70s</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>inbound</td>
<td>10s</td>
<td>40s</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>outbound</td>
<td>40s</td>
<td>80s</td>
</tr>
</tbody>
</table>

Fig. 10. The relative distance between the AGVs without the moving obstacle constraint. (AGV2 starts moving after $t=10s$) than the energy used for the inbound move. The energy reduction of the inbound move results from the increased velocity during the move.

4.3 Illustration of moving obstacle avoidance

To illustrate the avoidance of moving obstacles, a time table of two AGVs is given in Table 5. Following this time table, the trajectories of the AGVs without the moving obstacle constraint and with the moving obstacle constraint are generated. The relative distance of the AGVs over time in these two situations are shown both in Fig. 10 and Fig. 11. It can be seen from Fig. 10 that the relative distance of the AGVs is less than 10m from 50s to 70s. That means the two AGVs collide with each other without the moving obstacle constraint. In Fig. 11, the relative distance of the AGVs is consistently more than 10m that means the moving obstacle is prevented.

5. CONCLUSIONS AND FUTURE RESEARCH

This paper proposes an approach to generate the trajectories of AGVs in automated container terminals. The trajectory planning takes the dynamics and collision avoidance of AGVs into account, interacting with the schedule of different types of machines. The minimal-time is calculated for the interaction with the higher-level controller. A trajectory planning problem is formulated as a mixed integer linear programming problem (MILP), which is solved by the SCIP solver. The simulation results illustrate the potential of the proposed methodology in a case study.

Future research will consider a larger scale system of multiple AGVs. In this case, more AGVs will be employed to serve multiple quay cranes to maintain the high productivity of cranes. The scheduling problem of interacting machines and the trajectory planning problem will interact with each other in the proposed control architecture.

REFERENCES


