Simultaneous Adaptation of the Process and Measurement Noise Covariances for the UKF Applied to Nanosatellite Attitude Estimation

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Abstract: A common technique for improving the estimation performance of the Kalman filter and making the filter robust against any kind of faults is to adapt its process and measurement noise covariance matrices. Although there are numerous approaches for the adaptation such as full estimation or scaling, simultaneous adaptation of these two matrices is an ongoing discussion. In this paper, firstly, two common problems for the attitude estimation of a nanosatellite are solved by adapting the process and noise covariance matrices. Then these two adaptation methods are integrated with an easy to apply scheme and the matrices are simultaneously adapted. The newly proposed filtering algorithm, which is named Robust Adaptive Unscented Kalman Filter, considerably increases the estimation performance and is fault tolerant against the sensor malfunctions.

1. INTRODUCTION

In a broad perspective, the attitude determination problems for nanosatellites carrying magnetometers onboard are related to magnetic disturbance compensation which is necessary to guarantee the magnetic cleanliness of the spacecraft. The basic problem is finding an accurate estimation algorithm for the in-orbit and real time estimation of the magnetometer biases and that is an obligation. This is a recent topic for nanosatellite applications and since the magnetometers are popular sensors for this type of satellites there are many documented studies especially on magnetometer calibration. In (Inamori et al., 2009) the magnetometer biases are estimated as a part of the magnetic disturbance compensation for a nanosatellite. The Unscented Kalman Filter (UKF) is used as the estimator algorithm. Han et al. (2012) proposes both pre-launch on ground and post-launch in-orbit magnetometer calibration schemes for Chinese ZDPS-1A nanosatellite. In (Soken and Hajiyev 2012) along with the magnetometer biases the scale factors are also considered and a UKF based reconfigurable attitude estimation and magnetometer calibration method is presented. Lastly Vinther et al. (2011) investigates the effects of magnetometer and gyro calibration on the attitude accuracy and gives a simultaneous estimation algorithm using the full-order UKF.

The biggest difficulty that arises in case of in-orbit real time sensor bias estimation is tuning the process noise covariance (Q) matrix of the estimator. If prior assumptions about the covariance values are poor then the filter’s optimality will be affected and estimation performance will degrade (Almagbile et al., 2010). For the general case where the UKF is used for estimating only the attitude and gyro biases as a reduced-order filter the process noise covariance matrix can be approximated analytically (Crassidis and Markley 2003; Fosbury 2011). But this method fails if the magnetometer biases are also estimated as a part of the state vector. One possible solution technique is to use an adaptive algorithm to tune the Q matrix as discussed in this paper.

The adaptation of the UKF is also a necessity for building a filter which is robust against any kind of sensor malfunctions. Since the spacecraft is vulnerable against the external disturbances there is a high risk for the magnetometer measurements to be affected and give faulty outputs for a period of time. Unless the filter is built robust, sensor faults will deteriorate the estimation performance significantly. In this case the measurement noise covariance (R) matrix must be adapted (Soken and Hajiyev, 2010).

In literature there are several methods to adapt the linear Kalman filter (KF). Unquestionably, the pioneering methods in this area have been proposed by Mehra (Mehra 1970; Mehra 1972). Specifically the covariance matching technique discussed in (Mehra 1972) may be considered as the fundamental of the algorithms proposed in this paper. The main drawback of these studies, and as well their successors that examine the adaptation of the KF (Geng and Wang 2008; Kim et al., 2006; Odelson et al., 2006; Dunik et al., 2009; Fakharian et al., 2011), they are generally appropriate for discrete-time linear systems and cannot be used as a method for the adaptation of the UKF without any correction or modification.

In this sense, researches on the adaptation of nonlinear Kalman filters should be searched. In (Han et al. 2009), two distinct methods are described as the Adaptive Unscented Kalman Filter (AUKF) algorithms. In the first method, the MIT rule is used to derive the adaptive law and a cost function is defined in order to minimize the difference between the filter computed covariance and the actual
innovation covariance. The algorithm is used for the Q-adaptation and it is stated that a similar approach may be followed for the R-adaptation. As a deficiency, the presented algorithm requires calculation of the partial derivatives and that introduces a relatively large computational burden as it is also stated by authors themselves. In the second method, two UKFs are run in parallel within master and slave filter manner. Its computational demand is lower than the first method but as it is known (Soken, 2013), despite being free of the Jacobian calculations, the computational burden of the UKF is not very low because of the sigma point calculations. Therefore, using two UKF algorithms in a parallel manner still increases the required computation burden significantly. Hence the main problem for both of the methods presented in (Han et al. 2009) is high computational load.

There are also adaptation techniques, applied to the other nonlinear Kalman filters. In (Sebesta and Boizot, 2014) an adaptive high-gain Extended Kalman Filter (EKF) is introduced specifically to solve the convergence problem associated with the traditional EKF. Both the Q and R matrices are adapted based on the filter’s innovation. In (Karlgaard and Shen, 2013) a robust divided difference filtering approach based on the concept of Desensitized Kalman Filtering is proposed. The filter is adapted to reduce the sensitivity to the deviations in the assumed plant model parameters. The main drawback of these studies is the complexity which may increase the computational load unnecessarily.

In this study, first we examine two practical problems for a nanosatellite carrying magnetometers onboard and describe how to solve these problems using the adaptive Kalman filtering approach. In this sense, the proposed Q and R adaptation techniques, which are mentioned briefly in (Soken and Sakai, 2013), are discussed in details. Then as the next step, we propose an integration scheme for using these two adaptation techniques in a single UKF simultaneously. We address the applicability conditions of the new algorithm, which is named Robust Adaptive UKF (RAUKF). As the final step we demonstrate the RAUKF for attitude estimation of a hypothetical nanosatellite to validate its availability.

2. PROCESS AND MEASUREMENT NOISE COVARIANCE ADAPTATION FOR ATTITUDE ESTIMATION

2.1 The Q-Adaptation for In-flight Magnetometer Calibration

Accuracy

As mentioned, magnetometers are not accurate sensors. Moreover, on-ground calibration of these sensors is not sufficient since the spacecraft is small and there is a high interaction between the operating subsystems. For an effective calibration and increasing the overall ADCS performance the magnetometers must be in-flight calibrated (Soken, 2013).

In general, magnetometer calibration methods may be categorized in two: attitude-independent algorithms such as two-step and its varieties (Alonso and Shuster, 2002; Sakai et al., 2011); and attitude-dependent algorithms where usually a Kalman filter is used for estimation (Vinther et al., 2011). The latter is also what we prefer in this study. We use a reduced-order UKF, an UKF that is propagating the states via gyro-based model, for estimating the satellite’s attitude and calibrating the gyro and magnetometer measurements.

A nonlinear version of the Kalman filter must be used for the attitude estimation problem. The UKF algorithm is a relatively new nonlinear filtering method which has many advantages over the well known EKF (Julier et al., 1995). The biggest challenge, when we use the UKF for the simultaneous attitude and magnetometer bias estimation, is determining the process noise covariance matrix of the filter. If the a priori statistics selected as a constant do not match with the real values, then the filter characteristics such as the accuracy or convergence speed may be affected and even a serious mismatch may cause the filter to fail in practice (Dunik et al., 2009). The designer has always chance to tune the process noise covariance by trial-error method but this is a time consuming process and obtaining the optimal values is not guaranteed.

Our proposed solution for this problem is to use an adaptive algorithm to estimate the Q matrix based on the residual series. The process noise covariance is estimated via the following maximum likelihood based algorithm (Maybeck, 1982) to get the Q values that increase the overall estimation performance.

\[ Q(k+1) = \gamma Q^* + (1-\gamma)Q(k), \]  
\[ Q^* = \Delta x(k+1)\Delta x^T(k+1) + P(k+1|k) \]  
\[ -P(k+1|k+1)Q(k), \]  
\[ \Delta x(k+1) = \hat{x}(k+1|k+1) - \hat{x}(k+1|k). \]

where, \( Q(k+1) \) is the estimated process noise covariance matrix for step \( k+1 \), \( Q^* \) is the observation for the estimation, \( \hat{x}(k+1|k+1) \) is the estimated state vector, \( \hat{x}(k+1|k) \) is the predicted state vector, \( P(k+1|k+1) \) is the estimated covariance matrix, \( P(k+1|k) \) is the predicted covariance matrix and \( \gamma \) is the scale factor for low-pass effect.

The process noise covariance matrix estimated by (1) is not a diagonal matrix because of the state residual term. So specifically for this problem we propose modifying it such that it fits in the form given as

\[ Q = \begin{bmatrix} Q_{q} I_{3 \times 3} & Q_{q,gb} I_{3 \times 3} & 0_{3 \times 3} \\ Q_{q,gb} I_{3 \times 3} & Q_{gb} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & Q_{mb} I_{3 \times 3} \end{bmatrix}, \]  

when the attitude, gyro and magnetometer biases are estimated. Here \( Q_{q} \), \( Q_{gb} \) and \( Q_{mb} \) are the scalar process noise covariance terms which correspond to the attitude quaternion, gyro and magnetometer biases respectively and \( Q_{q,gb} \) are the scalar terms for the noise covariance in between the quaternion and gyro bias states.
The UKF is started with an initial guess for the Q matrix. Then, after each state estimation step, Q is updated using the residual based estimation procedure (1-4) and this updated matrix is used for the next step.

2.2 The R-Adaptation for Magnetometer Fault Tolerance

For the nanosatellite missions the measurement sensors may be easily affected from the other subsystems considering the size of the nanosatellite and necessity for placing the subsystems closely to each other. Furthermore the external disturbances such as the ionospheric currents may have a great deteriorating effect on the measurement performance. Such faulty measurements affect the stability and accuracy of the UKF even when they last for few samples.

Therefore, a robust algorithm must be introduced such that the filter is insensitive to the measurements in case of malfunctions and the estimation process is corrected without affecting the remaining good estimation behavior.

The robustness of the filter is secured by scaling the measurement noise covariance matrix in case of fault. For scaling we multiply the measurement noise covariance matrix with a scale matrix formed of multiple factors. The scale matrix is calculated based on the innovation series as (Soken and Hajiyev, 2010),

\[ S_n(k) = \left( \frac{1}{\xi} \sum_{j=1}^{\xi} e(j+1) e'(j+1) - P_{\psi}(k+1|k) \right) R^{-1}(k+1) . \]  

where \( S_n(k) \) is the scale matrix, \( e(j+1) \) is the innovation of the UKF, \( P_{\psi}(k+1|k) \) is the observation covariance matrix, \( R(k+1) \) is the measurement noise covariance matrix and \( \xi \) is the size of the moving window. The calculated \( S_n(k) \) matrix may not be diagonal and may have diagonal elements which are “negative” or lesser than “one”. In order to avoid such situations, composing the scale matrix by the following rule is suggested:

\[ S^* = \text{diag}\{s_1^*, s_2^*, \ldots, s_z^*\} \]  

\[ s_i^* = \max\{1, S_{n,i}\} i = 1, z . \]  

Here, \( S_{n,i} \) represents the \( i \)-th diagonal element of the matrix \( S_n(k) \), where \( z \) is the dimension of the innovation vector.

Apart from that point, if the measurements are faulty, \( S^*(k) \) will change and so affect the Kalman gain as;

\[ K(k+1) = P_{\psi}(k+1|k) \left[ P_{\psi}(k+1|k) + S^*(k) R(k+1) \right]^{-1} . \]  

Here \( K(k+1) \) is the Kalman gain.

The robust algorithm affects characteristic of the filter only when the condition of the measurement system does not correspond to the model used in the synthesis of the filter. Otherwise the UKF work with the regular algorithm. The fault detection is realized via a kind of statistical information. In order to achieve that, following two hypotheses may be proposed:

- \( \gamma_o \): the system is normally operating
- \( \gamma_f \): there is a malfunction in the estimation system.

Then we may introduce the following statistical functions for the R-adaptation

\[ \beta(k) = e^T(k+1) \left[ P_{\psi}(k+1|k) + R(k+1) \right]^{-1} e(k+1) , \]  

This function has \( \chi^2 \) distribution with \( z \) degree of freedom.

If the level of significance, \( \alpha \), is selected as,

\[ P\{\chi^2 > \chi^2_{\alpha,z}\} = \alpha ; \quad 0 < \alpha < 1 , \]  

the threshold value, \( \chi^2_{\alpha,z} \), can be determined. Hence, when the hypothesis \( \gamma_1 \) is correct, the statistical value of \( \beta(k) \) will be greater than the threshold value \( \chi^2_{\alpha,z} \), i.e.:

\[ \gamma_o : \beta(k) \leq \chi^2_{\alpha,z} \quad \forall k \]  

\[ \gamma_f : \beta(k) > \chi^2_{\alpha,z} \quad \exists k \].

3. THE RAUKF ALGORITHM

The Q and R-adaptation algorithms presented in Section 2 are solutions for different problems. The Q-adaptation is used as a tuning algorithm for the process noise covariance of the filter in order to ease the difficult tuning procedure and make the filter more efficient in the sense of estimation accuracy. On the other hand, the R-adaptation is performed as a measure against the possible measurement faults in the harsh space environment. In this section we integrate these two filters.

The integration of the Q and R-adaptation techniques is an open topic and there are numerous researches in the literature (Hajiyev and Soken, 2013; Almagbile et al. 2010). Indeed there is not any stable integration method when both the R and Q matrices are estimated based on the innovation covariance (Almagbile et al. 2010). In this case the Q must be estimated assuming full knowledge of the R and vice versa. Nonetheless, the Q-adaptation method presented in this paper estimates the Q matrix based on the residual covariance and the adaptation method for the R matrix is an innovation covariance based scaling method, not the direct estimation of the matrix itself. The adaptation methods use different information sources, the R-adaptation uses the innovation and the Q adaptation uses the residual. Moreover the Q-adaptation is performed by directly estimating the matrix whereas the R is adapted by scaling. Hence these two methods can be run at the same time. Fig. 1 shows the integration method with the key steps of the RAUKF.

There are two important points that should be regarded while designing the RAUKF:

- The R scaling is performed only when a fault is detected in the measurements as given with (11). In all other cases the filter runs with the regular algorithm only with the Q estimation (when there is no fault the algorithm is same as the UKF with Q estimation).
The scale factor for the Q-adaptation in (1) should be selected carefully. If more aggressive adaptation is performed (such that $\gamma \approx 1$) the stability of the RAUKF might be affected in case of a measurement fault when both R and Q adaptations are necessary.

Initial attitude errors are set to 30, 25 and 25 deg for pitch, yaw and roll axes respectively. The initial estimation values for the gyro and magnetometer biases are all taken as 0.

We tested the RAUKF for the continuous bias failure. A constant value is added to the measurements of the magnetometer aligned in the $x$ axis between the 30000th and 30200th seconds for a period of 200 seconds such that

$$B(x,q,t) = B(x,q,t) + 20000nT \quad t = 30000...30200\text{sec}$$

A deviation in the bias with this amount is reasonable when the values given in (Sakai et al., 2011) are taken into consideration.

4.1 Effects of the Q-Adaptation

In Fig. 2 the pitch angle estimation results that are obtained when we use the regular UKF or the proposed RAUKF are given in the same plot. As clearly seen, especially from the zoomed subplot, the results obtained by the RAUKF are far more accurate. This is mainly because of the nearly optimal values of the $Q$ matrix for the RAUKF that we cannot easily obtain by the trial-error method.

For the simulations the process noise covariance matrix for the UKF is

$$Q = \begin{bmatrix}
(1 \times 10^{-3})I_{3x3} & -(1.5 \times 10^{-7})I_{3x3} & 0_{3x3} \\
-(1.5 \times 10^{-7})I_{3x3} & (1 \times 10^{-10})I_{3x3} & 0_{3x3} \\
0_{3x3} & 0_{3x3} & (1 \times 10^{-12})I_{3x3}
\end{bmatrix}.$$
As seen the estimated Q values are absolutely different from the initial values and it is not easy to guess such values by trial-error method.

In case we use the RAUKF the sensor calibration performance is also highly increased as may be seen in Fig. 3, which presents the magnetometer bias estimation error for the magnetometer aligned in the z axis. Indeed, such increment is tightly related to the increased attitude estimation accuracy. The RAUKF itself increases both attitude estimation and sensor calibration performance as the estimated Q values match with the real values. Besides, when the magnetometer biases are estimated more precisely that brings about better attitude estimation results since the accuracy of the incoming measurements are increased.

As expected the UKF estimations deteriorate in case of measurement fault and it takes an additional 1000 seconds for the filter to converge again and satisfy estimation results with error less than 0.1deg. On the other hand the RAUKF maintains its good estimation performance even in case of the fault.

Fig. 4. Estimation of the roll angle via the RAUKF (red line) and UKF (black line) in case of measurement malfunction.

Similar behavior can be seen for the magnetometer bias estimation results (Fig.5). An additional bias that is experienced because of the sensor fault is considered as a variation in the estimated bias terms by the regular UKF. Therefore UKF bias estimations, specifically for the magnetometer with the fault, worsen. However the RAUKF keep providing the accurate bias estimation results.

5. CONCLUSIONS

In this study, first two practical problems for a nanosatellite carrying magnetometers onboard are examined and it is described how to solve this problems using adaptive Kalman filtering approach. In this sense, the process noise covariance (Q) and measurement noise covariance (R) adaptation techniques are presented. The Q-adaptation method is used to tune the Q matrix based on the residual series and obtain the optimal Q values. As a result the attitude estimation and sensor calibration performance of the UKF increased. The innovation based R-scaling method is used to adapt the R...
matrix and build an UKF that is robust against sensor malfunctions. Then as the next step, an integration scheme for using these two adaptation techniques in a single UKF simultaneously is proposed. The applicability conditions of the new algorithm that is named Robust Adaptive UKF (RAUKF) are discussed. As the final step the RAUKF is demonstrated for attitude estimation of a hypothetical nanosatellite. The simulation results show that the RAUKF perform well under all conditions including the sensor fault case and give better estimation results than the regular UKF algorithm. Besides the demonstrations prove that the proposed integration scheme for two different adaptation methods works properly.

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