A Novel Method for Modelling Cellular Response Genome Stress by Combined Gene-Environment Network (GEN) and Kinetic Theory Framework

Jin-Peng Qi*, Qing. Zhang*, and Jie. Qi *

Abstract: Under acute Ion Radiation (IR) perturbation from external environment, a normal cell can trigger a series of cellular self-defensive mechanisms. To model the complicated Cellular Response Genome Stress (CRGS, for short), a novel method is proposed based on combined Gene-Environment Network (GEN) and Kinetic Theory of Active Particle (KTAP) framework. In this method, the integrated CRGS model is divided into three subsystems, and each subsystem is composed of different particles with different discrete states at molecular level. Then the dynamic kinetics of integrated CRGS is dealt as outcomes of complicated mutual interactions among molecular particles from different subsystems and external environment cofactors at systematic level. The simulation results show that our method is efficient to illustrate a series of cellular activities, including stochastic Double Strand Break (DSB) generation and repair, switch-like Ataxia Telangiectasia Mutated (ATM) activation, oscillating P53-MDM2 feedback loop as well, and helpful to analyze the capability of cellular self-defensive mechanism, as well as genome stability in response to different IR perturbation circumstances.

1. INTRODUCTION

As one unit of bio-system, a cell, consists of a large number of active molecules, such as, DNA, mRNA, protein etc (Bellouquid and Delitala 2006; Bellouquid and Delitala 2006). Under acute perturbation from outer environment, a cell can start a series of inner activities in response to genome stress (Perez and Brady 1999; Li, Story et al. 2001). For example, under acute IR, Double Strand Breaks (DSBs) occur in a normal cell stochastically. Fortunately, a cell can fix these DNA damage by cellular self-repair mechanisms, and then keep genome stability. To investigate the complicated mechanisms of biological system, currently, the combined approaches of control theory, system biology and bioinformatics can stimulate new ideas, which provide a good link between diverging areas of biomedicine and mathematics (Chou 2004; Bellomo, Bellouquid et al. 2010). Recently, several existing methods are used to represent the complicated cellular activities in response to genome stress via P53 signal pathway models. For example, the stochastic models of p53 system were proposed in (Puszyński, Hat et al. 2008; Cai and Yuan 2009; Leenders and Tuszynski 2013). Also, other models of p53 were investigated, such as in (Batchelor, Loewer et al. 2011), (Kim, Rho et al. 2009), (Qi, Shao et al. 2007; Qi, Shao et al. 2008; Qi, Ding et al. 2009; Zhang, Liu et al. 2012), and (Ma, Wagner et al. 2005).

On the other hand, as a novel mathematical framework, KTAP can be used in modelling the overall system by evolution equations corresponding to the dynamics of all their elements (Bellomo and Delitala 2008; Brazzoli 2008). In terms of KTAP framework, a biological phenomenon can be dealt as an evolution of dynamics of several interacting modules (Bellouquid and Delitala 2006; Bellomo and Forni 2008), and the description of bio-system essentially means defining microscopic state of these interacting molecules and distribution function under the state above (Bellouquid and Delitala 2006). KTAP is motivated not only by applied mathematicians, but also by researchers in the field of biological science. The methodology of statistical mechanics and kinetic theory to model complex biological system is capturing the attention of more and more applied mathematicians (Degond, Pareschi et al. 2004; Brazzoli 2008; Yu, Wacholder et al. 2012). Moreover, KTAP is applied in various fields of social and life science, e.g. modelling social behaviour of interacting individuals (Bertotti and Delitala 2008), especially, multi-cellular systems (Bellouquid and Delitala 2006; Bellomo and Forni 2008; Bellomo, Bellouquid et al. 2010), tumour-immune system competition (Bellomo and Forni 2008), as well as cellular self-repair mechanisms (Qi, Ding et al. 2011).

To investigate a series of cellular self-defensive activities in response to acute IR perturbation from outer environment, in this paper, a novel method is proposed to model CRGS based on the combined GEN and KTAP framework. In this method, the integrated CRGS model is divided into three subsystem, including the stochastic DSB generation and repair, switch-like ATM activation, oscillating P53-MDM2 feedback loop as well. Each part is composed of the different molecular particles with different discrete states, and then the dynamic kinetics of the integrated CRGS is dealt as the consequences.

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of complicated mutual interactions among molecular particles within different subsystems and external environment cofactors. Comparing with the deterministic ODE methods, our method can well not only describe stochastic mutual interactions among large-scale active particles at molecule level, but also demonstrate the complicated and dynamic kinetics of cellular self-defensive activities in response to external environment perturbation cofactors at system level. In the following paragraphs, Section.2 gives the proposed method in detail, including a brief survey of CRGS model, and the detailed implementation of CRGS model. Section.3 simulates the dynamic kinetics of integrated CRGS model, and roughly analyzes cellular self-defensive capability and genome stability under different IR perturbations. Section.4 gives the conclusion of above research.

2. METHOD

2.1 Survey of CRGS model

Based on the combined GEN and KTAP framework, our CRGS model aims to investigate cellular self-defensive mechanisms in response to genome stress at molecule and single cell levels. Fig.1 is the profile of CRGS model under acute IR perturbation. It is composed of three subsystems: DBS generation and repair, ATM activation, and P53-MDM2 feedback loop. Each subsystem includes active particles with different microscopic active states. Moreover, acute IR and molecular interactions may modify either the state of each molecular or the number of molecular in each population by proliferation/destruction phenomena (Bellouquid and Delitala 2006; Bellouquid and Delitala 2006; Brazzoli 2008).

As acute IR is applied to a cell, the first part represents the stochastic kinetics of DSB generation and repair processes. The second part denotes switch-like ATM activation under DSBC signal transferring; and the third part illustrates the oscillating kinetics of P53-MDM2 feedback loop induced by ATM activation, which is essential to trigger cellular response in fighting against genome stress further (Oren 2003; Ma, Wagner et al. 2005). To simplify the model, we assume the active particles in different subsystems are homogeneously distributed in space, undergoing localized binary interactions, in which the test particle enters into the action domain of the field particle (Brazzoli 2008). Due to the complicated mechanisms in CRGS, the integrated model above is just a simple description and simulation for the complicated cellular self-defensive activities. The limited vital genes and environment cofactors, as well as their mutual interactions and regulating pathways are involved into our integrated model.

2.2 Implementation of CRGS model

The dynamic kinetics of integrated CRGS model is composed of three parts, including stochastic DSB generation and repair, switch-like ATM activation, as well as oscillating P53-MDM2 feedback loop, respectively. In the following paragraphs, \( f_x \) denotes the normalized concentration density of the molecular particle \( x \). \( \eta_{x_0} \) stands for the encounter rate of molecular particle \( x \) with particle \( y \). \( B_{x,y}^{xy} \) refers to the probability of a particle \( x \) transforming into the state \( x' \) from initial state under mutual interaction with particle \( y \). \( K_{x,y} \) refers to the probability of a molecular \( x \) transforming into state \( x' \) from its initial state without any interference. \( d_x \) is the degradation rate of molecular particle \( x \).

In the DSB generation and repair sub-system shown in Fig.1.(a), DNA in a cell is broken down in response to external IR perturbation, and then DSBs occur stochastically. For a normal cell, it can fix DNA damage through self-repair mechanisms (Rothkamm, Krüger et al. 2003). In this period, Repair Protein (RP), a repair enzyme, can bind into the nascent DNA end and synthesize DSB-protein complex (DSBC) (Budman and Chu 2005). As a main signal source to transfer genome damage, DSBC can relay DNA damage to downstream ATM gene, and then trigger switch-like kinetics of ATM activation (Kohn and Pommier 2005). Under mutual interaction between repair gene (the test particle) and DSB (the field particle), repair mRNA transcription is prompted, and then RP translation is accelerated by mutual interaction between repair mRNA and DSB. Providing that RP is available around DNA damage site, DSBC synthesis is promoted by mutual interactions of DSB with RP (Qi, Ding et al. 2011). The correct repair part of DSBCs (rDSBCs) is dealt as a main signal to transfer damage signal to downstream genes and regulation pathways. Whereas, the disrepair part of DSBCs (mDSBCs) and intact DSBs will be remained in a cell, which can seriously weaken genome stability and cellular viability, even lead to abnormal and cancerous eventually (Ma, Wagner et al. 2005). The
stochastic kinetics of DSB generation and repair can be represented by the following equations:

\[
\frac{df_{DSB}}{dt} = C_k \cdot C_{DNA,IR} \cdot B_{DNA-IR} \cdot \eta_{IR} \cdot f_{DNA}, \tag{1}
\]

\[
B_{DNA-IR} = K_{DSB} \cdot Poisson(a_{IR} \cdot g(r_{IR})), \tag{2}
\]

\[
\frac{df_{rate}}{dt} = S_{rate} \cdot f_{rate} + \eta_{rate} \cdot B_{rate-DSB} \cdot f_{rate} \cdot f_{DSB} - \eta_{rate} \cdot B_{rate-DSB} \cdot f_{rate} \cdot f_{DSB} - d_{rate} \cdot f_{rate}, \tag{3}
\]

\[
\frac{df_{IR}}{dt} = S_{IR} \cdot f_{IR} + \eta_{IR} \cdot B_{IR-DSB} \cdot f_{IR} \cdot f_{DSB} - 5 \cdot d_{IR} \cdot f_{IR}, \tag{4}
\]

\[
\frac{df_{DSB}}{dt} = \eta_{DSB-IR} \cdot B_{DSB-IR} \cdot f_{IR} \cdot f_{DSB} - \eta_{DSB-IR} \cdot B_{DSB-IR} \cdot f_{IR} \cdot f_{DSB} - d_{IR} \cdot f_{IR}, \tag{5}
\]

where \( C_k \) refers the rate of DNA damage generation in accordance with different cell types. \( C_{DNA,IR} \) is the encounter rate of DNA and external IR perturbation cofactor. \( K_{DSB} \) is the number of DSB generation within each time scale, and \( a_{IR} \) is the number of DSB generation induced by per IR dose.

In the **ATM activation sub-system** shown in Fig.1.(b), ATM, a detector of DNA damage, is extremely sensitive to genome stress induced by external perturbation (Bakkenist and Kastan 2003; Paulkin, Kristjuhan et al. 2005). In response to damage signal transferred from DSBC synthesis, inactive ATM dimmer is phosphorylated into active ATM (ATM*) after positive interaction with rDSBC particles. Generally, ATM activation is a reversible kinetics, that is, ATM monomer can convert into ATM dimmer, and active ATM (ATM*) can transform into ATM dimmer reversibly. In addition, the total concentration of ATM including ATM monomer and ATM* is supposed to be a constant in the cell (Ma, Wagner et al. 2005; Qi, Ding et al. 2011). The kinetics of ATM activation is represented by following formulas:

\[
\frac{df_{ATM}}{dt} = \frac{1}{2} K_{ATM-ATM} \cdot f_{ATM}^2 - K_{ATM-ATM} \cdot f_{ATM}, \tag{6}
\]

\[
\frac{df_{ATM}}{dt} = 2K_{ATM-ATM} \cdot f_{ATM} - K_{ATM-ATM} \cdot f_{ATM}^2 - \eta_{ATM-ATM} \cdot B_{ATM-ATM} \cdot f_{ATM} + K_{ATM-ATM} \cdot f_{ATM}, \tag{7}
\]

The other related equations used in the integrated CRGS model can be found in Appendix A in detail.

### 3. RESULT AND DISCUSSION

By using platform of Matlab 7.1, the dynamics of CRGS are illustrated under different IR perturbation circumstances. In our simulations, the different 1, 5, 10, and 15Gy IR dose are applied into a normal cell, respectively. The concentrations of active molecule particles are denoted by the marks [*] in all figures of simulation results. The dimensionless parameters are shown in Appendix B in detail, which are taken and revised from (Ma, Wagner et al. 2005; Qi, Ding et al. 2009; Qi, Ding et al. 2011; Zhang, Liu et al. 2012).

#### 3.1 Stochastic kinetics of DSB generation and repair

As shown in Fig.2, in response to external interaction with acute IR perturbation cofactors from outer environment,
different numbers of resulting DSBs occur stochastically, and the more DSBs are generated as the stronger IR is applied. As a result, RP is translated from repair mRNA under mutual interaction with resulting DSBs, and the more resulting DSBs induced by stronger IR, the less RP is available around damage site. Thereafter, resulting DSBs are synthesized into DSBC under mutual interaction with RP available, and fewer numbers of rDSBCs are synthesized as available RPs decrease against stronger IR Perturbation cofactor. These simulations show that RP available can be dealt as an indicator of cellular self-repair capability in a normal cell. Whereas, this capability begins to decrease as the strength of external perturbation cofactor overtakes the threshold that a normal cell can burden maximally.

3.2 Oscillations of P53-MDM2 under ATM activation

As a detector of genome stress, ATM is very sensitive to damage signal from DSBC synthesis. In response to mutual interaction with rDSBC molecular, ATM is activated from inactive ATM monomer (ATMm), as $f_{\text{coac}}$ overtake the minimal threshold that ATM can sense. As shown in Fig.3, in response to different IR perturbations, the switch-like ATM activation occur after a period of radiation time scales, and the stronger IR is applied, the shorter time is needed. After activated, the phosphorylated ATM (ATM*) rapidly rises to a high plateau and keeps dynamic equilibrium until $f_{\text{coac}}$ is below the minimal threshold. The shorter duration that ATM activation can keep when the stronger IR is applied, because limited RPs are available around increasing damage sites, and $f_{\text{coac}}$ is lower that ATM cannot be detected. Then, the dynamic equilibrium of P53-MDM2 feedback loop is broken, as a result, oscillations occur between P53 and MDM2, after mutual interaction with ATM* particles. The dynamic oscillations between P53 and MDM2 keep active similarly with ATM activation, and the period of oscillation is shorter as stronger IR is applied. To some extent, these results are quite similar to the experimental observations and in accordance with existing studies in (Ma et al., 2005; Qi et al., 2008; 2009). The simulations above indicate that switch-like ATM activation can relay genome stress signal induced by external perturbation cofactors, and then transfer to P53-MDM2 feedback loop further. In addition, the periodical oscillations between P53 and MDM2 suggest that once self-defensive mechanisms is triggered, a series of cellular activities can be started, e.g., cell cycle arrest, and cell apoptosis etc.

3.3 Genome stability analysis

Although a normal cell can defence external perturbations properly through a series of cellular activities, some inevitable toxins, such as, intact DSBs and mDSBC might be accumulated in a cell, which damage cellular response capability and genome stability, even lead to fatal cancer (Qi et al., 2011; Ma et al., 2005). Therefore, we propose that a number of toxins remained in a cell can decrease genome stability directly. In our simulations, Fig.4 shows that $f_{\text{adac}}$, a by-product of DSBs repair process, is fluctuated in accordance with RP available and rDSBC synthesis shown in Fig.2. On the other hand, as another part of toxin, intact DSBs keep increasing against radiation time, and the remained DSBs increase faster, especially when stronger IR is applied, because of the limited RP available around increasing damage cites. As a result, genome stability keeps decreasing against toxin accumulating in response to different IR perturbations, and the more strength of IR is applied, the more damage for genome stability occurs. These results suggest that, a normal cell can deal with genome stress efficiently by means of a series cellular self-defensive activities, whereas, cellular self-repair capability, and genome stability might be damaged, even lethal cancerous or apoptosis occur as the strength of external perturbation.
overwhelms the threshold that a normal cell can burden maximally.

![Graph showing genome stability in response to toxin accumulation under different IR perturbations.](image)

**Fig.4.** Analysis for genome stability in response to toxin accumulation under different IR perturbations. (a1-a4) The kinetics of mDSBC generation; (b1-b4) intact DSBs accumulation; as a result, (c1-c4) genome stability decreasing, under 1, 5, 10, and 15Gy IR, respectively.

4. CONCLUSION

Based on GEN and KTAP framework, an integrated model of CRGS is proposed to represent a series of cellular activities in response to genome stress under external IR perturbation. In this work, CRGS model is divided into three subsystems. Each subsystem is composed of different molecular particles with different discrete states. The systematic kinetics of cellular response activities are dealt as results of the complicated mutual interactions among different particles from different subsystems. The simulations show that our method can successfully illustrate the dynamics of CRGS, including stochastic DSB generation and repair, switch-like ATM activation, and oscillations of P53-MDM2 feedback loop, and analyze cellular self-defense capability, as well as genome stability in response to genome stress under different IR perturbation circumstances. The proposed method is flexible and efficient in capturing stochastic interactions in cellular activities at molecular level, as well as dynamic evolution of CRGS at systematic level; especially, it is very helpful to deeply investigate and analyze the complicated cellular self-defending mechanisms in response to acute perturbation cofactors from outer environment.

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REFERENCES


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Appendix A.

The other related equations used in the integrated CRGS model are as follows:

\[ \eta_{\text{repair-G-DSB}} = \frac{f_{\text{DSB}}}{f_{\text{DSB}} + f_{\text{quench}}} \cdot B_{\text{repair-G-DSB}} = \frac{k_{\text{repair}} \cdot f_{\text{quench}} \cdot f_{\text{P53}}}{f_{\text{DSB}} + f_{\text{quench}}} \],

\[ \eta_{\text{repair-R-DSB}} = \frac{f_{\text{R}}}{f_{\text{R}} + f_{\text{quench}}} \cdot B_{\text{repair-R-DSB}} = \frac{k_{\text{repair}} \cdot f_{\text{quench}} \cdot f_{\text{P53}}}{f_{\text{DSB}} + f_{\text{quench}}} \],

\[ \eta_{\text{DSB-RP}} = \frac{f_{\text{DSB}}}{f_{\text{DSB}} + f_{\text{RP}}} \cdot B_{\text{DSB-RP}} = \frac{k_{\text{DSB}} \cdot f_{\text{RP}} \cdot f_{\text{P53}}}{f_{\text{DSB}} + f_{\text{RP}}} \],

\[ \eta_{\text{ATMm-ATMn}} = \frac{f_{\text{ATMn}}}{f_{\text{ATMn}} + f_{\text{ATMn}}} \cdot B_{\text{ATMm-ATMn}} = \frac{k_{\text{ATM}} \cdot f_{\text{ATMn}}}{f_{\text{ATMn}} + f_{\text{ATMn}}} \],

\[ \eta_{\text{P5S-ATMn}} = \frac{f_{\text{P5S}}}{f_{\text{P5S}} + f_{\text{ATMn}}} \cdot B_{\text{P5S-ATMn}} = \frac{k_{\text{P5S}}}{f_{\text{P5S}} + f_{\text{ATMn}}} \],

\[ \eta_{\text{P5S-P5S}} = \frac{f_{\text{P5S}}}{f_{\text{P5S}} + f_{\text{P5S}}} \cdot B_{\text{P5S-P5S}} = \frac{k_{\text{P5S}}}{f_{\text{P5S}} + f_{\text{P5S}}} \],

\[ \eta_{\text{P5S-P5S-MDM2}} = \frac{f_{\text{P5S-P5S}}}{f_{\text{P5S-P5S}} + f_{\text{MDM2}}} \],

\[ B_{\text{P5S-P5S-MDM2}} = \frac{f_{\text{P5S-P5S}}}{f_{\text{P5S-P5S}} + f_{\text{MDM2}}} \].

Appendix B.

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