A clutch based transmission for mechanical flywheel applications

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Abstract: Continuously variable transmissions play an important role in mechanical kinetic energy recovery systems. In this work, a clutch based continuously variable transmission comprising two stages of three clutches is considered. A detailed mathematical model is derived and a control strategy for the load torque is given. The model covers losses in bearings, clutches and fixed transmissions as well as flywheel losses due to gas friction.

Keywords: continuously variable transmission; modeling; simulation; automotive control; torque control

1. INTRODUCTION

Flywheel based mechanical hybrid systems are known in literature for many years. They consist of an internal combustion engine, a flywheel storage system and a variable transmission that adapts the corresponding speed levels. The flywheel technology has been applied in motorsport for many years and is now adapted for passenger cars, see Douglas and Brockbank [2009]. Remarkable effort has been dedicated to exploit the fuel saving potential for different powertrain topologies, see e.g. Brockbank and Body [2010]. Plenty of papers exist dealing with the fuel optimal control of such systems, e.g. Pfiffner et al. [2003]. Typically, the model complexity is reduced to a minimum, in order to enable the development of energy management strategies (Serrarens et al. [1998], van Berkel et al. [2012]). Commonly, simplified clutch representations (on/off) are used. Additionally, the control of flywheel hybrid powertrains with continuously variable transmissions (CVT) is investigated extensively (Shen and Veldpaus [2004]).

Usually, metal belt and chain CVTs (Srivastava and Haque [2009]) are used for such powertrains. Alternatively, there are toroidal (Nakano et al. [2000]) and spherical (Kim et al. [2002]) CVTs as well as electric variable transmissions (Hoeijmakers and Ferreira [2006]) available.

In this work, a clutch based continuously variable transmission (C-CVT) is presented. It consists of 6 clutches as shown in Fig. 1. A first stage (clutches 1, 2, 3) is coupled with a flywheel, a second stage (clutches 4, 5, 6) is linked to a load.

The clutches are coupled via different fixed transmissions. As a consequence, 9 fixed transmission ratios are possible, if one clutch from the first stage and one from the second stage are used simultaneously. This 9 ratios split the possible range between the vehicle speed and flywheel speed into 10 segments, as shown in Fig. 2.

Two modes are possible during dynamic operation of the C-CVT. Energy can be transferred from the flywheel to the load (mode ‘unload flywheel’, solid line in Fig. 2) as well as
Fig. 1. MATHEMATICAL MODEL

The overall structure shown in Fig. 1 is split into three parts, which can be described by means of one differential equation each. Only positive speeds are considered.

2.1 Flywheel to first clutch stage

Fig. 3 shows the first part comprising the flywheel and the corresponding side of the first clutch stage. The moments of inertia of the flywheel resp. of the transmissions between flywheel and first clutch stage are symbolized by \( J_F \) and \( J_{FC} \). The moments of inertia of the flywheel side of the parallel clutches 1, 2 and 3 (including the clutch discs) are denoted by \( J_{1F}, J_{2F} \) and \( J_{3F} \).

Additionally, there are fixed transmissions for a first reduction with ratio \( i_{FC} \) (efficiency \( \eta_{FC} \)) and for all three parallel clutch branches (ratios \( i_{1F}, i_{2F}, i_{3F} \) and efficiencies \( \eta_{1F}, \eta_{2F}, \eta_{3F} \)).

All fixed transmissions considered in the C-CVT are modeled as follows. The ratio is defined by the quotient of input and output speed of the transmission, i.e.

\[
i = \frac{\omega_1}{\omega_2}.
\]

Using the input torque \( T_1 \) and output torque \( T_2 \), the input power \( P_1 \) and output power \( P_2 \) are calculated. If \( P_1 \) is greater (or equal) than \( P_2 \), the transmission is operated in positive power direction. A constant efficiency \( \eta \) is assumed for the positive power direction, an efficiency of

\[
\bar{\eta} = \frac{1}{\eta} \quad \text{for the negative power direction. Then, the resulting torques are given by}
\]

\[
T_2 = T_1 \cdot i \cdot \eta \quad \text{resp.} \quad T_1 = \bar{\eta} \frac{T_2}{i}.
\]

Furthermore, diverse torque losses act on the system as shown in Fig. 3. Losses \( T_{GF} \) due to gas friction within the flywheel housing are taken into account. Based on relations of the boundary layer theory of fluid mechanics, see e.g. Schlichting and Gersten [2000], \( T_{GF} \) is modeled as a function of the flywheel speed \( \omega_F \). It is approximated by a polynomial of second order.

Secondly, losses in bearings are considered. They are said to be affine in speed. For example, the losses in the flywheel bearings are modeled as

\[
T_{i,F} = b_F \omega_F + b_{F0} \quad ,
\]

where \( b_F \) and \( b_{F0} \) are constants. For the losses in clutch bearings, parameters \( b_{1F}, b_{2F} \) and the corresponding clutch speeds \( \omega_{k,F} \) are used. Index \( k = 1, 2, 3 \) stands for the number of the clutch.

Furthermore, viscous oil losses are taken into account. These losses are proportional to speed differences \( \Delta \omega_k \) of the plates of clutches \( k = 1, 2, 3 \) via constants \( v_{k,F} \). The torques

\[
T_{kv} = \begin{cases} v_{k,F} \Delta \omega_k & \text{if } |\Delta \omega_k| < \Delta \omega_{k,\text{max}} \\ \bar{v}_{k,F} \text{ sign} \Delta \omega_k & \text{else} \end{cases}
\]

are limited to values of \( \pm \bar{v}_{k,F} \), that are reached at speeds \( \pm \Delta \omega_{k,\text{max}} \). Here, the speed difference, e.g. of clutch 1, is defined by

\[
\Delta \omega_1 = \frac{\omega_F}{i_{FC} i_{1F}} - \omega_B i_{AB}.
\]

The rotary velocity \( \omega_B i_{AB} \) represents the speed of the right side of clutch 1. Ratio \( i_{AB} \) and speed \( \omega_B \) are quantities of the interconnection between the two clutch stages, according to Fig. 4. Torque losses (5) are positive, if the flywheel drives the second clutch stage, i.e. \( \Delta \omega_k > 0 \).

If the flywheel is driven by the second clutch stage, the coupling via the oil in the clutches acts as a propulsion for the flywheel, i.e. \( T_{kv} \) appears with the opposite sign.
With the combination of bearing and viscous oil losses
\[ T_{1,k,F} = b_{k,F} \omega_{k,F} + b_{k,F,0} + T_{k,v}, \quad (7) \]
the dynamics of the first C-CVT part is governed by differential equation
\[
J_{F,tot} \frac{d\omega_{F}}{dt} = -T_{GF} - T_{1,F} - \frac{1}{i_{FC} \eta_{FC}} \left( T_{c1} + T_{1,F} \right) + \frac{1}{i_{2F} \eta_{2F}} \left( T_{c2} + T_{1,F} \right) + \frac{1}{i_{3F} \eta_{3F}} \left( T_{c3} + T_{1,F} \right). \quad (8)
\]
Here, \( T_{c1}, T_{c2} \) and \( T_{c3} \) represent the clutch torques transmitted due to friction. The total moment of inertia
\[
J_{F,tot} = J_{F} + \frac{1}{i_{FC} \eta_{FC}} \left( J_{FC} + \frac{1}{i_{2F} \eta_{2F}} J_{c2,F} + \frac{1}{i_{3F} \eta_{3F}} J_{c3,F} \right). \quad (9)
\]
is dependent on the actual power direction.

### 2.2 First clutch stage to second clutch stage

The second part of the C-CVT is shown in Fig. 4. It consists of the interconnection between the first clutch stage (right sides of clutches 1, 2 and 3) and the second clutch stage (left sides of clutches 4, 5 and 6), where \( T_{c,k} (k = 1, 2, \ldots, 6) \) symbolize the friction torques at the clutches.

Each parameter \( J_{A}, J_{B} \) and \( J_{C} \) combines the moments of inertia of two coupled clutches, e.g. clutch 1 and 4 (including the clutch baskets), see Fig. 4. A fixed transmission \( i_{AB} \) between clutches 1-4 and 2-5 (efficiency \( \eta_{AB} \)) and a fixed transmission \( i_{BC} \) between clutches 3-6 and 2-5 (efficiency \( \eta_{BC} \)) are used.

Analog to Section 2.1, several kinds of losses are modeled. The losses in the bearings between baskets and housing are assumed to be affine in speeds \( \omega_{j} (j \in \{A,B,C\}) \); parameter \( b_{j} \). All losses in bearings between baskets and plates are affine in the according speed differences \( \Delta \omega_{k} (k = 1, 2, \ldots, 6) \); parameter \( b_{k,j}, b_{k,j,0} \). For example, \( b_{2B} \) represents the proportional factor of the losses in the bearing next to clutch 2 on shaft \( B \), see Fig. 4.

The viscous oil losses within the clutches \( T_{k,v} \) are proportional to speed differences \( \Delta \omega_{k} \) up to a maximum torque, according to (5).

Additionally, losses caused by the pumping effect of the clutch baskets are considered. This pumping losses have to be taken into account only if minimal speeds \( \omega_{j,min} \) are exceeded. They are negligible below this limit. For higher speeds, the power losses are assumed to be proportional to speeds \( \omega_{j} \) because of a constant oil flow. As a consequence, the pumping losses are modeled as
\[
T_{jp} = \begin{cases} 0 & \text{if } \omega_{j} < \omega_{j,min} \\ \frac{\alpha_{j1} - \alpha_{j2}}{\omega_{j}} & \text{else}\end{cases} \quad (10)
\]
The constant parameters \( \alpha_{j1} \) and \( \alpha_{j2} (j \in \{A,B,C\}) \) are dependent on the oil flow through the clutches.

Combining all introduced losses leads to the total torque losses acting on shaft \( A \), i.e.
\[
T_{l,A} = b_{A} \omega_{A} + b_{A0} + b_{1A} \Delta \omega_{1} + b_{lA0} + b_{4A} \Delta \omega_{4} + b_{A0} + T_{Ap} - T_{1,v} + T_{k,v}. \quad (11)
\]
The negative sign in (11) appears because of the propulsion effect of clutch 1 for the positive power direction. The total torque losses \( T_{l,B} \) and \( T_{l,C} \) at shafts \( B \) and \( C \) are modeled analog to (11).

The mathematical description of the second C-CVT part is stated as differential equation
\[
J_{B,tot} \frac{d\omega_{B}}{dt} = T_{c2} - T_{c5} - T_{l,B} + i_{AB} \eta_{AB} (T_{c1} - T_{c4} - T_{l,A}) + i_{BC} \eta_{BC} (T_{c3} - T_{c6} - T_{l,C}). \quad (12)
\]
for speed \( \omega_{B} \), where the total moment of inertia \( J_{B,tot} \) is given by
\[
J_{B,tot} = J_{B} + i_{AB}^{2} \eta_{AB} J_{A} + i_{BC}^{2} \eta_{BC} J_{C}. \quad (13)
\]

### 2.3 Second clutch stage to load

In Fig. 5, the third part of the C-CVT is depicted. All load sides of clutches 4, 5 and 6 (moments of inertias \( J_{c4L}, J_{c5L}, J_{c6L} \)) are coupled via three fixed transmissions with ratios \( i_{4L}, i_{5L}, i_{6L} \) to the final reduction ratio \( i_{CL} \) of the C-CVT. The corresponding transmission efficiencies \( \eta_{4L}, \eta_{5L}, \eta_{6L} \) and \( \eta_{CL} \) are included into the model as in Section 2.1.

Parameter \( J_{CL} \) symbolizes the moment of inertia of all parts between the second clutch stage and the load. The moment of inertia of the load
\[
J_{L} = \frac{m r^{2}}{i_{F,d}} \quad (14)
\]
represents an equivalent vehicle with mass \( m \), rolling radius \( r \) and final drive ratio \( i_{F,d} \). The speed of the equivalent vehicle \( v \) is linked to the speed \( \omega_{L} \) of the load shaft via
\[
v = \omega_{L} \frac{r}{i_{F,d}}. \quad (15)
\]
For the modeling of the losses in bearings (parameter $b_L$, $b_{L0}$, $b_{CL}$, $b_{CL0}$, $b_K$, $b_{KL0}$ with $k = 4, 5, 6$) and viscous oil losses $T_{ke}$, the same assumptions are made as in Section 2.1. As a result, these losses are given by

$$T_{i,CL} = b_L \omega_L + b_{L0} + b_{CL} \omega_L + b_{CL0}$$

(16)

and

$$T_{i,KL} = b_{KL} \omega_{KL} + b_{KL0} - T_{ke}$$

(17)

Additional losses $T_{loss}$ of the equivalent vehicle are considered as well, see e.g. Kiencke and Nielsen [2010]. These losses are modeled as

$$T_{loss} = \frac{r}{i_{Fd}} \left[ \frac{1}{2} \rho A_f c_d v^2 + c_r m g \cos(\alpha) + m g \sin(\alpha) \right]$$

(18)

with the following parameters: density of the ambient air $\rho$, frontal area of the vehicle $A_f$, aerodynamic drag coefficient $c_d$, rolling friction coefficient $c_r$, acceleration of gravity $g$ and road angle $\alpha$.

The inputs for the third part of the C-CVT are the clutch torques $T_{c4}$, $T_{c5}$, $T_{c6}$ as well as an additional torque $T_A$ that can be applied e.g. by means of an internal combustion engine.

Hence, the dynamics of the third part of the C-CVT can be expressed as

$$J_{L,tot} \frac{d \omega_L}{dt} = T_A - T_{loss} - T_{i,CL} +$$

$$i_{CL} \eta_{CL} \left[ i_{4L} \eta_{4L} (T_{c4} - T_{i,4L}) + i_{5L} \eta_{5L} (T_{c5} - T_{i,5L}) + i_{6L} \eta_{6L} (T_{c6} - T_{i,6L}) \right]$$

(19)

with a total moment of inertia of

$$J_{L,tot} = J_L + i_{CL}^2 \eta_{CL} (J_{CL} + i_{4L}^2 \eta_{4L} J_{c4} +$$

$$i_{5L}^2 \eta_{5L} J_{c5} + i_{6L}^2 \eta_{6L} J_{c6})$$

(20)

The load torque $T_L$ at the C-CVT interface equals

$$T_L = J_L \frac{d \omega_L}{dt} - T_A + T_{loss} + b_L \omega_L$$

(21)

### 2.4 Calculation of the transmitted clutch torques

A central point of the model presented in this paper is the calculation of the clutch torques $T_{ck}$ ($k = 1, 2, \ldots, 6$) based on the basic clutch model described in Appendix A. It covers the Coulomb friction and the stiction effect.

If only one clutch is closed (e.g. clutch 1), the speeds of the left and right side of the clutch are identical, i.e. $\omega_F \equiv \omega_B i_{AB} i_{IF} i_{FC}$. Consequently, the accelerations are also the same. This yields

$$\frac{1}{J_{F,tot}} T_1 = \frac{1}{J_{B,tot}} T_2 i_{AB} i_{IF} i_{FC}$$

(22)

where $T_1$ is the right side of (8), $T_2$ the right side of (12). An extraction of the unknown stiction torque $T_{c1}$ results in

$$\left( J_{F,tot} i_{IF} i_{FC} i_{AB}^2 t_{AB} + J_{B,tot} \frac{1}{i_{FC} \eta_{FC} i_{IF} \eta_{IF}} \right) T_{c1} = J_{B,tot} T_1 \{ \{ T_{c1} \} - J_{F,tot} T_2 \{ \{ T_{c1} \} i_{AB} i_{IF} i_{FC} \}$$

(23)

In (23), e.g. $T_1 \{ \{ T_{c1} \}$ is used as a short notation for $T_1$ according to (8), where all terms containing $T_{c4}$ are removed. All other clutches may slip while clutch 1 is closed.

For the case of two closed clutches (one from the first and one from the second stage), two speed identities have to be fulfilled, e.g. $\omega_F \equiv \omega_B i_{AB} i_{IF} i_{FC}$ and $\omega_B \equiv \omega_L i_{5L} i_{CL}$, if clutch 1 and 5 are closed. The corresponding stiction torques $T_{c1}$ and $T_{c5}$ are determined via

$$\left( J_{F,tot} i_{IF} i_{FC} i_{AB}^2 t_{AB} + J_{B,tot} \frac{1}{i_{FC} \eta_{FC} i_{IF} \eta_{IF}} \right) T_{c1} -$$

$$J_{F,tot} i_{IF} i_{FC} i_{AB} T_{c5} =$$

$$J_{B,tot} T_1 \{ \{ T_{c1} \} - J_{F,tot} T_2 \{ \{ T_{c1} \} \} i_{AB} i_{IF} i_{FC}$$

(24)

and

$$-J_{L,tot} i_{AB} \eta_{AB} T_{c1} +$$

$$\left( J_{B,tot} i_{5L}^2 \eta_{5L} i_{CL}^2 + J_{L,tot} \right) T_{c5} =$$

$$J_{L,tot} T_2 \{ \{ T_{c1} \} \} - J_{B,tot} T_3 \{ \{ T_{c5} \} i_{5L} i_{CL}$$

(25)

With the right side $T_3$ of differential equation (19), both stiction torques can be determined.

The stated procedure for the calculation of the stiction torques is realized for all admissible combinations of clutches 1 to 6. A clutch logic checks, which clutches are opened and closed depending on the current clutch actuations, torques and speeds.

### 3. CONTROL STRATEGY

For the operation of the C-CVT, a strategy for the switching between different clutch combinations is necessary.

In Fig. 6, a procedure for a switching between clutch combination 1-6 and 4-1 is shown exemplarily. Before starting the switching, clutches 1 and 6 are activated (see top plot in Fig. 6) and clutch 1 is closed (middle plot), clutch 6 is slipping. The switching is triggered if
a certain speed difference at clutch 6 is reached. Then, the actuation torques are reduced, where the rate is limited due to the used actuators. If the actuation torque of the closed clutch equals a predefined threshold, e.g. 20 Nm in Fig. 6, clutch 4 is actuated. The actual torques at the clutches depend on the current speed and torque levels in the system, see Fig. 6 (bottom). After closing clutch 4, clutch 1 has to be controlled in order to achieve a desired load torque $T_L$.

Please note that there are several strategies for the switching between two clutches of the same stage to reduce the torque drop. For the switching between two clutches of different stages, e.g. from clutches 1 and 4 to clutches 3 and 5, the torque gap cannot be avoided.

The torque controller is based on a simplified version of model (8,12,19). Here, the assumption of vanishing losses yields

$$J_1 \frac{d \omega_F}{dt} = - \frac{1}{i_{FC} i_{1F}} T_{c1} - \frac{1}{i_{FC} i_{2F}} T_{c2} - \frac{1}{i_{FC} i_{3F}} T_{c3}$$

$$J_2 \frac{d \omega_B}{dt} = i_{AB} T_{c1} + T_{c3} + i_{BC} T_{c5} - i_{AB} T_{c4} + T_{e5} + i_{BC} T_{e6}$$

$$J_3 \frac{d \omega_L}{dt} = i_{CL} i_{4L} T_{c4} + i_{CL} i_{5L} T_{c5} + i_{CL} i_{6L} T_{e6}$$

$$T_L = J_L \frac{d \omega_L}{dt}$$

where $J_1$, $J_2$ and $J_3$ represent $J_{F,tot}$, $J_{B,tot}$ and $J_{L,tot}$ under the assumption that all efficiencies are 1. The simplified model (26)-(29) can also be used for a rough estimation of the load torque.

A feedforward is derived by rearranging (26)-(29). For example, clutch 5 is closed, i.e. $\omega_B = \omega_L i_{5L} i_{CL}$, and clutch 1 is slipping, i.e.

$$T_{c1} = k_{c1} \text{sign} \left( \frac{\omega_F}{i_{FC} i_{1F}} - \omega_B i_{AB} \right).$$

4. SIMULATION

In Fig. 7, the results of a simulation without any clutch actuation are shown. The flywheel (initial speed $\omega_F = 60.000 \text{rpm}$) accelerates the parts between first and second clutch stage due to the viscous oil losses in clutches 1, 2 and 3. Furthermore, the power losses in bearings, flywheel housing and clutches are shown. The losses in fixed transmissions are negligible because of the low torques transmitted. The power transmitted to the load via viscous losses is not sufficient to accelerate the load, i.e. the load speed remains zero for the simulation.

As a second example, a vehicle acceleration maneuver is plotted in Fig. 8. The desired load torque $T_L = 100 \text{Nm}$ can be provided by the C-CVT except for the short switching periods. Because the presented model is designated for design studies, the achieved performance of the control loop is sufficiently good.

Fig. 9 illustrates the acceleration maneuver with respect to flywheel and vehicle speed. Four switchings are used to decelerated the flywheel resp. to accelerate the vehicle to speed $v = 36 \text{km/h}$ (x in Fig. 9). In contrast to the previous...
It is planned to design and manufacture a prototype C-CVT to investigate the behavior of the controlled flywheel system on a test rig.

REFERENCES


Fig. A.1. Scheme of the basic clutch model


Appendix A. BASIC CLUTCH MODEL

Fig. A.1 shows the scheme of a simple clutch model. Here, $J_1$ denotes the moment of inertia of the first clutch side (e.g. including the disks) and $J_2$ symbolizes the moment of inertia of the second clutch side (e.g. including the basket). The principle of angular momentum yields differential equations for the input speed $\omega_1$ and output speed $\omega_2$

$$J_1 \frac{d\omega_1}{dt} = T_1 - T_C \quad \text{and} \quad J_2 \frac{d\omega_2}{dt} = T_C - T_2 , \quad (A.1)$$

where the input torque $T_1$ and output torque $T_2$ are assumed to be known. Both equations are coupled via torque $T_C$ that is transmitted due to friction. For a slipping clutch, i.e. $\omega_1$ differs $\omega_2$, friction torque $T_C$ is modeled by

$$T_C = k \left| \frac{\omega_1}{\omega_2} \right| \omega_2 ,$$

where parameter $k$ depends on the hydraulic clutch actuation. This yields

$$T_C = \frac{T_1 J_2 - T_2 J_1}{J_1 + J_2} . \quad (A.2)$$

The clutch remains closed, if torque $T_C$ is smaller (or equal) than the maximal transmittable torque, i.e. $|T_C| \leq k \, k_{st}$. The stiction factor $k_{st}$ is assumed to be constant. If the maximal torque is exceeded, the clutch opens and the transmitted torque equals

$$T_C = k \left( \frac{T_1 J_2 - T_2 J_1}{J_1 + J_2} \right) . \quad (A.3)$$