Robust Engine Torque Control by Discrete Event Disturbance Observer

Takashi Nagata*, Masayoshi Tomizuka**

*Dept. of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, U.S.A. (e-mail: taka_me@me.berkeley.edu)
**Dept. of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, U.S.A. (e-mail: tomizuka@me.berkeley.edu)

Abstract: A model-based approach is applied to robustly control torque generation in four-stroke spark ignition (SI) engines. Discrete event engine model (DEM) is adopted to describe the torque generation process consisting of discrete combustion strokes. Disturbance observer (DOB) is utilized to achieve robust stability and performance of the torque generation process. For a single-input, single-output model with throttle air intake as input and generated torque as output, the desired plant behavior is stably realized by the DOB over a desired frequency band which is sufficient for powertrain control applications. Numerical and experimental results show the effectiveness of the proposed DOB scheme.

1. INTRODUCTION

Torque based integrated control of engine and powertrain has become an important topic of research in view of the increasing demand for improved driving capability and comfort. Direct and fast-response measurement of engine torque has become available by in-cylinder pressure sensors (e.g. Ulrich et al., 2001). The gross engine torque generated inside a cylinder, also known as indicated torque, is obtained by numerically integrating the measured in-cylinder pressures in real time. By utilizing these new technologies, this paper examines a model-based approach to control the engine torque output by adjusting the throttle air intake with robustness considerations. Disturbance observer (DOB) is utilized to achieve robust stability and performance of the torque generation process.

An important application of engine torque control is smooth gear shifting. A previous study (Nagata, Hur and Tomizuka, 2006) presents a collaborative scheme between engine control and automatic transmission (AT) gearbox control. For a given gear shifting from a certain gear position to another in a conventional automatic transmission gearbox, one can obtain an engine torque reference profile for the engine torque controller as well as hydraulics actuations reference profiles for the AT gearbox controller. By each individual controller conducting its own tracking control with respect to individually provided reference profiles, smooth torque and speed controls are realized at the wheel so that the driver and passengers feel little shock i.e. no abrupt changes in acceleration from longitudinal motion (Fig. 1). An example of engine torque reference profile for 1st-to-2nd gear shifting is shown in Fig. 2. The profile prescribes engine torque generation over a short duration in the order of 100ms since a short duration is preferred for gear shifting. This highlights the need for engine torque tracking with sufficiently fast response. Therefore, a model-based approach is under focus in this study.
2. ENGINE MODELING

2.1 Discrete event engine model

The dynamics that are important to power generation in engines are intake manifold air dynamics, fuel injection dynamics, and crankshaft dynamics. The mean value engine model (MVEM) (Hendricks and Sorenson, 1990) describes these dynamics by a set of three continuous time non-linear dynamical equations. However, an actual engine combustion process consists of discrete event stages such as intake, compression, combustion, and exhaust stages in a four-stroke engine. Therefore, a discretized model is more suitable in reflecting the discrete event nature of engine into design and implementation of a fast-response engine controller aiming to control torque generation in a stroke-by-stroke timescale.

For the above purpose, a discrete event engine model (DEM) is introduced (Guzzella and Onder, 2004). It is derived from piecewise integrations of the MVEM equations in crankshaft angle domain. The DEM is indexed by \( k \) for each 180 deg. crankshaft angle increment representing each stroke.

**Fuel injection dynamics:**

\[
\begin{align*}
    m_{f,k+1} &= Fm_{f,k} + Km_{\pi,k} \\
    m_{f,k} &= (1-F)m_{f,k} + (1-K)m_{\pi,k}
\end{align*}
\]

(1)

where \( m_{f,k} \) and \( m_{\pi,k} \) are the fuel masses injected to the intake port and flowed into the cylinder, respectively, within a 180 deg. angle segment between indices \( k-1 \) and \( k \). \( m_{f,k} \) is the mass of the fuel puddle on intake port wall at index \( k \). \( F \) is a parameter derived from the fuel evaporation time constant \( \tau_e \) in the MVEM, and it also depends on the engine speed \( \omega_e \) as \( F = \exp(-4/\omega_e\tau_e) \). \( K \) is a fuel adhesion parameter that represents the rate of injected fuel deposited to the fuel puddle, hence \( 0 \leq K \leq 1 \). \( K \) is unchanged from the MVEM.

**Intake manifold air dynamics:**

\[
m_{a,k+1} = \frac{1}{1+X} m_{a,k} + \frac{X}{1+X} m_{at,k+1}
\]

(2)

where \( m_{a,k} \) and \( m_{at,k} \) are the air masses flowed through the throttle and into the cylinder, respectively, within a 180 deg. segment between indices \( k-1 \) and \( k \). \( X \) is a parameter derived from the volumetric efficiency \( \eta_{vol} \) in the MVEM. For a 4 cylinder engine, \( X = \eta_{vol}(\omega_e, P_{man}) \times \lambda_{\text{air}} \text{ where } P_{man} \text{ is the intake manifold pressure, } V_d \text{ is the engine displacement, and } V_{man} \text{ is the volume of the intake manifold.} \)

**Torque generation dynamics:**

\[
T_{\text{gen},k} = \frac{H_u}{\pi} \eta_i (\lambda, \theta, \omega_e) m_{f,k-d}
\]

(3)

where \( T_{\text{gen},k} \) is the average torque generated within a 180 deg. segment between indices \( k-1 \) and \( k \). \( H_u \) is the specific enthalpy of fuel, \( \eta_i \) is the thermal efficiency of the engine, \( \lambda \) is air-fuel ratio, and \( \theta \) is spark timing angle. \( H_u \) and \( \eta_i \) are unchanged from the MVEM. The delay step \( d \) in the discrete event index of the \( m_{f,k-d} \) term represents the combustion delay \( \tau_e \) in the MVEM into the discrete event framework.

One important observation about the DEM is that the resulting model can be regarded as a linear dynamical system where the nonlinearities in the MVEM are absorbed in the time-varying system parameters of the linearized dynamics.

2.2 SISO LTI discrete event model

It is noted that in this engine torque control study, it is expected that air-fuel ratio (AFR) is kept close to the stoichiometric value 14.7. This means that in order to increase torque generation, an appropriate amount of air must be added corresponding to the amount of fuel, implying that air dynamics must play a key role in realizing quick response for desired torque generation. From this context, a SISO model is used in this study to describe dynamics from the throttle angle to the generated torque. In order to connect the output of the intake manifold air dynamics and the input of the fuel injection dynamics, a simple air-fuel ratio (AFR) regulator is applied as \( m_{f,k} = \frac{1}{14.7} m_{a,k} \).

A further approximation is applied to treat the DEM dynamics (1) ~ (3) as linear time invariant (LTI) systems by regarding the system parameters \( F, K, \) and \( \eta_i \) to be constant. This assumption is practically justified if the parameters are slowly varying with respect to crankshaft angle increment index \( k \). Such cases include when engine speed is varying slowly, or when the behavior of interest is in transient dynamics only and thus the scope of time is short.

Following the arguments above, a LTI SISO model from air mass at the throttle \( m_{at,k} \) to generated torque \( T_{\text{gen},k} \) is derived from the DEM in the transfer function form as shown below:

\[
G(z) = \frac{T_{\text{gen}}(z)}{m_{at}(z)} = Cz^{-d} \left( 1 - \frac{1}{1+\lambda} z^{-1} - \frac{F}{1+\lambda} z^{-2} + F \frac{1}{1+\lambda} z^{-3} \right)
\]

(4)

where \( C = \frac{H_u}{\pi} \eta_i \frac{1-K}{14.7} \frac{X}{1+X} \) and \( z \) represents a one-step advance of discrete steps where each step corresponds to 180 deg. crankshaft angle increment.

2.3 System identification with discrete event model

To test the validity of the LTI DEM, a simple parametric system identification is conducted for the fuel injection dynamics and the intake manifold air dynamics with an actual engine (Toyota 2AZ-FE). The measured quantities are the air mass intake per stroke at the throttle \( m_{at,k} \) and the air mass per stroke into the cylinder \( m_{f,k} \), which is derived from the measurements of intake manifold pressure \( P_{man} \). The fuel injection mass \( m_{f,k} \) is measured in terms of fuel injection duration, in unit \( \mu s \), which is an input command given to the fuel injector. Finally, the air-fuel ratio (AFR) is measured from the exhaust from which the fuel mass inside the cylinder...
is derived using \( m_{a,k} \) above. \( m_{a,k} \) and \( m_{f,k} \) are used for the identification of the intake manifold air dynamics under the DEM framework using least square methods. Likewise, \( m_{f,k} \) and \( m_{a,k} \) are used for another identification of the fuel injection dynamics under the DEM with the same methods.

The dynamics are identified through analyses of step input responses at engine speeds of around 1500 rpm. For the intake manifold air dynamics, it is obtained that the parameter \( X \) is about 0.07 i.e. the volumetric efficiency \( \eta_{vol} \) is about 0.8, which is an adequate value for the tested naturally aspirated engine. For the fuel injection dynamics, the parameter \( F \) is obtained to be about 0.7 which corresponds to fuel evaporation time constant \( \tau_f \) of about 0.2, an appropriate value for the gasoline fuel at normal intake manifold pressure and temperature. The fuel adhesion ratio \( K \) is obtained to be about 0.8 i.e. about 80% of the injected fuel goes to the fuel puddle and remaining 20% goes directly into the cylinder.

It is noted that for the fuel injection dynamics, there exist cases that are interpreted as non-minimum phase dynamics for some values of parameters \( F \) and \( K \). This is derived from (4) with its LTI approximated transfer function model:

\[
G_F(z) = (1 - K) + K \frac{1 - F}{z^4 - F} = (1 - K) \frac{1 - F + \frac{F}{z^4}}{1 - F z^4}
\]

Notice that the unstable zeros exist when

\[
\left| \frac{F - K}{1 - K} \right| > 1
\]

This condition is satisfied with large \( K \) values i.e. close to 1, and small \( F \) values. Application of the system identification results of \( K \) and \( F \) stated above shows that the observed fuel injection dynamics exhibit minimum phase characteristics. However, it should also be noted that a slightly different value of \( K \) (e.g. 0.9) results in a non-minimum phase dynamics, indicating that such a case can be an actual matter of concern when considering this model-based approach.

### 3. ROBUST CONTROL BY DISTURBANCE OBSERVER

#### 3.1 Key modeling uncertainties

Model discrepancies exist between the LTI DEM engine and the actual engine, no matter how extensively system identification is conducted. This is because the actual plant is in fact non-linear, and hence the LTI DEM is an approximation that is valid under certain limiting conditions.

The key modeling uncertainties that appear in the DEM are the parameters \( F, K, X, \) and \( \eta \). Of these, \( X \), which is related to volumetric efficiency \( \eta_{vol} \), and \( F \), which comes from fuel evaporation time constant \( \tau_f \), both represent natural physical properties of the plant. On the other hand, the fuel adhesion parameter \( K \) can vary with intake valve timing settings, and the thermal efficiency \( \eta \) is heavily dependent upon spark timing settings. In practical engine control schemes, the intake valves and spark timings are manipulated to guide the engine characteristics toward a desired performance i.e. a nominal plant of the engine. Robust control techniques are therefore essential in compensating for model uncertainties and realizing the desired nominal plant with intended fuel dynamics and energy conversion characteristics.

#### 3.2 Disturbance observer for realizing nominal plant

In order to cancel the discrepancy between the nominal plant with desired plant characteristics and an actual engine plant, disturbance observer (DOB) technique (Ohnishi, 1987 and White et al., 2000) is applied. The DOB scheme regards the discrepancy as an equivalent disturbance input, which is estimated and cancelled. Figure 3 shows the structure of DOB. It has been adapted to discrete event dynamics which is assumed for both nominal and actual plants.

![Disturbance observer for the discrete event engine.](image)

The dynamics from the air mass intake at the throttle \( m_{at,k} \) to the generated torque \( T_{gen,k} \) is represented as \( G_{DOB} \):

\[
T_{gen}(z) = G_{DOB}(z)m_{at}(z)
\]

where \( G_{DOB} \) has the following expression derived from the DOB structure in Fig. 3:

\[
G_{DOB} = \frac{G_{at}G_{nom}}{(1 - Qz^{-1})G_{nom} + Qz^{-d}G_{act}}
\]

In (8), \( G_{act} \) is the actual plant and is not necessarily LTI, and \( G_{nom} \) is the nominal plant that has the LTI DEM expression:

\[
G_{nom}(z) = C_{nom} z^{-d} \frac{1 - \frac{F_{nom} - K_{nom}}{X_{nom}} z^{-4}}{1 + \frac{1}{14.7 X_{nom}} z^{-1} - \frac{F_{nom}}{X_{nom}} z^{-4} + \frac{1}{1 + X_{nom}} z^{-5}}
\]

where \( C_{nom} = \frac{H}{\pi} \eta_{nom} \frac{1 - K_{nom}}{14.7} \frac{X_{nom}}{1 + X_{nom}} \). The subscript "nom" attached to the parameters \( F, K, X, \) and \( \eta \) represent the desired nominal parameter values. \( Q(z) \) is LTI and is called \( Q \) filter. It is a design parameter for the DOB and is normally chosen to be of minimum phase. When \( Q(z)z^{-d} \approx 1 \),

\[
G_{DOB} \approx G_{nom}
\]

i.e. the nominal plant is virtually realized at frequencies where the \( Q \) filter has magnitude close to unity. Once the nominal plant is realized up to a sufficient bandwidth, a tracking controller for desired engine torque reference profiles is designed based on the nominal plant characteristics. The advantage of this scheme is that one can effectively rule
out model uncertainty considerations in the design phase of the tracking controller since the DOB takes care of compensations for effects due to model uncertainties.

3.3 Feasibility of the disturbance observer loop

Assume that the actual plant \( G_{\text{act}} \) is expressed as:

\[
G_{\text{act}} = G_{\text{nom}} (1 + \Delta)
\]

(11)

where \( \Delta \) represents the multiplicative model uncertainty dynamics. \( \Delta \) is not necessarily LTI since \( G_{\text{act}} \) is not LTI. Using \( \Delta \), \( G_{\text{DOB}} \) takes another expression:

\[
G_{\text{DOB}} = \frac{G_{\text{act}}}{1 + Q z^{-d} \Delta}
\]

(12)

In this study, the actual plant \( G_{\text{act}} \) is assumed to be stable by inspection of the actual engine in its air intake and torque output relationships. Then, depending upon the stability properties of \( \Delta \), the following results are obtained:

If \( \Delta \) is stable:

From the small gain theorem, the DOB loop is stable if \(|Q| < 1\). This is valid even when \( G_{\text{act}} \) is non-linear, hence, \( \Delta \) is also non-linear. The condition above implies that the \( Q \) filter must roll off at high frequencies to suppress model uncertainties that exist in shorter transient dynamics.

If \( \Delta \) is unstable:

This case occurs when the desired nominal plant \( G_{\text{nom}} \) is prescribed to have non-minimum phase characteristics, for instance, its fuel injection dynamics being non-minimum phase by a particular choice of the nominal fuel adhesion parameter \( K_{\text{nom}} \). Assuming that the actual plant \( G_{\text{act}} \) is well approximated by an LTI \( G_{\text{act}}(z) \) of the form of (4), it is stated from the Nyquist plot of \( Q(z)z^{-d} \Delta(z) \) and the Nyquist stability criterion that a high frequency passing \( Q \) filter may stabilize the DOB (Nagata and Tomizuka, 2007).

It should however be noted that a nominal model with non-minimum phase dynamics is not a practical choice of DOB design. This is because (a) the DOB approach requires \( G_{\text{nom}}^{-1} \) so it is favorable that \( G_{\text{nom}} \) is of minimum phase, and (b) there exists increased difficulty in controlling non-minimum phase behavior in the resultant \( G_{\text{DOB}} \) constructed with a non-minimum \( G_{\text{nom}} \). Therefore, the selection and use of a non-minimum phase nominal model should be avoided.

4. EVALUATIONS OF MODEL MATCHING

4.1 Minimum phase requirement for the nominal plant

Here, numerical tests are conducted to examine what nominal values for the fuel adhesion parameter \( K \) i.e \( K_{\text{nom}} \) are feasible choices to realize a DOB that satisfies robust stability and performance requirements. Following the previous section, the desired nominal plant \( G_{\text{nom}} \) should be of minimum phase. Therefore, the permissible value range of \( K_{\text{nom}} \) is given as:

\[
K_{\text{nom}} \in \left[ 0, \frac{1 + F_{\text{nom}}}{2} \right]
\]

(13)

from the existence condition of an unstable zero given by (6).

4.2 Performance requirement for torque control

The performance requirement for engine torque control in this study is derived from the engine torque reference profile for smooth gear shifting mentioned before. By inspection of the desired torque profile for 1st-to-2nd gear shifting in Fig. 2, the profile is assumed to have significant frequency contents up to 1.25 Hz. This is equivalent to discrete time frequency of 0.05\( \pi \) rad at a sampling rate of 50 Hz, which corresponds to the frequency of each stroke i.e. increment of index \( k \), at an engine speed of 1500 rpm. Since the 1st to 2nd gear shifting is assumed to occur at around that engine speed, it is appropriate to set the bandwidth requirement for the torque controller scheme as \( \omega_{\text{bandwidth}} = 0.05 \pi \).

It is noted that a quick drop in the engine torque reference profile (Fig. 2) is expected to be realized by faster responding spark timing control, whereas a more gradual increase of torque must be achieved by coordinated air and fuel control which the DOB based torque control intends to achieve.

4.3 Bode plot analysis of model matching and stability

Bode plot analysis has been performed to evaluate the model matching performance of the DOB loop \( G_{\text{DOB}} \) for realizing the desired nominal plant \( G_{\text{nom}} \). Here, the actual plant \( G_{\text{act}} \) is assumed to be \( \text{LTI} \) and has the transfer function expression:

\[
G_{\text{act}}(z) = C_{\text{act}} z^{-d} \frac{1 - \frac{p_{\text{act}} - K_{\text{act}}}{1 + K_{\text{act}}} z^{-4}}{1 - \frac{1}{1 + X_{\text{act}}} z^{-3} - F_{\text{act}} z^{-4} + F_{\text{act}} \frac{1}{1 + X_{\text{act}}} z^{-5}}
\]

(14)

where \( C_{\text{act}} = \frac{H_u}{\pi}, \eta_{\text{act}} = \frac{1 - K_{\text{act}}}{14.7 + 1 + X_{\text{act}}} \), and the parameters \( F_{\text{act}}, K_{\text{act}}, X_{\text{act}} \) and \( \eta_{\text{act}} \) are assumed to include parametric uncertainties. \( H_u \) is normalized to 1 without loss of generality, and \( d \) is set to 2 to reflect the two strokes delay from intake to combustion. The desired \( G_{\text{nom}} \) is \( \text{LTI} \) and has the transfer function expression in (9).

Figure 4 shows the variations of the Bode magnitude plots for \( G_{\text{act}}(z) \)'s when \( K_{\text{act}} \) is perturbed between 0 and 1 with a step size of 0.1. Other parameters are fixed as \( X_{\text{act}} = 0.07 \), \( F_{\text{act}} = 0.7 \), and \( \eta_{\text{act}} = 0.4 \). The dotted vertical lines represent the required bandwidth \( \theta_{\text{bandwidth}} = 0.05 \pi \). The plots of \( G_{\text{act}}(z) \)'s spread about 10dB at \( \theta_{\text{bandwidth}} \) when \( K_{\text{act}} \) is perturbed.

Figure 5 shows the corresponding Bode plots for \( G_{\text{DOB}} \)'s with \( K_{\text{nom}} = 0.5 \) while \( K_{\text{act}} \) is perturbed similarly. Other nominal parameters are \( X_{\text{nom}} = 0.07 \), \( F_{\text{nom}} = 0.7 \), and \( \eta_{\text{nom}} = 0.4 \). The \( Q \) filter for the \( G_{\text{DOB}} \)'s is chosen as a low pass filter \( Q(z) = \frac{1}{1 - 0.7 z^{-1}} \) that maintains unit magnitude for \( \omega \) up to about \( \theta_{\text{bandwidth}} \) at 0.05\( \pi \).
5. EVALUATIONS OF CONTROL STRATEGY

5.1 Evaluation by simulation

The DOB based scheme is tested with a simple control structure (Fig. 7). The DOB \( G_{DOB} \) aims to virtually realize the desired nominal plant \( G_{nom} \), which is prescribed to have minimum phase characteristics. Therefore, one can readily implement a feed-forward block prior to \( G_{DOB} \) which is in essence a stable inverse of \( G_{nom} \). A delay factor \( z^{-\phi} \) may be applied to the feed-forward block to resolve acausality issues in implementing \( G_{nom}^{-1} \), although this measure is unnecessary if the desired engine torque profile is computed offline.

Figure 8 shows the simulation results with the control structure above. The control structure is given a desired nominal plant \( G_{nom} \). The unit of torque is arbitrary in this simulation. Fuel adhesion parameter \( \eta_{act} \) is constructed with the nominal fuel adhesion \( K_{act} \) as well as thermal efficiency parameter \( \eta_{nom} \) is perturbed in the simulation. The desired engine torque profile is computed offline. Though still under study, it is expected that this strategy is beneficial in realizing more detailed balancing between robust performance and stability.

5.2 Experimental evaluation

The DOB scheme is tested on an actual engine setup. First, the actual engine torque measurement is compared with the nominal plant generated torques for the same air mass intake at the throttle. Generated torque measurements \( T_{gen,k} \) are...
obtained from in-cylinder pressure sensor readings and are shown in Fig. 9 together with measurements of air mass intake at the throttle $m_{at,k}$. The engine is first set operating at 1500 rpm with the throttle angle at 5 degrees. Then the throttle angle is increased to 7.5 degrees in a stepwise manner at $k$ (crankshaft angle increment index) about 520 in Fig. 9.

![Fig. 9: Actual $m_{at,k}$ (top) and actual $T_{gen,k}$ (bottom) measured from an actual engine setup.](image)

Figure 10 shows the $T_{gen,k}$'s from both the actual and nominal plants. The nominal plant in this case has the nominal fuel adhesion parameter $K_{nom} = 0.5$ and the nominal thermal efficiency $\eta_{nom} = 0.46$. The trends of two $T_{gen,k}$'s match closely, except during the onset of the rise in $m_{at,k}$ when the actual $T_{gen,k}$ takes higher values than nominal plant output.

![Fig. 10: $T_{gen,k}$ from actual and nominal plants plotted for $k = 0 \sim 2000$ (top), and enlarged for $k = 500 \sim 600$ (bottom).](image)

The actual $T_{gen,k}$ is then converted into equivalent input by an inverse nominal plant $G_{nom}^{-1}$. Figure 11 shows the equivalent input together with the actual input $m_{at,k}$. The difference between the two plots corresponds to the equivalent disturbance input, the detection of which is one of the primal objective of the DOB. Figure 11 shows a particular peak in equivalent input that is localized during the rise of the actual input $m_{at,k}$. This indicates that the DOB scheme is effective under the tested conditions with the actual engine setup.

![Fig. 11: Comparing equivalent input with actual input $m_{at,k}$.](image)

6. CONCLUSIONS

A disturbance observer (DOB) has been constructed based on the discrete event model (DEM) of a four-stroke 4 cylinder engine. Model matching performance of the DOB scheme has been tested under perturbations in fuel adhesion properties. It has been shown that the DOB scheme realizes desired plant characteristics up to certain bandwidth required for smooth gear shifting control while maintaining stability. A permissible range of choice for the desired nominal plant has also been shown from the stability analysis of the DOB structure. Both numerical and experimental evaluations show that the DOB scheme is promising for realizing robust engine torque control. Further verification studies are in progress.

ACKNOWLEDGMENTS

This work was supported by Toyota Motor Corporation. The authors would like to express their gratitude to Mr. Akira Ohata, Dr. Tomoyuki Kaga and Mr. Noriyasu Adachi of Toyota for valuable technical discussions.

REFERENCES


