Novel Anti-windup PID Controller Design under Holonomic Endpoint Constraints for Euler-Lagrange Systems with Actuator Saturation

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Abstract: A novel anti-windup PID controller design method under holonomic constraints is proposed for nonlinear Euler-Lagrange systems with actuator saturation. The controller design is based on passivity, quasi-natural potential and saturated-position feedback. According to four saturation cases, switching of four integrating functions in the control law is utilized and four Lyapunov functions, such as hybrid control, are derived. Global asymptotic stability is ensured by energy dissipation between the four Lyapunov functions. The control performance is verified by numerical simulations using a two-link robot arm.

1. INTRODUCTION

Every plant system is subject to input limitations. When the controller requires an input that exceeds these limitations, the control performance is degraded and becomes unstable. Actuator saturation caused by exceeded input limitations gives rise to windup phenomena, which degrade the control performance. Over the past decade, several anti-windup controller design methods have been proposed for linear systems to suppress windup degradation (e.g., Kanamori et al., 2007). However, studies of specifically nonlinear robot systems are few. Several studies have investigated anti-windup controller design for robot systems, including those by Kanamori (2011, 2013a,b), Khan et al. (2010), Lopez-Araujo et al. (2012), Loria et al. (1997), Morabito et al. (2004), Suntibanez et al. (1996), Teo et al. (2009), and Zavara-Rio et al. (2009). Kanamori (2011) proposed an anti-windup PID position controller for nonlinear Euler-Lagrange systems with input saturation. In this work, global asymptotic stability was guaranteed by the Lyapunov theorem based on the passivity described by Arimoto (1996). This anti-windup method was extended to adaptive tracking control considering input saturation (Kanamori, 2013a), and the control performance was demonstrated by experimentation (Kanamori, 2013b). A series of such works based on passivity imply that passivity is extremely suitable to stabilization that considers input saturation for nonlinear Euler-Lagrange systems. Since, in general, holonomic constraints have given rise to almost all robot systems (Khan et al., 2010), an anti-windup control method based on passivity is expected.

In the present paper, the anti-windup adaptive law by Kanamori (2013a) is extended to anti-windup PID position control under holonomic constraints for nonlinear Euler-Lagrange systems with input saturation. According to four saturation cases, switching of four integrating functions in the control law is utilized and four Lyapunov functions, such as hybrid control, are derived. Global asymptotic stability is ensured by energy dissipation between the four Lyapunov functions. The control performance of the proposed controller is verified by numerical simulations using a two-link robot arm.

2. PRELIMINARY

Let us consider the case where the endpoint of the manipulator is constrained on a surface (Arimoto, 1996). The surface is described by a scalar function as

\[ \phi(x_1, x_2, x_3) = 0, \]

where

\[ x = [x_1, x_2, x_3]^T \]  \hspace{1cm} (2)

denotes the Cartesian coordinates fixed at the internal reference frame. The contact force arises in the direction of the normal vector to the surface at point \( x \) and the contact friction arises in the direction of \( -\dot{x} \) with magnitude \( \xi(\|x\|/\|x\|) \), where \( \xi(\cdot) \) is a positive scalar function. Then, the robot dynamics are described by

\[ H(q)\ddot{q} + B_0 + \frac{1}{2}H(q) + S(q, \dot{q})\dot{q} + g(q) = J_\phi(q)^T f - \xi(\|x\|/\|x\|) J_s(q)^T \dot{x} + \sigma(u), \]

where

\[ J_\phi(q) = \left( \frac{\partial \phi_i(q)}{\partial q_j} \right)_{i=1,2,3, j=1,...,n}, \]  \hspace{1cm} (4)

\[ J_s(q) = \frac{\left( \frac{\partial \phi}{\partial x} \right)^T J_s(q)}{\|\frac{\partial \phi}{\partial x}\|}, \]  \hspace{1cm} (5)
and \( f \) is the contact force. In addition, \( q = [q_1, \ldots, q_n]^T \) represents the angular vector of each joint, \( H(q) \in \mathbb{R}^{n \times n} \) is the positive definite inertia matrix, \( B_q \in \mathbb{R}^{n \times n} \) is the positive diagonal constant matrix derived from the Raleigh dissipation function, \( S(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the skew-symmetric matrix, and \( g(q) \) is the gravity term derived from the relation \( q \in \mathbb{R}^n \). Furthermore, \( u = [u_1, \ldots, u_n]^T \) is the control input torque vector, and \( \sigma(u) = [\sigma(u_1), \ldots, \sigma(u_n)]^T \) is the saturated input torque vector, defined as follows:

- if \( u_{\min, i} \leq u_i \leq u_{\max, i}, \sigma(u_i) = u_i \),
- if \( 0 < u_{\max, i} < u_i \), \( \sigma(u_i) = u_{\min, i} \),
- if \( u_i < u_{\min, i} \), \( \sigma(u_i) = u_{\max, i} \),

where \( u_{\min, i} = u_{\min, i} \) for \( i = 1, \ldots, n \). The estimation value of the input saturation \( \psi(u) = [\psi(u_1) \ldots \psi(u_n)]^T \) is introduced as

\[
\psi(u) = u - \sigma(u),
\]

where

- if \( u_{\min, i} \leq u_i \leq u_{\max, i}, \psi(u_i) = 0 \),
- if \( 0 < u_{\max, i} < u_i \), \( \psi(u_i) = u_i - u_{\min, i} > 0 \),
- if \( u_i < u_{\min, i} \), \( \psi(u_i) = u_i - u_{\max, i} < 0 \).

The closed-loop system is described as

\[
L(q, \dot{q}, \ddot{q}) + F(g(q), \sigma(u), f) = 0,
\]

where

\[
L(q, \dot{q}, \ddot{q}) = H(q)\ddot{q} + [B_q + \frac{1}{2} \dot{H}(q) + S(q, \dot{q})]\dot{q} + \sum_{j=1}^n \|J_j(q)^T J_j(q)\| q_j \dot{f}_j,
\]

\[
F(g(q), \sigma(u), f) = g(q) - \sigma(u) - J_f\dot{f}.
\]

To achieve the state \((q, \dot{q}, f) \rightarrow (q_i, \dot{q}_i, f_i)\), where \( q_i \) and \( f_i \) are the target vectors of each joint angle and target contact force, respectively, the following assumption is introduced.

**Assumption 1:** Assume that the following conditions are satisfied for \( i = 1, \ldots, n \) for any target posture:

\[
u_{\min, i} < g_j(q), u_{\max, i} \]

\[
u_{\min, i} < [g_j(q) + J_f q_j]^T f_i < u_{\max, i}.
\]

Take the appropriate output \( y \) and assume that the inner product of \( y \) and the closed-loop system (9) has the following form:

\[
y^T L(q, \dot{q}, \ddot{q}) = \frac{d}{dt} V_0 + W_0,
\]

\[
y^T F(g(q), \sigma(u), f) \geq \frac{d}{dt} V_c + W_c,
\]

where \( V_c \) and \( W_c \) are switched for \( i = 1, \ldots, l \). Adding (14) and (15), the following relation holds:

\[
0 \geq \frac{d}{dt} V_i + W_i,
\]

where

\[
V_i = V_0 + V_c, \quad W_i = W_0 + W_c.
\]

Assume that \( V_i \) and \( W_i \) for \( i = 1, \ldots, l \) are positive definite and \( V_j \) is the minimum function, \( V_j = \min(V_1, \ldots, V_l) \). Let us denote the difference between \( V_i \) and \( V_j \) for \( i \neq j \) by \( U_i = V_i - V_j \). For asymptotic stability, the following lemma may be introduced (Kanamori, 2013a).

**Lemma 1:** If we have the following two states for \( i = 1, \ldots, l \):

\[
V_i > V_j > 0 \quad \text{and} \quad V_i > 0 \quad \text{for} \quad i \neq j : \text{state 1}
\]

\[
V_i > 0 \quad \text{and} \quad V_i < 0 : \text{state 2}
\]

the minimum function \( V_j \) is a Lyapunov function of the whole trajectory of the closed-loop system and the equilibrium state is asymptotically stable.

**Outline Proof of Lemma 1:** For the proof, it is sufficient to show that \( V_j \) is dissipative on state 1 shown in (18). Taking the time integral of (16), the following relation holds:

\[
V_j(t) - V_j(0) \leq U_i(0) - U_i(t) - \int_0^t W_i dt.
\]

If \( U_i \) increases, then \( V_j \) decreases. If \( U_i \) decreases, then \( V_j \) might increase; however, the increase value is bounded by \( U_i(0) \). Since \( W_i \) is positive definite, the time integral of \( W_i \), which is the dissipated energy, always increases with the integral of time so that the increase in \( V_j \) turns into a decrease soon. This implies that \( V_j \) is a Lyapunov function and the equilibrium state of the closed-loop system is asymptotically stable.

Let us choose the following output \( y \) for the evaluation in Arimoto (1996):

\[
y = \dot{q} + \alpha Q_j(q)\Delta \dot{q},
\]

where \( \alpha \) is a positive definite scalar, \( Q_j(q) \) is defined as

\[
Q_j(q) = I - J_f(q)^T J_f(q) \frac{\|J_f(q)\|}{\|J_f(q)\|},
\]

which is regarded as a projection that projects vectors in joint space onto the plane tangent to the surface \( \phi(q) = 0 \) at the contact point \( q \), and \( I \) denotes the identity matrix. The function \( s(\Delta \dot{q}) \) is the output saturation function described in Arimoto (1996) as follows:

\[
s(\Delta \dot{q}) = \frac{1}{2} \|\Delta \dot{q}\|^2.
\]
holds so that the margins between input limits and the gravity term are utilized for the stability.

**Property 4:** The following inequality holds:

If \( s(\Delta q)^T Q_\sigma(q)\sigma(u) < 0 \), then

\[
s(\Delta q)^T Q_\sigma(q)F(g(q),\sigma(u),f) \geq c_{0\min} s(\Delta q)^T Q_\sigma(q)s(\Delta q),
\]
where \( c_{0\min} \) is the positive minimum value of \( c_{0i} \) for \( i = 1,...,n \) as follows.

\[
c_{0\min} = \min\{c_{01} \ldots c_{0n}\}.
\]

**Property 5:** The following inequalities are satisfied for saturation cases (I) through (IV) so that the input saturation function is eliminated by the inequalities (Kanamori, 2013a):

(I) If \( \dot{q}^T \psi(u) \geq 0 \) and \( s(\Delta q)^T Q_\sigma(q)\psi(u) \geq 0 \), then \( y^T F(g(q),\sigma(u),f) \geq y^T F(g(q),u,f) \).

(II) If \( \dot{q}^T \psi(u) \geq 0 \) and \( s(\Delta q)^T Q_\sigma(q)\psi(u) < 0 \), then \( y^T F(g(q),\sigma(u),f) \geq \dot{q}^T F(g(q),u,f) + \alpha c_{0\min}s(\Delta q)^T Q_\sigma(q)s(\Delta q) \).

(III) If \( \dot{q}^T \psi(u) < 0 \) and \( s(\Delta q)^T Q_\sigma(q)\psi(u) \geq 0 \), then \( y^T F(g(q),\sigma(u),f) \geq \dot{q}^T F(g(q),u,f) \).

(IV) If \( \dot{q}^T \psi(u) < 0 \) and \( s(\Delta q)^T Q_\sigma(q)\psi(u) < 0 \), then \( y^T F(g(q),\sigma(u),f) \geq \alpha c_{0\min}s(\Delta q)^T Q_\sigma(q)s(\Delta q) \).

The inner product, as shown in (14), is determined by the given robot system. \( V_0 \) and \( W_0 \) are independent of the controller design and are obtained as follows (Arimoto, 1996).

**Property 6:** \( V_0 \) and \( W_0 \) as shown in (14) become

\[
V_0 \geq \frac{1}{4} \dot{q}^T H(q)\dot{q} + \alpha(b_{0\min}c_{\min} - \alpha\gamma_M)\left|\left| s(\Delta q) \right|\right|^2,
\]

\[
W_0 \geq \{b_{0\min} - \alpha c_{\min} + \frac{1}{2}\xi \left|\left| J_s(q)^T J_s(q) \right|\right| \} \frac{1}{2} \Delta B_0
+ \frac{1}{2} \alpha^2 s(\Delta q)^T Q_\sigma(q)s(\Delta q),
\]

where \( c_{\min} \) is the minimum value of \( c_i \) as follows:

\[
c_{\min} = \min\{c_1 \ldots c_n\},
\]

\( b_{0\min} \) is the minimum value of the diagonal component of the positive diagonal matrix \( B_0 \) in (3). \( \gamma_M \) is the maximum eigenvalue of \( H(q) \), and \( \Delta B_0 \) is the difference between \( B_0 \) and \( b_{0\min}I \) as follows:

\[
\Delta B_0 = B_0 - b_{0\min}I \geq 0.
\]
Assumption 2: There exists positive real numbers $\alpha$ and $c_{\min}$ such that the following inequalities are satisfied:

$$b_{\text{min}} - c_{\min} \alpha > 0,$$  \hspace{1cm} (44)

$$b_{\text{min}} - c_{\min} \alpha \sigma > 0.$$

(45)

$$2c_{\text{min}} I - \alpha \left( \frac{1}{2} \Delta B_0 + \xi \|J_s(q) \|^2 \right) > 0.$$  \hspace{1cm} (46)

Property 7: There exists a positive real number $\alpha$ such that the following inequalities are satisfied:

$$U(q) - U(q_d) - \Delta q^T g(q_d) + \frac{a}{2} \|\Delta q\|^2 > 0,$$  \hspace{1cm} (47)

$$s(\Delta q)^T Q_s(q) \{ g(q) - g(q_d) \} + \frac{a}{2} \|Q_s(q) s(\Delta q)\|^2 > 0.$$  \hspace{1cm} (48)

Property 8: The contact force $f$ is given by the following form:

$$f = \left[ J_\theta(q) H(q)^{-1} J_\theta(q) \right]^{-1} \left[ - J_\theta(q) H(q)^{-1} X - J_\theta(q) q \right],$$

(49)

where

$$X = -\{ B_0 + \frac{1}{2} H(q) + S(q, \dot{q}) \} \dot{q} - g(q) + \sigma(\dot{u}) - \xi \| \dot{J}_s(q) \|^2 \dot{x}.$$  \hspace{1cm} (50)

3. CONTROLLER DESIGN

Let us consider the following proportional, integral and derivative (PID) control law:

$$u = -a \Delta q - b \dot{q} - c Q_s(q) \int_{0}^{t} \ddot{q}(\tau) d\tau$$

$$- J_\theta(q)^T f_d + \beta J_\theta(q)^T \int_{0}^{t} \dddot{q}(\tau) d\tau,$$  \hspace{1cm} (51)

where $a$, $b$, $c$, and $\beta$ are positive definite real numbers that are regarded as feedback gain, and $\Delta f$ is the error between contact force $f$ and its target force $f_d$ represented as $\Delta f = f - f_d$. Using the notation

$$\Delta F = \int_{0}^{t} \Delta f(\tau) d\tau,$$  \hspace{1cm} (52)

$$z = \int_{0}^{t} \ddot{q}(\tau) d\tau - \int_{0}^{t} \dot{q}(\tau) d\tau \text{ and } \int_{0}^{t} q(\tau) d\tau = -c^{-1} g(q_d).$$

(53)

$F(g(q), \sigma(u), f)$ defined by (11) becomes

$$F(g(q), \sigma(u), f) = g(q) - Q_s(q) g(q_d) + a \Delta q + b \dot{q} + Q_s(q) \dot{z} - J_\theta(q)^T \left( \Delta f + \beta \Delta F \right) + \psi(u).$$  \hspace{1cm} (54)

Let us take the following $\dddot{q}(t)$ according to the four saturation cases (Kanamori, 2013a):

In CASE (I),

$$\dddot{q}(t) = \dot{q} + a Q_s(q) s(\Delta q),$$  \hspace{1cm} (55)

In CASE (II),

$$\dddot{q}(t) = \dot{q},$$  \hspace{1cm} (56)

In CASE (III),

$$\dddot{q}(t) = a Q_s(q) s(\Delta q),$$  \hspace{1cm} (57)

In CASE (IV),

$$\dddot{q}(t) = 0.$$  \hspace{1cm} (58)

The inner products, as shown on the right-hand side of (36) through (39), can be taken easily because $\sigma(\cdot)$ is eliminated and $V_c$ and $W_i$ in (15) are also easily derived. In CASE (I), the inner product $y^T F(g(q), \sigma(u), f)$ in (36) is represented as follows:

$$y^T F(g(q), \sigma(u), f) \geq y^T F(g(q), u, f) = \frac{dV_{ci}}{dt} + W_i,$$  \hspace{1cm} (59)

where $V_{ci}$ and $W_i$ are represented as

$$V_{ci} \geq U(q) - U(q_d) - \Delta q^T g(q_d) + \frac{a}{2} \|\Delta q\|^2$$

$$+ abc_{\min} \|s(\Delta q)\|^2 + \frac{\alpha c_{\min}}{2} \|\Delta q\|^2,$$  \hspace{1cm} (60)

$$W_i \geq c \|Q_s(q) s(\Delta q)\| + b \|\Delta q\|.$$  \hspace{1cm} (61)

The suffix $i = 1$ of $V_{ci}$ and $W_i$ means saturation CASE (I). In the derivation, the following relation, (26) and (28) are utilized:

$$y^T Q_s(q) c z = c y^T z = c z^T z = c \frac{d}{dt} \| z \|^2.$$  \hspace{1cm} (62)

In the same manner, $V_i$ and $W_i$ for $i = 2, 3$ are obtained by using (56) and (57), respectively. Notice that the second term $abc_{\min} s(\Delta q)^T Q_s(q) s(\Delta q)$ on the right-hand side of (37) or (39) must be included in $W_{c2}$ or $W_{c4}$, respectively. In CASE (IV), there is no inner product on the right-hand side in (39); however, it is regarded that there is the inner product of zero and $F(g(q), u, f)$ as follows:

$$0^T F(g(q), u, f) = \frac{dV_{ci}}{dt} = 0,$$  \hspace{1cm} (63)

where

$$V_{c4} = \frac{c}{2} \| u \|^2 : \text{constant.}$$  \hspace{1cm} (64)

Then, $W_{c4}$ becomes

$$W_{c4} \geq a c_{\min} \|s(\Delta q)^T Q_s(q) s(\Delta q)\|.$$  \hspace{1cm} (65)

It is confirmed that $V_{ci}$ for $i = 1, \ldots, 4$ is positive definite when the conditions (47) are satisfied, and $V_i$ in the first equation in (17) is also positive definite, since $V_o$ is positive definite under (44). $W_i$ for $i = 1, \ldots, 4$ in the second equation in (17) is also positive definite under (45), (46), (48) and the following condition:

$$a I - \alpha \left[ \frac{1}{2} \Delta B_0 + \xi \| \dot{J}_s(q)^T J_s(q) \| \right] > 0.$$  \hspace{1cm} (66)
The above results imply that we have the following two states according to the saturation cases:

\[ V_i \geq V_{i} > 0, \ \dot{V}_i < 0, \quad \text{for} \ i = 1, 2, 3 ; \ \text{state 1} \]  
\[ V_i > 0, \ \dot{V}_i < 0. \ \quad \text{state 2} \]  

where \( V_i \) is switched according to saturation cases (I) through (IV). Based on Lemma 1, we have the following main theorem.

**Main Theorem (Anti-windup PID control law):** Assume that conditions (44) through (46) are satisfied. Choose feedback gain \( a \) as the positive real number such that the conditions (47), (48) and (66) are satisfied. Adopt the switching function as (55) through (58) for the control law (51). Then, the equilibrium state of the closed-loop system is globally asymptotically stable based on Lemma 1.

**Proposition (Practical anti-windup PID control law):** Assume that the conditions in the Main Theorem are satisfied. For the control law (51), choose the function \( \tilde{f}(t) \) as follows:

\[ \tilde{f}(t) = \tilde{f}_1(t) + \alpha \tilde{f}_2(t), \]  

where

\[ \text{if} \ \dot{q}_j \psi(u_j) \geq 0, \ \text{then} \ \tilde{f}_1 = \dot{q}_j, \]  
\[ \text{if} \ \dot{q}_j \psi(u_j) < 0, \ \text{then} \ \tilde{f}_1 = 0, \]  
\[ \text{if} \ \{ s(\Delta q)Q_\phi(q) \}_{\psi(u_j) \geq 0} > 0, \ \text{then} \ \tilde{f}_2 = \{ s(\Delta q)Q_\phi(q) \}_{\psi(u_j) < 0} > 0, \]  
\[ \text{if} \ \{ s(\Delta q)Q_\phi(q) \}_{\psi(u_j) < 0} = 0, \ \text{then} \ \tilde{f}_2 = 0, \]  

for \( i = 1, \ldots, n \), where \( \{ \cdot \}_i \) denotes the \( i \)-th component of the vector, \( \tilde{f}_k \) denotes the following vector component:

\[ \tilde{f}_k = [\tilde{f}_{k1}, \ldots, \tilde{f}_{ki}, \ldots, \tilde{f}_{kn}]^T \]  

and \( k = 1, 2 \).

Then, the equilibrium state of the closed-loop system is globally asymptotically stable.

### 4. CONTROL PERFORMANCE

The control performance of the proposed anti-windup PID controller is examined by numerical simulations using a two-link robot arm. A schematic two-link robot arm with contact force is depicted in Fig. 1, where \( L_1 \) and \( L_2 \) denote the length of the arm. \( L_0 \) is the coordinate value of \( x_1 \) and the start point of the constraint surface. In this system, the constraint surface is represented as (75) and (1) becomes (76).

\[ x_2 = a_1 x_1 + b_1, \quad L_0 \leq x_1, \]  
\[ \phi = x_2 - a_1 x_1 - b_1 = 0, \]  

where \( a_1 \) and \( b_1 \) are real constants for the constraint surface.

The system equation with actuators and the detail of the two-link robot system have been described in (48) through (51) and TABLE 1 in Kanamori (2013b), respectively. Output saturation function \( s(x_i) \) in (23) has also been shown in (52) in Kanamori (2013b). Input limits \( \pm 2 \) V are given for the shoulder joint input and \( \pm 1 \) V are given for the elbow joint input. The following values are used for the simulations.

\[ f_d = 10 \text{ N}, \quad \xi = 0.3 \text{ Nm/s}, \quad \gamma_M = 6.38, \]  
\[ \alpha = 3.0, \quad a = 10.0, \quad b = 2.0, \quad c = 5.0, \quad \beta = 1.0. \]

Fig. 2 shows the control performance using the normal PID controller. For the strict input limits, the windup phenomenon is observed at elbow joint input \( u_2 \), as shown in the upper plots in Fig. 2. As a result, considerable overshoots occur in shoulder joint angle \( q_1 \) and elbow joint angle \( q_2 \), as shown in the lower plots in Fig. 2. As shown in Fig. 3, the contact force also degrades for the input saturation. In contrast, the windup phenomenon is restrained by using the proposed anti-windup PID controller, as shown in the upper plots in Fig. 4. As a result, considerable overshoots are restrained very well.
Fig. 4 Control performance using the proposed controller

Fig. 5 Contact force using the proposed PID controller

Fig. 6 Confirmation plot of conditions (32), (47) and (48)

as shown in the lower plots in Fig. 4. As shown in Fig. 5, the control performance of the contact force is improved in comparison with that in Fig. 3. Fig. 6 shows the validity of conditions (32), (47), and (48). The top plot shows the left-hand side of (32), where the right-hand side is transposed to the left-hand side. The middle and the lower plots correspond to the left-hand sides of (47) and (48), respectively. Since these values are positive or zero, it is confirmed that conditions (32), (47), and (48) are satisfied. These simulation results validate the analysis and design of the proposed anti-windup PID controller.

5. CONCLUSIONS

An anti-windup PID controller design for Euler-Lagrange systems under holonomic endpoint constraints has been presented. The control performance was examined by numerical simulations using a two-link robot arm with contact force. The simulation results validate the analysis and design of the proposed anti-windup PID controller.

REFERENCES


