A Dynamic Market Mechanism for Markets with Shiftable Demand Response

Jacob Hansen ∗ Jesper Knudsen ∗ Arman Kiani ** Anuradha Annaswamy *** Jakob Stoustrup ∗

Abstract: In this paper, we propose a dynamic market mechanism that converges to the desired market equilibrium. Both locational marginal prices and the schedules for generation and consumption are determined through a negotiation process between the key market players. In addition to incorporating renewables, this mechanism accommodates both consumers with a shiftable Demand Response and an adjustable Demand Response. The overall market mechanism is evaluated in a Day Ahead Market and is shown in a numerical example to result in a reduction of the cost of electricity for the consumer, as well as an increase in the Social Welfare.

Keywords: Smart grids, Power systems stability, Demand Response, Shiftable demand, Electricity market, Day Ahead Market

1. INTRODUCTION

Two dominant features of a smart grid are a high penetration of renewable energy resources (RERs) and Demand Response (DR), the flexibility to adjust power consumption (see Annaswamy et al. [2013]). The introduction of both renewable energy sources as well as efforts to integrate DR-compatible consumption brings in a set of dynamic interactions between the major components of a smart grid. Electricity markets, the entity that carries out power balance by scheduling power using bids from various generating companies, are crucial components that can facilitate such dynamic interactions. This paper proposes a dynamic market mechanism in a smart grid that includes the behavior of key market players such as generators, consumers, and Independent System Operator (ISO), together with DR-compatible participants.

Demand Response consists of systems, services, and strategies that enable demand resources to adjust their consumptions in response to economic signals from a competitive wholesale market. Given these varied flexibilities, DR can be grouped into two major categories, DRs, which corresponds to DRs that adjust their consumption (see for example, Sioshansi, Short [2009], Staff [2008]) and DRs that shift their consumption (see for example, Roscoe, Ault [2010]), both in response to wholesale energy prices. Both types of consumption can provide relief from capacity constraints and promote more economically efficient uses of electrical energy. In Kiani, Annaswamy [2011], a market model was proposed that only included DRs. In this paper, an extension is proposed to include the second category DRs, where the demand is shiftable.

Various methods have been proposed in the literature to determine market models, based on market equilibrium, Equilibrium Programming with Equilibrium Constraints (EPECs), Variational and Complementarity Problem (CP) formulation, and game theory (see for example, Yao et al. [2008], Ruiz, Conejo [2009], Hu, Ralph [2007], Morales et al. [2009], Klemperer, Meyer [1989], Visuddhiphan, Ilic [1999], Alvarado et al. [2000], Cunningham et al. [2002]). In contrast to the above, a dynamic approach has been proposed in Kiani, Annaswamy [2011], Kiani, Annaswamy [2012] where iterative negotiations are proposed between the key market players so as to arrive at the desired market equilibrium. Similar to, Arrow et al. [1958], the idea is to use gradient approach to determine these iterative strategies. And similar to Alvarado et al. [2000], price is used as a feedback control input for power balance.

The approach in, Kiani, Annaswamy [2011], Kiani, Annaswamy [2012] included Demand Response that was assumed to be directly responsive to prices; that a proportional increase or decrease was assumed to occur for a decrease or increase in the price. This class is expanded much further in this paper to include DR that is shiftable over a 24-hour interval rather than adjustable at each instant. This introduces a significant challenge of integral constraints. A two-step approach is proposed to address this challenge; in the first, a desired demand profile is generated that accommodates the integral constraint over

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a period of interest and in the second, a dynamic market mechanism is proposed, similar to Kiani, Annaswamy [2011], which suitably allows the flexible demand to converge to the desired demand profile. Using the resulting model, conditions of stable convergence are defined.

This paper has been organized as follows: In Section 2 the desirable demand response profile for DRs is presented. In Section 3, a wholesale energy market structure is introduced, and the underlying dynamic model is presented including both DRu and DRs. In Section 3.6 stability properties are derived and the region of attraction is established. In Section 4 numerical studies of an IEEE 4-bus are reported and finally in Section 5, we provide summary and concluding remarks.

2. A DESIRABLE DEMAND PROFILE

The main goal of electricity markets is to ensure efficient power balance. A dominant factor that may impede this is the variation in demand. Given the paradigm shift that is occurring at present with Demand Response, and the possibility of a shiftable demand (see Roscoe, Ault [2010]), we discuss the optimal demand profile in this section, and introduce constraints to represent the shiftable component.

Denoting the shiftable and non-shiftable demand components as $P_{Ds}$ and $P_{Df}$, respectively, we represent the following constraints:

$$\sum_{t \in T} P_{Ds_t} = P_{D_{tot}}$$

where $P_{Ds_t}$ is the shiftable demand at time $t$, $T$ denotes the time period of interest, $P_{D_{tot}}$ is the total shiftable demand over $T$, and $P_{D_{max}}$ is the maximum allowable shiftable demand at time $t$.

One can then generate the desired demand profile by constructing an objective function

$$\sum_{t \in T} (P_{Ds_t} + P_{Df_t})^2$$

where $P_{Df_t}$ is the fixed demand component at time $t$, and then pose the overall problem as the minimization of (2) subject to constraints in (1). The resulting solution, denoted as $D_{ref}$, can be seen to be one where the demand is more uniform over $T$.

Fig. 1 illustrates a desired demand profile using typical consumption data over a 24-hour period where it is assumed that 10% of the power consumption is shiftable at each hour $t$. That is, $P_{Ds_t} = 0.1(P_{D_{tot}} + P_{Df_t})$ for $t \in [1, 24]$. With the constraints presented in (1) chosen as $P_{D_{max}} = 117.1$ MWh and $P_{D_{ref}} = 60$ MWh, the solutions $P_{D_{ref}}$ (blue) and $P_{D_{max}}$ (black) at each $t \in (2)$ subject to (1) are determined. For $t \in [9, 23]$ peak reductions (gray) are achieved.

1 From ISO New England at http://www.iso-ne.com/markets/ with a scaling factor of 0.01.
2 Such a percentage is within projected DR penetration found in studies of demand (Milligan, Kirby [2010]).

Fig. 1. 90% of hourly demand distribution (black), remaining 10% optimally distributed as shiftable consumption (blue) and peak reduction (gray) from the original distribution in NEMASSBOST on 07/19/13.

3. WHOLESALE ENERGY MARKET STRUCTURE

The electricity market that we consider in this paper is wholesale and is assumed to function as follows: First, each consumer (ConCo) submits the bidding stacks of each of its demands to the pool. Similarly, each generating company (GenCo) submits the bidding stacks to the pool. Then, the ISO clears the market using an appropriate market-clearing procedure resulting in prices and consumption and production schedules. In what follows, we model each of the components (ConCo, GenCo and ISO) together with their constraints and the overall optimization goal.

In the modeling, the following nomenclature will be used. $N$, $N_1$, $N_D$, $N_{Gc}$ and $N_{Gw}$ denote the total number of buses, transmission lines, consuming units, dispatchable generating units and non-dispatchable generating units, respectively. Further, $\theta_{n}$, $\varphi_{n}$ and $\phi_{n}$ denote the set of indices of dispatchable generating units, non-dispatchable generating units and consuming units at node $n$, respectively. $\Omega_n$ denote the set of indices of nodes connected to node $n$ and finally, $D_g$, $G_c$ and $G_w$ denote the set of indices of consuming units, dispatchable generating units and non-dispatchable generating units, respectively.

3.1 Consumer Modeling

For consumer company $j \in D_g = \{1, 2, \ldots, N_D\}$ the consumption is divided into three classes: fixed, adjustable and shiftable, denoted $P_{D_{fj}}, P_{D_{aj}}$, and $P_{D_{sj}}$, respectively. Each consumer company is assumed to consist of one unit of each class of consumption and the total consumption $P_{D_j} = P_{D_{fj}} + P_{D_{aj}} + P_{D_{sj}}$ where $P_{D_{fj}} = P_{D_{aj}} = P_{D_{sj}}$. The use of each class of electricity for the consumer is represented by the associated utility functions in which the marginal utilities are decreasing linear functions of power consumption. The utility functions are

$$U_{D_{aj}}(P_{D_{aj}}) = b_{D_{aj}}P_{D_{aj}} + c_{D_{aj}}P_{D_{aj}}^2$$

where $b_{D_{aj}}, b_{D_{sj}}, c_{D_{aj}}$, and $c_{D_{sj}}$ are consumer utility coefficients. The utility of the total consumption $U_{D_j}(P_{D_j}) = U_{D_{aj}}(P_{D_{aj}}) + U_{D_{aj}}(P_{D_{aj}}) + U_{D_{sj}}(P_{D_{aj}})$.

The consumption values are constrained; $P_{D_{aj}}$ and $P_{D_{sj}}$ must evolve such that in equilibrium, they reach a value no smaller than the derived reference and reach the derived reference, respectively, where both references are determined as in Section 2.
3.2 Generator Modeling

The generating companies are separated into conventional dispatchable units e.g. coal plants, and non-dispatchable renewable energy resource units e.g. wind energy, and modeled separately. It is assumed that all GenCos bid their marginal cost, rather than performing strategic biddings. The non-dispatchable generator companies are furthermore assumed to submit their bids to the market like conventional GenCos.

**Conventional Dispatchable GenCos** Each dispatchable generator company $i \in G_{c} = \{1, 2, \ldots, N_{Gc}\}$ is assumed to consist of only one generating unit and the production of each generator company is denoted $P_{Gc_i}$. The costs of operation are assumed quadratic functions of generated power, implying a linear marginal cost i.e. cost of changing production one unit. The operating cost of dispatchable GenCo $i$ is given by

$$C_{Gc_i}(P_{Gc_i}) = b_{Gc_i}P_{Gc_i} + \frac{c_{Gc_i}}{2}P_{Gc_i}^2,$$

where $b_{Gc_i}$ and $c_{Gc_i}$ are generator cost coefficients and $P_{Gc_i}^\text{min}$ and $P_{Gc_i}^\text{max}$ are lower and upper bounds on GenCo $i$, respectively. Rate constraints, startup and shutdown costs on the dispatchable generation are not included in this model.

**Non-Dispatchable RER GenCos** Each non-dispatchable generator company $l \in G_{w} = \{1, 2, \ldots, N_{Gw}\}$ is assumed to consist of only one generating unit and the production of each generator company is denoted $P_{Gw_l}$. The costs of operating are given by

$$C_{Gw_l}(P_{Gw_l}) = \frac{c_{Gw_l}}{2}P_{Gw_l}^2,$$

where $b_{Gw_l}$ and $c_{Gw_l}$ are generator cost coefficients, $b_{Gw_l}$ and $c_{Gw_l}$ are reserve cost coefficients. $C_{Gw_l}(P_{Gw_l})$ denotes the traditional generation cost, which is very close to zero. $C_{Gw_l}^r(\Delta_{w_l})$ denotes the cost of committing specific generators as reserves due to the wind uncertainty and $\Delta_{w_l}$ is given by

$$\Delta_{w_l} = P_{Gw_l} - \Delta_{Gw_l},$$

where $0 < \Delta_{Gw_l} < 1$ corresponds to overestimated wind energy which implies the assumption that the power can be purchased elsewhere or demand can be adjusted. Underestimation is present when $-1 < \Delta_{Gw_l} < 0$ which implies a surplus in power that is assumed to be handled alternatively.

The non-dispatchable generation is constrained by

$$0 \leq P_{Gw_l} \leq P_{Gw_l}^\text{max},$$

where $P_{Gw_l}^\text{max}$ is the maximum achievable generation of non-dispatchable unit $l$, limited by external factors e.g. wind speeds. Rate constraints, startup and shutdown costs on the non-dispatchable generation are not included and the reserve market is assumed cleared independently of the energy market.

3.3 ISO Market-Clearing Model

The ISO clears the electricity market optimizing a cost function subject to network constraints. Generally, the most significant constraints in the network are due to network losses and line capacity limits. Technical limitations are constraining the power flow through lines, which is said to be congested when approaching these upper limits. Congestion is explicitly included in the model, whereas ohmic losses are not modeled.

The market-clearing procedure acts on behalf of ConCos and GenCos by maximizing the utility of consumption for the ConCos and minimizing the cost of generation for the GenCos. The cost function to optimize is often termed Social Welfare and is defined as

$$S_W = \sum_{j \in D} U_j(P_{Dj}) - \sum_{i \in G_{c}} C_{Gc_i}(P_{Gc_i})$$

Expressed as an optimization problem, the market-clearing procedure is given by

$$\text{Maximize } S_W = \text{Minimize } -S_W$$

subject to

$$-\sum_{j \in D} P_{Gc_i} - \sum_{l \in G_{w}} (P_{Gw_l} + \Delta_{w_l}) + \sum_{j \in D} P_{Dj}$$

$$+ \sum_{m \in G_{w}} B_{nm} [\delta_{n} - \delta_{m}] = 0; \rho_{n}^l \forall n \in N \text{ (12a)}$$

$$B_{nm} [\delta_{n} - \delta_{m}] \leq p_{nm}^\max \gamma_{nm} \forall n, \forall m \in \Omega \text{ (12b)}$$

$$P_{Daj} - P_{Dnj}^\text{ref} = 0; \lambda_{j} \forall j \in D_{g} \text{ (12c)}$$

$$P_{Dfj}^\text{ref} \leq P_{Dfj}^l \forall j \in D_{g} \text{ (12d)}$$

$$P_{Gw_l} \leq P_{Gw_l}^\text{max}; \zeta_{l} \forall l \in G_{w} \text{ (12e)}$$

where $\delta_{n}$ is the voltage angle of node $n$, $B_{nm}$ is the susceptance of line $n$ to $m$, $P_{nm}^\max$ is the power capacity limit of line $n$ to $m$ and $P_{Dnj}^\text{ref}$ and $P_{Dnj}^\text{ref}$ are the references for shiftable and fixed demand, respectively, defined in Section 2. Power balance and capacity limits are respected through constraint (12a) and (12b), respectively. Constraint (12c) and (12d) dictates the desired profile for the shiftable and fixed demand, respectively. Constraint (12e) corresponds to the limit on RERs. The associated Lagrange multipliers, $\rho_{n}^l$, $\gamma_{nm}$, $\lambda_{j}$, $\zeta_{l}$ and $\xi_{l}$ are indicated for each constraint.

3.4 Game-Theoretical Dynamic Model of Wholesale Market

As argued in Kiani, Annaswamy [2011], an optimization problem as in (11)-(12) behaves as a game between the ConCos, GenCos and the ISO where each player seek maximization of their own benefit. The solution to this state-based game can be arrived at using gradient play, which leads to differential equations describing the path of $(P_{Gc_i}, P_{Gw_l}, P_{Daj}^l, P_{Daj}^r, P_{Dnj}^l, \delta_{n}^l, \rho_{n}^l, \lambda_{j}, \gamma_{nm}, \zeta_{l}, \xi_{l})$ from a perturbed state to the equilibrium $(P_{Gc_i}^*, P_{Gw_l}^*, P_{Daj}^l, P_{Daj}^r, \delta_{n}^l, \rho_{n}^l, \lambda_{j}, \gamma_{nm}, \zeta_{l}, \xi_{l})$ and is given by

$$\tau_{Gc_i} P_{Gc_i} = \rho_{n}^l - c_{Gc_i} P_{Gc_i} - b_{Gc_i}$$

$$\tau_{Gw_l} P_{Gw_l} = \rho_{n}^l - (c_{Gw_l} + b_{Gw_l} A_{Gw_l})P_{Gw_l} - (b_{Gw_l} + b_{w} A_{Gw_l} - \xi_{l})$$

$$\tau_{Daj} P_{Daj} = c_{Daj} P_{Daj} + b_{Daj} - \rho_{n}^l + \zeta_{l}$$
\[ \tau_{Dsj} \dot{P}_{Dsj} = c_{Dsj} P_{Dsj} + b_{Dsj} - \rho^\prime_{n(j)} - \lambda_j \]  
\[ \tau_{Ds} \dot{\delta}_n = - \sum_{m \in \Omega_n} B_{nm} [\delta_n - \delta_m] \]  
\[ \tau_{p_n} \dot{\rho}_n = - \sum_{i \in \delta_n} P_{Gci} - \sum_{l \in \delta_n} P_{GWL}(1 + \Delta_{GWL}) \]  
\[ + \sum_{j \in \Omega_n} P_{Dj} + \sum_{m \in \Omega_n} B_{nm} [\delta_n - \delta_m] \]  
\[ \tau_n \dot{\delta}_n = - \sum_{m \in \Omega_n} B_{nm} [\rho_n - \rho_m + \gamma_{nm} - \gamma_{mn}] \]  
\[ \tau_{\rho_n} \dot{\rho}_n = \rho_n \mu_t \]  
where \( \rho_n \) is the true LMP and \( \mu_t \) is a scalar that will be suitably defined later. Additionally, with \( d_{y} = d_t - \epsilon \),

\[ \text{Proj}_y(f(x), d_y, \epsilon) = \begin{cases} \
\frac{d_y - y^2}{d_y - (d_y)^2} f(x) & \text{if } d_y \leq y \leq d_y, \\
\frac{y^2}{\epsilon^2} f(x) & \text{if } 0 \leq y \leq \epsilon, \\
(\text{otherwise}) & \end{cases} \]  
ensures non-negativity of \( \gamma_{nm}, \xi_j, \) and \( \xi_t \) as required by the KKT conditions and bounds them by \( d_{\gamma_{nm}} \), \( d_{\xi_j} \), and \( d_{\xi_t} \), respectively, as stated in Lemma 1.

**Proof.** Let \( V(y) = \frac{1}{2} y^2 \) such that \( \dot{V}(y) = g(x) \). It is easily seen that if \( f(x) \leq 0 \) then \( \dot{V} \leq 0 \) and if \( f(x) > 0 \) then (15) ensures that \( \dot{V} \) will gradually decrease towards zeros when \( y \) approaches \( d_y \).

One can view the dynamics in (13) as a market mechanism with the evolution of the differential equations representing a continuous negotiation process between market players. The time it takes to complete this negotiation is shown in Fig. 2 with an overall market mechanism time scale where \( T_{neg} \) denotes the interval over which negotiations take place and \( T_{dpc} \) denotes the period over which the desired demand profile is calculated. The premise of our dynamic market mechanism is that once the desired demand profile is calculated within the time frame \( T_{dpc} \), continued negotiations take place between the market players over a period \( T_{neg} \), with these periods chosen such that \( T_{dpc} + T_{neg} < T \), the period of interest. For example, \( T = 24 \) hours in a Day Ahead Market (DAM) and \( T = 5 \) minutes in a Real-Time Market (RTM). This implies that the time-scales of the negotiations are faster than other events and hence we represent the underlying interactions using a differential equation as in (13).

The remaining problem that should be addressed is if these negotiations are well-behaved, i.e. if the underlying dynamic model is stable and all solutions converge to the desired equilibrium. This is analyzed in the next section.

### 3.5 Equilibrium of Wholesale Market

Using the market mechanism proposed in (13), a dynamic model can be written compactly as

\[ \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix}^T \]  
where

\[ x_1(t) = [P_{Gci}, P_{GWL}, P_{Dsj}, \delta_n, \rho_n, \lambda_j]^T \]  
\[ x_2(t) = [\gamma_1, \cdots, \gamma_n, \xi_j, \xi_t]^T \]  
\[ A_{11} = \begin{bmatrix} -\tau_{\rho_g}^{-1} A_{gc} - \tau_{\rho_g}^{-1} A_{gw}(I + \Delta_{gw}) - \tau_{\rho_g}^{-1} A_d & \tau_{\rho_g}^{-1} A_d \\ 0 & 0 \end{bmatrix} \]  
\[ A_{12} = \begin{bmatrix} 0 & -\tau_{\rho_g}^{-1} A_d \\ 0 & -\tau_{\rho_g}^{-1} A_d \end{bmatrix} \]  
\[ b_{11} = \begin{bmatrix} -b_{\rho_g} T_{\rho_g}^{-1} (b_{gw} + b_{u} \Delta_{gw}) T_{\rho_g}^{-1} b_{\rho_g} T_{\rho_g} \end{bmatrix} \]  
\[ b_{12} = \begin{bmatrix} 0 & -P_{GWL}^{-1} \end{bmatrix} \]  
A denotes the \( N_t \times N \) bus incidence matrix and \( A_r \) is the \( N_t \times N - 1 \) reduced bus incidence matrix with the column corresponding to the reference bus removed. \( A_d \)
denotes the $N \times N_D$ consumers incidence matrix, where $A_{ij} = 1$ if the $j$th consumer is connected to the $i$th bus and $A_{ij} = 0$ if the $j$th consumer is not connected to the $i$th bus. Similarly, $A_{g}$ denotes the $N \times N_G$ dispatchable generator incidence matrix and $A_{ng}$ denotes the $N \times N_G$ non-dispatchable generator incidence matrix. $B_{line}$ denotes the $N_L \times N_I$ diagonal line admittance matrix. $P_{max}$ is the vector of maximum capacity limits $P_{max} = \text{Ref}_{P}$ denotes the vector of fixed demand references $P_{ref}$. $P_{max}$ is the vector of maximum available non-dispatchable power $P_{G}$.

Furthermore, it is apparent that in hours with high LMP no adjustable power demand arises. Costs are computed as shown in Fig. 1. Table 1 summarizes the results for the 24-hour period.

<table>
<thead>
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Table 1. Results for 24-hour period.

Because we have that $V \leq -||y_1||^2 + [||y_1||]^2$, and by using (26), we can get

\[ V \leq -||y_1||^2 ||y_1|| - \alpha. \]

(31) Boundness of $y_1$ and the convergence of all trajectories to $\Omega_{min}$ follow from (31).

4. ILLUSTRATIVE EXAMPLE

We illustrate the dynamic market mechanism with a shiftable demand response proposed in Sections 2-3 using a 4-bus network (see Kiani, Annaswamy [2010]). The network consists of three generating units located at Bus 1 and 2. $G_1$ is a base-load dispatchable generator with low cost coefficients and slow dynamic response. $G_2$ is a peak-load dispatchable generator acting like a spinning reserve with high cost coefficients and fast dynamic response. $G_3$ is a non-dispatchable generator with very low cost coefficients and fast dynamic response. Power consumption is at Bus 3 and 4. All cost coefficients and time constants for both generating and consuming units and transmission line data, namely, susceptance $B_{nm}$ and capacity limit $P_{max}$ are given in Section 4.1.

In the following, two cases are considered, named Case 1 and Case 2. Case 1 is the base case which incorporates zero DR, i.e. includes only fixed and adjustable demand, $P_{DF} = P_{DA}$. Case 2 incorporates fixed demand, DR$_a$ and DR$_s$. $P_{DF}$, $P_{DA}$, and $P_{DS}$. As in Section 2, the amount of shiftable demand is set to 10% of the total demand. It is assumed that $P_{DS}$, and $P_{DF}$ are computed as shown in Fig. 1. With this profile, and the cost and utility coefficients as in Section 4.1 the dynamic market model in (13) was simulated. A $\mu_t = [1 1.02 1.03 1.03 1.04 1.04 1.04 0.99 0.89 0.89 0.90 0.97 1]^T$ was chosen as this resulted in the corresponding LMP profile to match a typical LMP profile as in ISO New England (see http://www.iso-ne.com/markets/). The resulting Social Welfare and total consumer costs are shown in Table 1.

Table 1 demonstrates an increase in Social Welfare and a decrease in cost per unit of consumption and hence a positive effect of introducing DR. The distribution of the demand profile between fixed, adjustable, and shiftable demand is shown in Fig. 3 for each hour of a 24-hour period. This figure also show the LMPs at the consumer nodes. From Fig. 3 we see that $P_{DA}$ adjusts according to the utility function and as a result, in hours with a large quantity of low-cost RERs, i.e. low LMP, the amount of adjustable power demand rises. Furthermore, it is apparent that in hours with high LMP no adjustable power demand rises. Costs are computed as shown in Fig. 1. Table 1 summarizes the results for the 24-hour period.

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Hurwitz, we obtain that

\[ V \leq -||y_1||^2 + 2||y_1|| \cdot \beta_1 d_t + \beta_2 d_c + \beta_3 d_s, \]

and by using (26), we can get

\[ V \leq -||y_1||^2 \cdot (||y_1|| - \alpha). \]

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power is consumed. A comparison between Case 1 and Case 2 shows a more even distribution of demand which is preferable since this aids in lowering the prices for consumers as a direct result of the ability to shift power from high demand hours to low demand hours.

4.1 Coefficients

The generator cost and demand utility coefficients are 

\[ b_{Ge} = [47.2, 48.2] \text{$/MWh}, \quad c_{Ge} = [0.25, 0.53] \text{$/MW}^2 h, \]

\[ b_{Gw} = 1 \text{$/MW}, \quad b_{w1} = 50 \text{$/MW}, \quad c_{Gw1} = 0.02 \text{$/MW}^2 h, \]

\[ c_{w1} = 0.7 \text{$/MW}^2 h, \quad \Delta_{Gw1} = 0 \text{$/MW}^2 h, \quad b_{Dw} = [67, 67] \text{$/MW}, \]

\[ b_{Ds} = [60, 60] \text{$/MW}, \quad c_{Da} = [-0.21, -0.21] \text{$/MW}^2 h, \]

\[ c_{Ds} = [-0.41, -0.41] \text{$/MW}^2 h, \quad \tau_{Ge} = [2.8, 0.7] \text{$/MW}, \]

\[ \tau_{Gw1} = 0.7 \text{$/MW} \text{ and } \tau_{Da} = \tau_{Ds} = [0.8, 0.8] \text{$/MW}. \]

The transmission line coefficients are \( P_{13}^{\text{max}} = P_{24}^{\text{max}} = 50.5 \text{ MW}, \quad P_{14}^{\text{max}} = P_{23}^{\text{max}} = 50 \text{ MW}, \quad B_{13} = B_{24} = 0.0372 \text{ p.u. and } B_{14} = 0.0504 \text{ p.u. , } B_{23} = 0.0336 \text{ p.u.} \)

5. SUMMARY

Increasing demand for electrical power generation and the current energy crisis have created an urgent need in incorporating RERs into the power grid. In this paper, we begin with the model of the players including dispatchable and RER GenCos, ConCo, and ISO, together with their constraints and the optimization goal and then capture the dynamics of the RTM using an interactive framework which introduces a state-space structure to the static market. A gradient play is used to derive the dynamic evolution of the actions for players and underlying states of the game as dual variables to reach the optimum solution of the RTM. In addition to the dynamic model this paper includes the incorporation of shiftable demand response evaluated over 24 hours. The stability of the resulting dynamical model of the RTM is investigated and the region of attraction around the equilibrium is established.

Numerical results are included that validate the theoretical results using a IEEE 4-bus system. The simulation results show that for a 10% shiftable demand, an 4% cost reduction to the consumer and a 1% increase in the Social Welfare is possible.

REFERENCES


