Declutching Control of Wave Energy Converters using Derivative-Free Optimization

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Abstract:
We propose a novel formulation for declutching control of wave energy converters with the power take-off time as the only decision variable. The optimal control problem is modeled as a single-variable optimization problem, thereby making real-time implementation a possibility. We present a derivative-free optimization algorithm that exploits the quantization of the decision variable in order to reduce the number of function evaluations needed to compute a solution. We propose two receding horizon closed-loop strategies: the first one uses past wave information and can increase the energy generation by 42% compared to the uncontrolled case, while the second formulation uses predictions of future waves and results in a further 40% increase in energy generation. For irregular waves with peak periods longer than 6 s, one can generate at least four times more energy when co-designing the physical system and controller, compared to a controlled system that was optimized without a controller in the feedback loop.

1. INTRODUCTION

The energy absorption of wave energy converters (WECs) can be significantly improved by various control strategies, as suggested by Falnes [2007], Budar [1975] and Abraham and Kerrigan [2013a]. This paper will study a control implementation for WECs known as declutching control. Declutching control can increase the power take-off (PTO) of WECs in a wide frequency range, particularly for devices whose resonant period is longer than the incident wave’s peak period. Declutching control works by bypassing the PTO mechanism for certain time intervals, thereby effectively altering the resonant frequency of the device to match the frequency of the incident wave. Detailed physical implementations can be found in Babarit et al. [2009] and Clément and Babarit [2012]. Declutching control is passive, easy to implement and more importantly, does not have violent cut-off actions as with latching control. Hence, declutching control is of great interest to many practitioners [Falnes, 2001, Salter, 1979]. Furthermore, designing an algorithm for controlling a two-mode system provides control theoreticians with an interesting example problem.

In this paper, we will modify the latching control algorithm in Feng and Kerrigan [2013], which is based on derivative-free optimization (DFO), to declutching control. Unlike latching control, for declutching control the cost function is continuous in most cases. However, the analytical gradient is very hard to obtain and a finite difference method will struggle due to the fact that the decision variable is quantized (this will be discussed in Section 2.2). Thus, reliable performance of derivative-based optimization methods is not guaranteed. Furthermore, since we have a univariate cost function, the computational cost of derivative-free optimization methods is low. Two formulations will be studied: a past data formulation, which uses recorded past wave data, and a future data formulation, which requires the prediction of future waves.

The paper is organized as follows: in Section 2 a declutching controlled WEC model will be described and we will demonstrate some properties of the cost function. In Section 3 we will briefly introduce the coordinate-search optimization method. We will discuss two closed-loop problem formulations in Section 4. Section 5 will present some simulation results, where we will demonstrate the benefit of cyber-physical co-design of the WEC, as well as compare our optimization method against other algorithms for univariate optimization. Finally, we will end with conclusions and suggestions for future work.

2. MODEL AND PROBLEM FORMULATION

2.1 Model of Wave Energy Converter

The model of a WEC used here is based on linear wave theory, due to its simplicity and sufficient accuracy for control applications. In the time domain, the model is given by:

\[
(M + \mu_m)\ddot{\zeta}(t) + \int_0^t K(t - \tau)\dot{\zeta}(\tau)d\tau + B_{pto}\dot{\zeta}(t) + (k_s + \rho g S)\dot{\zeta}(t) = F_e(t),
\]

where \(\zeta\) is the displacement of the WEC buoy from the equilibrium, \(M\) is the mass of the buoy, \(\mu_m\) is the infinite frequency added mass, \(K(\cdot)\) is the impulse response kernel, \(B_{pto}\) and \(k_s\) are the PTO damping coefficient and the spring constant of the PTO mechanism, respectively, \(\rho\) is the water density, \(g\) is the gravity constant, \(S\) is the submerged cross-sectional area of the buoy and \(F_e(\cdot)\) is the excitation force caused by incident waves. The model (1) is widely used in WEC control studies, for example in Babarit and Clément [2006] and Clément and Babarit [2012].
The system (1) is a differential-integral equation. The integral term causes difficulty in analysis and can be simplified, as suggested by Yu [1995], Loan [1978] and Kristiansen [2005]. The integral $B_{rad}(t) = \int_{t_0}^{t} K(t-\tau) \zeta(\tau) d\tau$ can be approximated as the output of a state-space system:

\[
\dot{q}(t) = A_r q(t) + B_r \zeta(t),
\]
\[
B_{rad}(t) = C_r q(t) + D_r \zeta(t),
\]

with an approximation order of $n_d$, where $q(t) \in \mathbb{R}^n$ is the approximation state, and $A_r, B_r, C_r, D_r$ are the coefficient matrices of the approximating system. These coefficient matrices are calculated via the impulse to state-space technique proposed in Kristiansen [2005].

According to Babarit et al. [2009], if we activate the PTO mechanism when $v(t) \neq 0$, the system may undergo high stresses. On the other hand, in simulation results, the optimal time instance for activating the PTO mechanism is mostly at $v(t) = 0$ [Abraham and Kerrigan, 2013a, Babarit et al., 2009]. It is clear that $v(t) = 0$ is often the best practical switching point for the PTO mechanism. Therefore, we propose a sub-optimal declutching control scheme that is similar to latching control: we activate the PTO mechanism whenever $v(t) = 0$ and declutch the PTO mechanism after $t_d$ seconds.

The dynamics of such a declutching-controlled system can be described as a hybrid system in terms of a timed automaton [Lee and Seshia, 2011, Chap. 4]. We assume that when the system is declutchled, the damping $B_{pto}$ becomes zero, i.e. no energy is generated in the declutched mode. Figure 1 shows the timed automaton representation of the declutching-controlled system, where $v(t) := \zeta(t)$ is the velocity of the buoy. First, define a clock with dynamics $\Xi(t) = 1$ and let $z(t) := [\zeta(t) v(t) q^T(t)]^T$ be the state of the system. Let the dynamics of the two different modes be:

\[
g_1(z, F_c; t) := \begin{bmatrix}
\frac{v(t)}{M_T} \left[-k_T \zeta(t) - D_r v(t) - C_r q(t) + F_r \right] \\
A_r q(t) + B_r v(t)
\end{bmatrix},
\]
\[
g_2(z, F_c; t) := \begin{bmatrix}
\frac{v(t)}{M_T} \left[-k_T \zeta(t) + B_{pto} + D_r v(t) \right] \\
A_r q(t) + B_r v(t)
\end{bmatrix},
\]

where $M_T := M + \mu_a$ and $k_T := (k + \rho gS)$. $g_1$ describes the dynamics in the declutched mode. The resonant period in this mode is slightly shorter than the PTO-active mode. $g_2$ describes the PTO-active mode, i.e. when the system’s PTO damping $B_{pto}$ is engaged and the power is generated. Whenever the velocity $v(t) = 0$, the clock is reset $\Xi(t) = 0$, the system transits from the declutched mode to the PTO-active mode and stays in this mode for $t_d$ seconds. The time during which the system is in the PTO-active mode is termed the PTO-active time.

We assume full state feedback. However, a state estimator can be constructed by modelling the WEC as a linear time-varying system, where $B_{pto}$ is a known function of time. Alternatively, one can model the system as a linear differential inclusion and follow the same procedure outlined in Abraham and Kerrigan [2013b].

![Fig. 1. Declutching-controlled WEC system.](image1)

![Fig. 2. Average power generated against $t_d$ under an irregular wave with park period 7 s for a 2000 s simulation.](image2)

2.2 Cost Function and the Optimization Problem

With a given WEC system, one aims to maximize the energy absorption over a time horizon, say from $t = a$ to $t = b$. The power extracted by the system depends on the damping coefficient of the PTO mechanism and the velocity of the buoy, i.e. $\hat{B}_{pto}(t; t_d) = \hat{B}_{pto}$, where $\hat{B}_{pto}(t) = B_{pto}$ if the system is in the $g_2$ mode and $\hat{B}_{pto}(t) = 0$ if the system is in the $g_1$ mode.

In declutching control, the optimal control problem over a horizon starting at $t = a$ and ending at $t = b$ can be defined as:

\[
\max J(z_a, t_a, F_c, a, b) := \int_{t_d}^{b} \hat{B}_{pto}(t) v^2(t; z_a, t_a, F_c) dt, \quad (4)
\]

where $J: \mathbb{R}^d \times \mathbb{R}_+ \times \mathbb{L}_c^p \times \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ depends on the initial state $z_a := z(a)$, control variable $t_a$ and $F_c_{[a,b]} : [a,b] \rightarrow \mathbb{R}$, the excitation force function over the integration interval.

Figure 2 shows the energy absorption against $t_d$ when the WEC described in Section 5 is subjected to a Joint North Sea Wave Project (JONSWAP) spectrum-generated excitation force with a peak period of 7 s. Such an excitation force is often used to simulate irregular wave conditions [Babarit and Clément, 2006]. Unlike the discontinuous cost function for latching control observed in Feng and Kerrigan [2013], the cost function of declutching control appears to be continuous. However, note that this is not always the case: the function can be discontinuous, as we will show later.

We will first provide a condition when the cost function is continuous. Consider that the declutching-controlled system (3), which can be written as:

\[
\dot{z}(t) = A_0 z(t) + B_0 F_c(t), \quad \forall t \in (t_0, t_1) \cup (t_1 + t_i, t_{i+1}), \quad (5a)
\]
\[
\dot{z}(t) = A_1 z(t) + B_1 F_c(t), \quad \forall t \in (t_i, t_{i+1}), \quad (5b)
\]

where $t_i, i = 1, …, N_i$, are the $N_i$ zero-crossing points of $v(t)$ over a time horizon and $v(t)$ is the second element in the state $z(t)$. $A_0, A_1$ and $B$ are the coefficient matrices.

The two solutions of the linear sub-systems (5a) and (5b) are:

\[
\Phi_0(z(a), a, b) := e^{A_0(b-a)} z(a) + \int_{a}^{b} e^{A_0(b-\tau)} B_0 F_c(\tau) d\tau,
\]
\[
\Phi_1(z(z(a), a, b) := e^{A_1(b-a)} z(a) + \int_{a}^{b} e^{A_1(b-\tau)} B_1 F_c(\tau) d\tau.
\]

Without loss of generality, assume the system starts in mode (5b), then the solution of system (5) is given by
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Gradient-based optimization methods are clearly not suitable for this application. Although it seems that the cost function is smooth in most of the cases, it is possible that the cost function will be discontinuous. Moreover, even in the smooth case, the analytical derivative is hard to find. Consequently, one cannot expect gradient-based methods to be numerically robust.

3. DERIVATIVE-FREE OPTIMIZATION

As discussed in Section 2.2, derivative-based optimization methods are not appropriate in this application. We choose to adapt the derivative-free coordinate search (CS) method discussed in Feng and Kerrigan [2013]. Here we only briefly present the algorithm specialized for declutching control; a detailed discussion can be found in Feng and Kerrigan [2013].

For declutching-control considered here, the generic CS algorithm becomes a simple two-directional line search procedure, which is summarized in Algorithm 1. The problem solved is

\[
\max f(x) := \max J(z_0, z_1) = \max_{t_a, t_0, t_1, \alpha} f(t_a, t_0, t_1, \alpha),
\]

where the decision variable \( x \) in this case is the PTO active time \( t_a \). Each iteration of Algorithm 1 evaluates points that are \( \alpha \) away from the current iterate and compare their values. If a cost increase is detected, the current iterate is replaced by the new point and the step size is increased. Otherwise, the algorithm stays at the current point and reduces the step size in order to try and find a point with higher cost.

Note that Algorithm 1 is different from the algorithm presented in Feng and Kerrigan [2013]. In Algorithm 1, \( \alpha_0 \in S_a \backslash \{0\} \) means that \( \alpha_0 \) is drawn from \( S_a \) but not 0. We take \( \alpha_{i+1} := \left[ \frac{\alpha_i}{h} \right] h \) to round \( \alpha_{i+1} \) to the nearest number in \( S_a \) that is larger than \( \gamma \alpha_i \); or we take \( \alpha_{i+1} := \left[ \frac{\alpha_i}{h} \right] h \) to round \( \alpha_{i+1} \) to the nearest number in \( S_a \) that is smaller than \( \beta \alpha_i \). Therefore, the sequence generated by the algorithm ensures \( x_i \in S_a \), so the algorithm does not evaluate points that are not feasible. The tolerance is \( h \), which is the smallest change in \( t_a \). By contrast, the algorithm presented in Feng and Kerrigan [2013] evaluates points in an uncountable set of reals, most of which may not be feasible and hence results in redundant function evaluations.

### Algorithm 1 One-dimensional coordinate search (CS)

**Require:** \( f() \): objective function; \( x_0 \): initial guess; \( \alpha_0 \in S_a \backslash \{0\} \): initial step size; \( \gamma > 1 \), \( \beta \in (0, 1) \); step size update parameters; \( h \): sampling time (tolerance);

**Ensure:** Locally optimal \( x^* \).

Define

\[
\hat{f}(x) := \begin{cases} f(x) & \text{if } x \in [0, f_{\max} h] \\
-\infty & \text{if } x < 0 \text{ or } x > f_{\max} h
\end{cases}
\]

Set \( i := 0 \). repeat

1. **Poll step:** Let the poll set be defined as \( P_i := \{ x_i + \alpha_t, x_i - \alpha_t \} \). If \( \exists x \in P_i \) s.t. \( \hat{f}(\tilde{x}) > \hat{f}(x_t) \), then set \( x_{i+1} := \arg \max_{x \in P_i} \hat{f}(\tilde{x}) \) and claim the poll step as successful.

2. **Parameter update:** If the poll step was successful, set \( \alpha_{i+1} := \left[ \frac{\alpha_i}{h} \right] h \). If the poll step was unsuccessful, set \( x_{i+1} := x_i \) and \( \alpha_{i+1} := \left[ \frac{\alpha_i}{h} \right] h \).

3. **Increment** \( i \) until \( \alpha_i < h \).

Gradient-based optimization methods are clearly not suitable for this application. Although it seems that the cost function is smooth in most of the cases, it is possible that the cost function will be discontinuous. Moreover, even in the smooth case, the analytical derivative is hard to find. Consequently, one cannot expect gradient-based methods to be numerically robust.
Algorithm 2 Closed-loop Receding Horizon Implementation

Require: Equivalent discrete-time systems $z((k + 1)h) = \hat{g}_1(z(kh), F_e; k)$ and $z((k + 1)h) = \hat{g}_2(z(kh), F_e; k)$; initial model $\Xi(0) := 0$; initial PTO active time $\hat{\xi}(0) := 0$; initial point for optimization solver $t_{a0} := 0$.

for each sampling instance $k = 0, 1, \ldots$ do
  if $v((k - 1)h) - v(kh) \leq 0$ then
    Compute $\xi(k) := t_{a0}^*(z, F_e, kh)$, where $s \in \{p, f\}$
    Reset timer $\Xi(kh) := 0$
  end if
  if $\Xi(kh) \leq \hat{\xi}(k)$ and $\hat{\xi}(k) > 0$ then
    $z((k + 1)h) = \hat{g}_1(z(kh), F_e; k)$
  else
    $z((k + 1)h) = \hat{g}_2(z(kh), F_e; k)$
  end if
  Increase timer $\Xi((k + 1)h) := \Xi(kh) + h$
  Replace $t_{a0}$ with $\hat{\xi}(k)$
end for

4. CLOSED-LOOP IMPLEMENTATION

We define two closed-loop formulations for the CS algorithm: the past data formulation and the future data formulation. If we record past wave data and estimate the state of the system, we have a cost function (7) w.r.t. PTO-active time $t_a$. Using the model (3), one can apply Algorithm 1 to determine the best PTO-active time $t_{ap}$ for the recorded horizon. Because the peak period of ocean waves is stable over a short interval, $t_{ap}$ can be considered as a good estimate of the optimal PTO-active time [Clément and Babarit, 2012; Babarit et al., 2004]. We refer to the first formulation as the past data formulation; mathematically,

$$\mathbb{P}\text{D}: t_{ap}(z, F_e, t) := \arg\max_{t_a \in S_a} J(z(t - T); t_a, F_e, t - T, t), \quad (8)$$

where $t$ is the current time and $T$ is the horizon length.

On the other hand, if we can predict the future excitation force, then we propose a formulation based on the current state and the predicted excitation force. One can use Algorithm 1 to find the best PTO-active time $t_{af}^*$ over the prediction horizon. We refer to this as the future data formulation; mathematically,

$$\mathbb{F}\text{D}: t_{af}(z, F_e, t) := \arg\max_{t_a \in S_a} J(z(t); t_a, F_e, t + T), \quad (9)$$

As in Figure 2, there often exists a unique global maximum close to zero. Thus, we propose $0$ as the initial guess $t_{a0}$ of the first optimization problem (at the first sampling instant). This makes it likely that the algorithm will converge to the global maximum in practice. For both formulations, one can solve the optimization problem at each sampling instant using a warm start procedure to speed up convergence. If the algorithm has converged to the global maximum, then it is likely that at each subsequent sampling instant the algorithm will converge to the global maximum.

For the closed-loop, one can solve problem (8) or (9) every $h$ seconds and implement the solution in a receding horizon fashion, as is standard in predicitive control. This procedure is detailed in Algorithm 2, where $s = p$ or $s = f$ depending on whether past or future wave data is available, respectively.

5. SIMULATION RESULTS

For the simulations, the parameters for the system (1) are $n_e := 5$, $M := 707 \text{ t (ton)}$, $\mu_o := 244 \text{ t}$, $g := 9.81 \text{m/s}^2$, $\rho := 1000 \text{ kg/m}^3$, $S := 78.5 \text{ m}^2$, $k_s := 240 \text{kN/m}$. The parameters for Algorithm 1 are $\gamma = 1.5$, $\beta = 0.5$. We set $h := 0.1 \text{s}$ and $T := 20 \text{s}$ which is the shortest horizon length that can achieve the best PTO and $t_a := 10 \text{s}$. Note that we assume that the switching can only occur at each sampling instance. Unless indicated otherwise, the future data formulation is used.

5.1 Cyber-Physical Co-design

We briefly discuss how co-design can affect the choice of system parameters. Traditionally, the system is designed without considering the control strategy; the uncontrolled system is tuned to its optimal performance against certain working conditions. Following this, the controller is designed to maximize the system’s performance. Co-design takes a different approach and designs the physical system and controller (cyber system) at the same time. In other words, the optimal physical parameters are a function of the control parameters, and vice versa.

Assume the to-be-designed parameter is the PTO damping $B_{pto}$. Figure 4 shows the power generated over 2000 s, subjected to irregular JONSWAP waves of peak periods $T_p$ from 5–12 s with different values of $B_{pto}$. In this case, no control has been applied. Figure 4 suggests the optimal value for $B_{pto}$ should be about 58–87 kNs/m for a significant range of periods. Thus, if we followed the standard procedure, we would take $B_{pto}$ around 78 kNs/m. If declutching control is applied, the maximum power is about 82 kW, obtained for waves with peak periods of $T_p = 6.2 \text{s}$.

Figure 5 shows the power take-off of the device under declutching control against $B_{pto}$ for different wave periods. The range for $B_{pto}$ is now increased to 1MNs/m to show the power take-off for very large $B_{pto}$. Figure 5 reveals that, unlike the uncontrolled case, for a declutching-controlled system, more power take-off can be achieved with a larger value of $B_{pto}$. Also, the peak power take-off wave period is slightly longer than for the uncontrolled case. Figure 6 shows the ratio of the power generated by the controlled system compared to the uncontrolled system. Note that there is little improvement for long peak periods. However, for short peak periods, declutching control with $B_{pto} = 900 \text{kN/m}$ is able to achieve at least 27% improvement, compared to an uncontrolled WEC, and a maximum of 170% improvement at about a peak period of 6 s. Therefore, after co-design we conclude that a larger $B_{pto}$ will give a better performance under declutching control, hence we choose $B_{pto} = 900 \text{kN/m}$. Note that this value is 3 times
Simulated Annealing is a widely-used global optimization method and does not require any initial guess for the golden ratio ensures a constant convergence rate. However, the gradient ascent method often got trapped in the flat region, as in Figure 2. The golden section search guarantees the convergence to a minimum of 6, are shown in Figure 11(a).

5.3 Comparison with Other Univariate Optimization Methods

Algorithm 1 was compared against four other methods, namely gradient ascent, golden section search, simulated annealing, and exhaustive search, in terms of the number of function evaluations. At each sampling time, we solved the PD formulation with an auto-regression (AR) model for predicting the future incoming wave [Fusco and Ringwood, 2010]. The window length (order) of the AR predictor is 100 samples, the prediction horizon 20 s and the past data length used to train the AR predictor is 50 s. The prediction was considered to be sufficiently accurate up to $4 - 5$ s into the future. Note that the future data formulation with AR predictor has a similar performance compared to using past data, despite relatively inaccurate predictions beyond 5 s.

Gradient Ascent A gradient ascent method, along with a backtracking line search [Nocedal and Wright, 1999, Chap. 3], was tested. The gradient was calculated through a central difference method with step size of 0.2 (different step sizes were tried, but the results were similar). The gradient ascent method often got trapped in the flat region, as in Figure 2.

Golden Section Search The golden section (GS) search process by repeatedly comparing and cutting the feasible interval into subsections according to the golden ratio (in order to ensure a constant convergence rate), until the termination tolerance is met. The golden section search guarantees the convergence to a local saddle point [Gill et al., 1986, Chap. 4.1.2]. The GS search took 11 function evaluations for all problems, because one cannot choose an initial guess for the golden-section search algorithm, the GS search was often trapped in the flat region, as in Figure 2.

Simulated Annealing Simulated annealing (SA) is a widely-used global optimization method and does not require any
Co-design of the physical system and controller can significantly increase the performance and robustness of a WEC, compared to the usual method of first optimizing the physical system without a controller, followed by controller design. This is because the controller significantly alters the closed-loop behaviour of a WEC, both qualitatively and quantitatively. Not only does the controller allow one to increase the power take-off for a given typical peak period, but the controller also allows one to increase the range of wave periods for which one can generate more energy, thereby extending the set of profitable operating conditions of the device. Advanced optimal control methods, coupled with co-design, could therefore become key technologies in determining the economic viability of generating power from ocean waves.

Future research could consider the PTO-active time at all of the predicted zero crossing points as independent decision variables, hence the optimization problem will no longer be univariate and more challenging to solve. It will be interesting to determine how much more power may be generated by solving this generalized problem.

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