Spacecraft Parameter Estimation by Using Predictive Filter Algorithm

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Abstract: An approach to estimate spacecraft system parameters is proposed in this paper. Oftentimes, spacecraft is under uncertainties of the system as well as internal and external disturbances which have different aspects each other. The moment of inertia of spacecraft is unknown especially when it is under considerable fuel consumption or equipped with deployable structures. This study aims to estimate the moment of inertia of the spacecraft body as well as its attitude and angular rate by using predictive filtering algorithm. Crassidis and Markley developed a predictive filtering algorithm for nonlinear estimation under a large system model error. This approach focuses not on the specific sources of model error described in the equations of motion but the resultant model error vector driving errors on the system dynamics. Therefore, we are not able to update the system model since the estimated model error has no additional information about the system. This paper establishes a method to apply the predictive filtering algorithm for nonlinear spacecraft parameter estimation by defining a new model error vector for parameters. This study shows different sources of the model error can be separated for estimation, and reveals excellent estimation results. Proposed algorithm is verified by numerical simulation studies.

1. INTRODUCTION

Conventional linear filtering algorithms such as a Kalman filter have shown their excellent usefulness in wide applications even under various noises and uncertainties (Crassidis et al., 2007, Pittelkau, 2001). The most common one, the Kalman filter, addresses uncertainties and noises of a system by the combination of the process and the measurement noises. This approach simplifies how to treat all the results by many sources of errors into adjusting the covariances of the process and measurement noises appropriately according to the accuracy of the system information. In order to utilize this simplicity, the process and measurement noises are assumed to be zero-mean Gaussian. In actual practices, however, the zero-mean Gaussian assumption of noises is far from its real features. There is variety of non-Gaussian uncertainties attributing to the filter performance degradation such as nonlinearity, the mass moment of inertia uncertainty, actuator and sensor misalignments, internal and external disturbance torques, control time delay, etc. In order to compensate the effects of uncertainties, parameter estimation as well as state estimation has been performed by the extended Kalman filter (EKF) (Iwasaki and Kataoka, 1989). The EKF propagates the estimates by the nonlinear system model while the measurements still updates the estimates based on the linearized model.

On the other hand, linear and nonlinear model errors have also been addressed from a control standpoint, by which system state variables successfully track and converge to the given references. Yoon and Tsiotras (2005) took into consideration of actuator misalignment, and designed Lyapunov function based adaptive control with a parameter adaptation rule. Singla et al. (2006) established adaptive output feedback control for spacecraft rendezvous and docking under system and measurement uncertainties. Both studies have verified the algorithms to show good tracking results of states. The parameter estimations, however, are not guaranteed to converge to their true ones. In these papers, parameter estimation is a bypass for state regulation or tracking control.

The predictive filtering algorithm was proposed by Crassidis and Markley (1997a, b), who were hinted at an idea of estimating model error vector from the study of Mook and Junkins (1988). Also, the predictive filter is based on the concept of duality with the predictive control initially suggested by Lu (1994). We can predict the contribution of the unknown model error to the states or the outputs by the Taylor series approximation to the differential relative degree between the model error and the states or outputs. Reversely, we can determine the model error minimizing predictive states or output estimation error for given measurements. The nonlinear approximation is known to show higher performance than the EKF does, even though it is not proved yet. The last outstanding character is the predictive filter can be implemented in real time as the EKF. Meanwhile, this notion does not make use of any zero-mean Gaussian assumption of model error present in the equations of motion. The model error may be in any form. This is one of the major advantages of the predictive filter.

Estimating and including the model error is one way of compensating the unmodeled error effects. Therefore, the model error estimate acts like an additional control input vector in the system model dynamics. The system model, however, is not updated so that we cannot extract any useful information for other purposes. This can be a large drawback.
since a separate estimator is needed in case we want to exploit 
an updated system model.

This study applies the extension technique of the extended Kalman filter which can be modified easily for unknown parameter estimation, to the predictive filter algorithm. As a result, the model error vector is augmented by the length of the parameters to be estimated. In the extended Kalman filter, the key role is upon the initial covariance matrices of the augmented states while the predictive filter has it on the weight matrix of the quadratic cost function corresponding to the model error of the parameter.

This article introduces the equations of motion of spacecraft in the following section. In section 3, follows the predictive filter algorithm and the application to parameter estimation. Numerical simulation results show the validity of the new method in section 4.

2. EQUATIONS OF MOTION OF SPACECRAFT

The total angular momentum of spacecraft, \( H \), is defined by the product of the moment of inertia and the body angular velocity.

\[
H = J \omega
\]

where \( \omega \in R^3 \) is the angular velocity of the spacecraft, and \( J \) is the mass moment of inertia matrix of the spacecraft body expressed in the body frame, respectively.

The equation of motion is

\[
\dot{H} + \omega \times H = L
\]

where \( L \in R^3 \) is the total external torque exerted on the spacecraft. Assuming no external disturbance torque, and substituting (1), we can rewrite (2) as

\[
\dot{\omega} = -J^{-1} \left[ \omega \times (J \omega) \right] + J^{-1} u,
\]

where \( u \in R^3 \) is the control torque from a attitude control actuator. It can be driven by the angular acceleration of a momentum wheel. In this paper, however, we assume that the angular momentum of the wheel is negligible with respect to that of the spacecraft body.

The attitude kinematics can be expressed by attitude quaternion such as

\[
\dot{q} = -\frac{1}{2} \Omega(q) q = -\frac{1}{2} \Xi(q) \omega
\]

where

\[
q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = \begin{bmatrix} q_{13}^T \mid q_4 \end{bmatrix}^T,
\]

and,

\[
\Omega(q) = \begin{bmatrix} -\omega \times & \omega & 0 \\ -\omega & 0 & \omega \end{bmatrix}, \quad \Xi(q) = \begin{bmatrix} q_1 I_3 + [q_{13}]_x \\ -q_{13} \end{bmatrix}.
\]

We can express the equations of motion of (3) and (4) in a general form:

\[
\dot{x}_i = f_i(x) + g_i(x) u.
\]

Definitely,

\[
f_i(q, \omega) = \frac{1}{2} \Omega(q) \omega - \frac{1}{2} \Xi(q) \omega
\]

\[
f_2(\omega, p) = -J^{-1} \omega \times J \omega
\]

and

\[
g_i(x) = J^{-1}.
\]

where \([\alpha \times]\) is a matrix indicating the cross product such as

\[
[\alpha \times] = \begin{bmatrix} 0 & -\alpha_1 & \alpha_2 \\ -\alpha_1 & 0 & -\alpha_3 \\ -\alpha_2 & \alpha_3 & 0 \end{bmatrix}.
\]

3. PARAMETER ESTIMATION

3.1 Predictive Filtering Algorithm

In this section, we briefly introduce the predictive filter algorithm derived by Crassidis and Markley (1997b). Let’s consider a nonlinear system model given by

\[
\dot{x}(t) = f(x(t)) + G(x(t)) d(t)
\]

\[
y(t) = c(x(t))
\]

where \( x(t) \in R^n \), \( d(t) \in R^r \), and \( y(t) \in R^m \) are the state estimate vector, the model error vector, and the estimated output vector, respectively. The measurements are assumed to be

\[
\tilde{y}_k = c(x(t_k)) + v_k
\]

where \( \tilde{y}_k \in R^m \) is the measurement vector at time \( t_k \), and \( v_k \) is assumed to be a zero-mean, Gaussian white-noise distributed process with

\[
E[v_k] = 0
\]

\[
E[v_k v_k^T] = R \delta_{ij}
\]

where \( R \in R^{m \times m} \) is a positive definite covariance matrix.

The predictive output estimate at time \( t + \Delta t \) is approximated by the Taylor series expansion to the relative degree \( \rho_i \) in terms of differentiation of each component with respect to the model error.

\[
\tilde{y}(t + \Delta t) \approx \tilde{y}(t) + \sum_{i} \frac{\rho_i}{i!} d^i \tilde{y}(t)
\]

Or,
where the \( i \)th element of \( z(\hat{x}(t), \Delta t) \) is

\[
z_i(\hat{x}(t), \Delta t) = \sum_{j=0}^{\Delta t} \frac{\Delta t^j}{k!} L_j^i(c_i)
\]

with the Lie derivative of

\[
L_j^i(c_i) = \left\{ \frac{\partial L_{j-1}^i(c_i)}{\partial \hat{x}} \right\} f \quad \text{for } k \geq 1.
\]

\( z(\hat{x}(t), \Delta t) \) is a nonlinear approximation term to the order of the relative degree, and \( D(\hat{x}(t), \Delta t) \) is understood to be the 1st order approximation of generalized sensitivity of the model error with respect to the predictive output estimate. \( D(\hat{x}(t), \Delta t) \) is to be obtained in the following section.

A cost function is defined by weighted combination of the square of predictive output estimate error and the square of the model error vector, which is expressed as

\[
J = d^T(t)Wd(t) + \frac{1}{2} \left[ \hat{y}(t + \Delta t) - \hat{y}(t + \Delta t) \right]^T R^{-1} \left[ \hat{y}(t + \Delta t) - \hat{y}(t + \Delta t) \right] \quad (12)
\]

where \( W \in \mathbb{R}^{n \times n} \) is a positive semidefinite weight matrix. Since the cost \( J \) is quadratic with respect to \( d(t) \), we can find the model error estimation minimizing the cost by substituting (11) into (12) and \( \partial J / \partial d(t) = 0 \):

\[
\hat{d}(t) = -D(\hat{x}, \Delta t) R^{-1} D(\hat{x}, \Delta t) + W \right]^{-1} \times D(\hat{x}, \Delta t) R^{-1} \left[ z(\hat{x}, \Delta t) - \hat{y}(t + \Delta t) + \hat{y}(t) \right] \quad (13)
\]

The predictive filter works like this: if the measurement \( \hat{y}(t + \Delta t) \) is given at time \( t + \Delta t \), the model error estimation \( \hat{d}(t) \) is produced by (13), which is used to propagate the system state at time \( t + \Delta t \) during which the model error is assumed constant. The predictive filter is considered to be given the word ‘predictive’ because the algorithm takes into consideration of distribution of model error to the output after the time interval \( \Delta t \).

The weighting matrix in (12) can be determined on the basis that the covariance of the measurement minus estimate error must match that of the measurement minus truth error. This condition is referred to as the covariance constraint as

\[
\frac{1}{m_{\text{tot}}} \sum_{i=1}^{m_{\text{tot}}} \left[ \hat{y}(t_i) - \hat{y}(t_i) \right]^T \left[ \hat{y}(t_i) - \hat{y}(t_i) \right] \approx R \quad (14)
\]

where \( m_{\text{tot}} \) is the total number of measurement data.

### 3.2 Spacecraft Parameter Estimation

Aforementioned predictive estimation is for state estimation purpose. Therefore, let’s modify the equations of motion in (3) for parameter estimation. Define the estimation \( \hat{p}(\in \mathbb{R}^p) \) assuming an unknown parameter for the spacecraft. Then, dynamics of the parameter estimate is assumed to be a new model error \( d_z(\in \mathbb{R}^p) \):

\[
\begin{align*}
\dot{x}_i &= f_i(\hat{x}) \\
\dot{z}_i &= f_i(\hat{x}, \hat{p}) + g_i(\hat{x}, \hat{p})u + d_i \\
\dot{\hat{p}} &= d_z
\end{align*}
\quad (15)
\]

where \( \hat{p} \) is a parameter vector to be estimated. Since the first equation of (15) represents kinematics of the system, we may assume that the parameter appears only in dynamics equation.

Now the problem is to obtain the cost minimizing model error vector \( d = [d_z^T, d_z^T] \in \mathbb{R}^{n+p} \). In the formulation of (15), the parameter estimation error and the measurement noise are expected to be separated properly. The measurement equation is same as (9):

\[
\hat{y}_k = c(x(t_k)) + v_k 
\quad (16)
\]

Let’s denote the relative degrees of the output with respect to the model error \( d \) as \( \rho_i^j \), then,

\[
\rho_i^j = \rho_i + 1,
\]

which means the predictive output and the parameter model error have a relationship such as

\[
\dot{\hat{y}}(t + \Delta t) \approx \hat{y}(t) + \sum_{i = 0}^{\Delta t} d_i \dot{\hat{y}}(t).
\]

Or,

\[
\dot{\hat{y}}(t + \Delta t) \approx \hat{y}(t) + z(\hat{x}(t), \Delta t) + D(\hat{x}(t), \Delta t)d(t) \quad (17)
\]

where the \( i \)th element of \( z(\hat{x}(t), \Delta t) \) is

\[
z_i(\hat{x}(t), \Delta t) = \sum_{j=0}^{\Delta t} \frac{\Delta t^j}{k!} L_j^i(c_i)
\]

with the Lie derivative of

\[
L_j^i(c_i) = \left\{ \frac{\partial L_{j-1}^i(c_i)}{\partial \hat{x}} \right\} f \quad \text{for } k \geq 1.
\]

The cost function is defined same as (12) except the weight matrix of the model error vector is of \( (q + p) \times (q + p) \) dimension.

Since the cost function is still quadratic with respect to the augmented model error, the cost minimizing model error estimation is finally obtained in the same form of (13):
\[
\dot{d}(t) = -\left[D(\dot{x}, \Delta t)Y R^{-1}D(\dot{x}, \Delta t) + W\right]^{-1}
\times D(\dot{x}, \Delta t)Y R^{-1}\{z(\dot{x}, \Delta t) - \dot{y} + \dot{y}(t)\}
\] (18)

3.3 Spacecraft Moment of Inertia Estimation

The moment of inertia (MOI) matrix is a representative parameter in spacecraft attitude dynamics. In this section, we solve the above formulation for the MOI estimation problem. We define a parameter vector such as

\[
p = [J_{11}, J_{22}, J_{33}, J_{12}, J_{13}, J_{23}]^T.
\]

The equations of motion are rewritten as

\[
\dot{q} = \frac{1}{2} \Omega(\dot{\omega}) \dot{q} = \frac{1}{2} \Xi(\dot{q}) \dot{\omega}
\]
\[
\dot{\omega} = -J^{-1}(\dot{\omega}x \dot{\omega} + \dot{J}^{-1}u + d_i)
\]
\[
\dot{p} = d_t
\] (19)

In this paper, we consider both the quaternion and the angular rate as measurements. The relative degrees \(\rho\) of both outputs are 3 and 2 with respect to the parameter model error. The predictive output estimates in (17) are expressed as

\[
\hat{q}(t + \Delta t) = q(t) + \Delta t \left\{ \Omega(\omega) + \frac{\Delta t}{4} \Omega^2(\omega) + \frac{\Delta t^2}{3} \left(\frac{1}{8} \omega^2 \omega \right) \Omega(\omega) \left\} q \right.
\]
\[
+ \frac{\Delta t^2}{4} \left( \Xi(q) + \frac{\Delta t}{3} \left[ \frac{1}{2} \Xi(q) \omega \omega \omega \omega - \omega \omega \omega \omega + \Xi(q)F_2\omega \right] \right\} (f_2 + gw) \}
\] (20)
\[
+ \frac{\Delta t^2}{4} \left[ \Xi(q) \left( F_{2p} + G_p \right) d_2 \right]
\]
\[
\omega(t + \Delta t) = \omega(t) + \Delta t \left\{ I_3 + \frac{\Delta t}{2} F_{2w} \left\} (f_2 + gw) \right.
\]
\[
+ \Delta t \left\{ I_3 + \frac{\Delta t}{2} F_{2u} \right\} d_2 + \frac{\Delta t^2}{2} \left( F_{2p} + G_p \right) d_2
\] (21)

The newly introduced terms are defined by

\[
F_{2w} = \left[ \frac{\partial f_2}{\partial \omega}, \frac{\partial f_2}{\partial \omega_1}, \frac{\partial f_2}{\partial \omega_2}, \frac{\partial f_2}{\partial \omega_3} \right] = J^{-1} \{(J \omega \omega) - \omega \times J\},
\]
\[
F_{2p} = \left[ \frac{\partial f_2}{\partial \omega}, \frac{\partial f_2}{\partial \omega_1}, \frac{\partial f_2}{\partial \omega_2}, \frac{\partial f_2}{\partial \omega_3} \right]
\]
\[
G_p = \left[ \frac{\partial g}{\partial \omega}, \frac{\partial g}{\partial \omega_1}, \frac{\partial g}{\partial \omega_2}, \frac{\partial g}{\partial \omega_3} \right]
\]

where

\[
\frac{\partial f_2}{\partial \omega} = J^{-1} \left( \frac{\partial J}{\partial \omega} J^{-1} \omega \times J \omega - \omega \times \frac{\partial J}{\partial \omega} \omega \right),
\]
\[
G_p = \left[ \frac{\partial g}{\partial \omega}, \frac{\partial g}{\partial \omega_1}, \frac{\partial g}{\partial \omega_2}, \frac{\partial g}{\partial \omega_3} \right]
\]

where

\[
\frac{\partial g}{\partial \omega} = -J^{-1} \frac{\partial J}{\partial \omega} J^{-1}.
\]

\(\partial J / \partial \omega\) is a 3\times3 matrix whose \((j,k)\) and \((k,j)\) components corresponding to \(\rho_j = J_{jk}\) are unity and others zero. \(I_3\) means 3\times3 identity matrix.

Equations (20) and (21) are rewritten in the form of (11) by

\[
\hat{q}(t + \Delta t) \approx \hat{q}(t) + \hat{z}(q, \omega, J, u) + D_q d_1 + D_{qq}d_2
\]
\[
\dot{\omega}(t + \Delta t) \approx \dot{\omega}(t) + \hat{z}(\omega, J, u) + D_{\omega}d_1 + D_{\omega\omega}d_2
\]

Or,

\[
\hat{y}(t + \Delta t) \approx \hat{y}(t) + z + Dd
\]

where \(z = [z_1, z_2, \ldots]^T\), and

\[
D = \begin{bmatrix}
D_{q1} & D_{q2} \\
D_{\omega1} & D_{\omega2}
\end{bmatrix}.
\]

Now, we can find the model error estimation \(\hat{d}(t)\) by the solution in (18).

4. SIMULATION STUDIES

Shown in this section are the simulation results for parameter estimation by predictive filtering algorithm. First, we need to select an attitude controller due to the control signal appearing in (19). In this paper, an estimation maneuver is assumed.

For simulation purposes, applied is a predictive controller for spacecraft attitude control by Crassidis \textit{et al.} (1997). Given reference trajectories to follow, the predictive control synthesizes control command based on nonlinear state prediction strategy using the Taylor series expansion.

4.1 Reference maneuver

In order to estimate the inertia matrix, so called ‘persistent excitation’ should be guaranteed. A constant body rate vector or one with constant direction will not satisfy this requirement. As one of the reference trajectories satisfying the ‘persistent excitation’ condition (Pittelkau, 2001), following rate trajectory is proposed:

\[
\omega = \phi l - (1 - \cos \phi) l + l \sin \phi,
\] (22)

where

\[
\phi = 50 \pi t (rad),
\]
\[
l = \begin{bmatrix}
\sin \omega t \sin \omega t \\
\cos \omega t \sin \omega t \\
\cos \omega t \end{bmatrix},
\] (23)
\[
\omega_1 = 0.01 rad/s, \omega_2 = 0.04 rad/s.
\]

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The reference trajectory in (22) is shown in Fig. 1. The overall maneuver time is 10min. The quaternion reference trajectory is given as in Fig. 2 by integrating (4). Taking the attitude as the measurement of the system as well as the body angular velocity, we utilize both for generating control input signals.

\[
R = \begin{bmatrix}
10^{-5} I_x & 0_{4	imes 3} & 10^{-4} I_z \\
0_{3	imes 4} & 10^{-4} I_z & 0_{3	imes 3} \\
5\times 10^4 I_3 & 0_{3	imes 3} & 5\times 10^{-4} I_3
\end{bmatrix},
\]

Initial attitude and rate errors are also adopted in this simulation:

\[
\Delta q_0 = [0.1925, -0.1925, 0, 0.9623]^T, \\
\Delta \omega_0 = [0.01, 0, 0.02]^T (rad / s)
\]

Fig. 3 to Fig. 5 shows the simulation result. The attitude quaternion and the body angular rate are filtered to have smoother histories. Large initial estimation error diminishes in 5 min. We see the MOI estimation error converge in Fig. 5. All the parameter components of the moment of inertia have its error less than 1kgm/s² at the end of simulation. But, the measurement noise is not fully filtered. Comparing the state convergence, the angular rate error converges faster than that of quaternion. The relative error with respect to the measurements is also better for the angular rate than for the quaternion. This is the same result as one by Singla (2006).

Fig. 1. Body angular rate reference trajectory

Fig. 2. Quaternion reference trajectory

4.2 Simulation results

Simulation is performed for 10min. Measurement sampling is given every 0.2s. The time interval of prediction \( \Delta t \) is 0.2s. The true moment of inertia and the initial estimate are assumed as

\[
J_{true} = \begin{bmatrix}
160 & -50 & -30 \\
-50 & 200 & -20 \\
-30 & -20 & 180
\end{bmatrix}, \\
J = \begin{bmatrix}
170 & -25 & -15 \\
-25 & 175 & -35 \\
-15 & -35 & 190
\end{bmatrix} (kgm / s^2).
\]

The measurement noise covariance and the weighting matrix are

Fig. 3. Quaternion measurement and estimation errors

Fig. 4. Body angular rate measurement and estimation errors
5. CONCLUSIONS

The predictive filtering method is applied to parameter estimation of spacecraft. Time derivative of parameters is assumed to be a model error vector, and the system model is augmented by the length of the parameter to be estimated as in the extended Kalman filter. As an example, estimation problem of the moment of inertia is solved and its simulation is performed. Parameter estimation error converges to zero within the bound of $1 \text{kgm}^2/s^2$. But, the measurement noise is not fully filtered by only the PF algorithm, and quaternion estimate should be enhanced further. Newly proposed method can extend application of the predictive filtering.

REFERENCES


