Flooding Prevention of the Demer River using Model Predictive Control

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Abstract

In order to prevent flooding along the Demer in Belgium, the local water administrations has installed hydraulic structures and flood reservoirs along the river. Though these actions have reduced the damage and frequency of flooding events, simulations have shown that an improved regulation could further decrease the flood risk. In this study a real time control procedure is being developed by means of model predictive control. For this purpose a full hydrodynamic model of the river basin has been simplified and a conceptual river model built in order to limit the model computational times. Afterwards a model predictive controller is built and used for flood control. A comparison is made between the performances of the model predictive controller and the currently used controller.

1. INTRODUCTION

Flooding of rivers are worldwide the cause of great economic losses. This is also the case along rivers in Belgium. This study focuses on the Demer river, where severe floodings have occurred in the past during periods of heavy rainfall. In order to prevent these flooding events the local water administration installed hydraulic structures (with movable gates) in order to control the discharges in the river systems. Extra storage capacity for periods of heavy rainfall was provided through flood control reservoirs. Structures to control the flow from and into the reservoirs were also installed. At this moment, the hydraulic structures are controlled by a three position controller. The three position controller determines the control actions of the gates based on some very simple rules. The main disadvantage of these rules is that they are only based on the current measurements of the water levels but don’t take the future rain predictions into account. This causes the control actions to be far from optimal. The local water administration has verified that the damage of past flooding events could have been significantly reduced and even completely avoided if different control decisions would have been applied than the ones obtained by the three position controller. The main objective of the study presented in this paper is to come up with a better control strategy.

1.1 Model Predictive Control

In this work the control strategy to be investigated is model predictive control (MPC) ([Camacho],[Rossiter]). MPC is a control technique that in the past decennia has become more and more popular in the process industry. Because the dynamics of river systems are relatively slow, because to prevent flooding input and state constraints need to be considered and because future rain predictions need to be taking into account model predictive control is a suitable control strategy in order to solve the flooding problem. In the literature several studies can be found in which automatic control techniques are used to control a river system ([Brian et al.],[Burt et al.],[Litrico et al.]). A good overview of the different controllers can be found in [Malaterre(1998)]. There are also several studies available in which a MPC is used to control river systems ([Overloop(2006)],[Rutz et al.],[Rodellar et al.]). These works however have as main goal to control the different water levels to some desired target value and not to prevent flooding. In these applications it is usually sufficient to linearize the system around the desired steady state value in order to obtain good results. This simple linearization does however not work when trying to avoid flooding. The main reason is that during periods of heavy rainfall the complete nonlinear dynamics of the system are excited. So in this application it is really important to use a MPC that is capable of taking the nonlinear model behaviour into account. In the sequel of the paper such a MPC will be discussed. As to the authors knowledge, there are no works published in which MPC is used in order to avoid flooding, with exception from [Thai(2005)]. However, in [Thai(2005)] the nonlinear behaviour introduced by the presence of the gates is not considered which is a strong simplification of the actual problem.

1.2 Modelling

MPC is a control paradigm that needs the model of the system in order to determine the optimal control inputs. So the first step in any application in which an MPC will be used, is to determine an appropriate model of the system.
2. MODELLING

In figure 1 a schematic overview is shown of the river system in the study area. The local water administration has an accurate full hydrodynamic model of this river system generated in the software package InfoWorks [OBM-Demer(2003)]. Because of the high computational times and because it is not straightforward to establish a software coupling with this software package, it was necessary to derive a simplified conceptual model, implemented in an own software (Matlab). The conceptual model is calibrated based on simulation results with the detailed InfoWorks model for 2 historical floods. Since this work is the first step towards the use of MPC for flooding the focus was limited to control only the part indicated on the figure by the (red) circle. A more detailed view of this part is depicted in figure 2. The river system considered in this work consists of 10 states (three water levels, four discharges and three volumes) and three inputs. The outputs of the system are the three water levels. The water level upstream is hopw, the water level downstream is hafw. There is one flood control reservoir that can be used to control the water levels during heavy rainfall; its water level is hs. There are three gates that need to be controlled by the controller, namely A, D and K7. There are two disturbance inputs qman en qopw to model the inflow of rain storm water(by means of smaller rivers) in the river system.

The conceptual model derived here is of the reservoir type. The equations are determined according to the methodologies described in [Vaes et al.(2002)]. The resulting model is a discrete time model with a simulation time step of 1 hour.

3. CONTROLLER DESIGN

MPC has a typical structure that can cope with all issues related to controlling a river system. The main issues are the following:

(1) The calculation time of a MPC controller limits its use to control systems with relatively slow dynamics. Because river systems have slow dynamics MPC can be applied to them.

(2) The gates of the control structures in the water system have some important physical limitations that have to be taken into account. The gates have upper and lower limits that can never be violated in reality. There is also a restriction on the speed of the gate movement as the gates cannot move infinitely fast in real time. MPC is capable of taking both constraints into account.

(3) In order to prevent flooding it is necessary to impose upper limits on the different water levels. In an MPC framework this can be imposed by means of constraints.

(4) Taking the rainfall predictions into account is a very important issue when trying to prevent flooding. MPC is capable of taking this into account by modelling the rainfall as a known disturbance input into the system.

(5) The model of the considered river system in this work is nonlinear. MPC is well suited to tackle this.

In the remainder of this section the principles of MPC will be explained as well as the implementation for the case study.
3.1 Principles of MPC

MPC is a control strategy that uses the model of the system in order to make future predictions on which an optimal input sequence is determined in order to minimize an objective function. The three basic components of MPC are the following:

1. A process model is used to determine the future outputs within a time window with length N, the prediction horizon. These future outputs are determined by the future control actions and the current state of the system.

2. An objective function is minimized. The objective function is typically a quadratic function that tries to minimize the water level errors and the gate movement by adjusting the unknown control inputs. Typically the objective function is also subject to constraints.

3. Once the sequence of future control actions that minimizes the desired objective function is determined, only the first set of control actions is implemented on the system. The system is then updated by measuring (estimating) the new state of the system and the process is repeated.

3.2 Implementation of MPC

The best known MPC is the linear MPC in which the process model is a linear time invariant system. This may seem restrictive but since in most control applications the goal is to steer the output to some predefined reference output and keep it there, linear MPC seems to work very well in practice. Furthermore, most nonlinear MPC strategies for nonlinear process models are based on linear MPC. Therefore, in the following a further outline of the linear MPC will be given and afterwards will be discussed how to extend this to come to the nonlinear MPC used in this work.

The linear time variant state space system of interest in this work has the following form:

\[ x(k + 1) = A_k x(k) + B_k u(k) + D_k d(k), \]
\[ y(k + 1) = C x(k + 1). \]

with \( x(k) \) the state vector of the system at time \( k \), \( u(k) \) the input vector (gates) at time \( k \), \( d(k) \) the disturbance vector (rainfall) at time \( k \), \( y(k) \) the output vector (water levels) of the system at time \( k \), \( A_k, B_k \) and \( D_k \) time variant system matrices and \( C \) a time invariant system matrix. Now, assume

\[ Y_p^T = [y_{k+1} \ y_{k+2} \ \cdots \ y_{k+N}] \]

with \( Y_p \) the vector containing the future outputs. Taking the equations of the linear time variant system into account it is possible to calculate the future outputs as follows:

\[ Y_p = G x_k + H u + J d \]

with the matrices \( G, H \) and \( J \) constant matrices determined by the time variant model. The second component of MPC is the objective function to be minimized. The objective function typically has the following form:

\[ \min_u \|Y_p^T(u) - Y_r\|_Q + \|u - u_r\|_R \]

with \( \|x\|_Q = x^T Q x \)

\( Y_r \) the desired output references, \( u_r \) the desired input references and \( Q \) and \( R \) positive definite symmetrical cost matrices.

By taking (4) into account the cost function can be written as a function of the unknown input vector \( u \) and initial state \( x_k \), this leads to a quadratic objective function which together with the constraints imposed to the system leads to the following (constrained) quadratic program (QP) that has to be solved at each time instant:

\[ \min_u u^T (H' Q H + R) u + 2 (u_k^T G' Q H + d^T J' Q H - Y_r^T Q H - u_r^T R) u \]
\[ A u \leq b \]  

(5)

In this work the process model is not a linear time variant but a highly nonlinear one. However, the results of the linear time variant system can be used in order to solve the control problem with the nonlinear process model by means of the following steps:

1. Simulation of the nonlinear model within the prediction horizon \( N \) with the inputs obtained by solving the QP in the previous time instant. This leads to a trajectory of future states.

2. At each time instant within the prediction horizon a linearization around the simulated future states is done. The linearization in this work was done iteratively by use of forward differences. The linearization gives rise to different linear systems at each sampling time which are the characteristics of a linear time variant system.

3. The QP (5) related to this linear time variant system is solved and a sequence of optimal inputs is obtained.

4. The previous steps are repeated with the recently computed optimal input sequence until convergence or until time runs out. After convergence the first input is applied to the system, the systems gets an update and the MPC strategy is repeated.

In literature [Allgöwer et al.] it has been shown that this procedure converges to a local minimum of the nonlinear control problem. In this work this procedure was used in order to obtain the results presented in section 4.

3.3 Local uncontrollability

An important problem that arises when controlling the gates is local uncontrollability. The river model discussed in section 1.2 is hybrid. The system consists of several modes that each have their own discharge equation. In some modes the discharge equations are independent from the regulating gate. In this situation the row of the \( B_k \)
matrix in (1) corresponding to that gate is a zero row. Such a mode renders the gate locally uncontrollable which can lead to serious complications as the MPC controller will not be capable of determining a suitable control action for that gate.

An example of an uncontrollable mode is the mode where the gate is much higher than its adjacent water levels. A small movement of the gate doesn’t change the discharge over it as the discharge stays equal to zero causing the gate to be locally uncontrollable. Most of the time gates regulating the discharges to the reservoirs are in this mode. Such gates only become controllable if the water level before the gate rises above the gate position by which a change to a controllable mode is realized. But waiting until the water level rises above the gate position before the MPC controller can control the gate can turn out to be too late for avoiding flooding. Similar uncontrollability issues arise in other modes.

In order to deal with this important problem the matrices $A_k$, $B_k$ and $D_k$ of the time variant system (1) are calculated on basis of a fuzzified model of the conceptual model. This fuzzified model is exactly the same as the conceptual model in the controllable modes. In the uncontrollable modes the equations are fuzzified in order to make them controllable. Suppose $f_i^j(x, u)$ is the discharge equation for uncontrollable mode $i$ in the fuzzified model and $f_i^j(x)$ the discharge equation for the same mode in the conceptual model then:

$$f_i^j(x, u) = f_i^j(x) + \alpha f_i^j(x, u)$$

(6)

with $f_i^j(x, u)$ a modified version of model equation $f_i^j(x, u)$ from the controllable mode $j$ and $\alpha$ a tuning parameter. Remark that mode $j$ is the mode adjacent to mode $i$, meaning that the system can switch from mode $i$ to mode $j$ without having to enter any other mode first. By doing this all the modes are controllable but the drawback is that predictions of the MPC controller are less accurate. The role of the tuning parameter $\alpha$ is to reduce this inaccuracy of the predictions but without losing the gained controllability. Also remark that the modified equation $f_i^j(x, u)$ is modified in such a way that it is a logical extension to $f_i^j(x, u)$. For example, in case of the uncontrollable mode where the gate is much higher than its adjacent water levels, suppose the adjacent controllable mode has the following equation

$$q = f_1(x, u)$$

(7)

then the fuzzified model is as follows:

$$q_z = 0 - \alpha f_1(x, u).$$

(8)

The logical extension in equation (8) is ensured due to the minus. In order to see this suppose the gate is lowered until the system is in the controllable mode where equation (7) holds. By raising the gate the discharge will decrease until at some point the discharge is equal to zero. The logical extension of further raising the gate is that the discharge switches from sign and increases in absolute value, even if in reality the discharge will stay equal to zero if the gate is raised more. This is important because by this logical extension the MPC controller knows that if the discharges over the valve should be positive, the gate should be lowered. Without the minus an increase in discharge in the fuzzified model could also be achieved by further raising the gate, causing the MPC controller to make wrong decisions.

### 3.4 Constraint and cost function strategy

The local water administration has some specific desires that should be achieved as close as possible during normal periods. However, during periods of heavy rainfall these desires change and the focus shifts more to flood prevention. This means that the objective function and constraints should change also during operation. Another reason for introducing a variable cost function and variable constraints is whenever the QP (5) turns out to be infeasible. Constraints with less priority will be discarded and their corresponding weights in the cost function will be modified. In the following this constraint and cost function strategy is discussed in more detail.

#### Constraints

The constraints in this work are all linear and can be formulated in the following way

$$A u \leq b$$

(9)

with

$$A = \begin{bmatrix} A_u & A_{\Delta u} \\ A^*(\rho, j) \\ \end{bmatrix}, \quad b = \begin{bmatrix} b_u \\ b_{\Delta u} \\ b^*(\rho, j) \\ \end{bmatrix}$$

(10)

and

$$A^*(\rho, j) = \begin{bmatrix} A_u^*(\rho) \\ \vdots \\ A_j^*(\rho) \\ \end{bmatrix}, \quad b^*(\rho, j) = \begin{bmatrix} b_u^*(\rho) \\ \vdots \\ b_j^*(\rho) \end{bmatrix}$$

(11)

with $\rho \in \{1, \ldots, n_\rho\}, j \in \{0, \ldots, n_\rho\}, [A_u^*(\rho) \ b_u^*(\rho)] = [\ ]$, $[A_u \ b_u \ b_{\Delta u} \ \Delta u] \Delta u$ upper bounds on the inputs, $[A_u \ b_u \ b_{\Delta u} \ \Delta u]$ upper bounds on the maximal gate movement and $[A^*(\rho, j) \ b^*(\rho, j)]$ constraints on the future water levels of the system. Remark that the constraints $A^*(\rho, j)$ $b^*(\rho, j)$ are variable in size depending on the value of $j$. For $j = 0$ the constraints $[A^*(\rho, j) \ b^*(\rho, j)]$ are empty. The variable $j$ defines a priority in the sense that if $j_1 \leq j_2$ then constraints $[A^*(\rho, j_1) \ b^*(\rho, j_1)]$ have a higher priority than constraints $[A^*(\rho, j_2) \ b^*(\rho, j_2)]$. The variable $\rho$ corresponds to a condition on some water levels. Basically the constraints $[A^*(\rho, j) \ b^*(\rho, j)]$ define an upper bound on the water levels. In this work there are 3 types of upper bounds namely surveillance levels, alarm levels and flood levels, by which $n_\rho = 3$. The actual value of $\rho$ depends on the value of the water levels downstream and upstream. During normal operation and small rainfall events $\rho = 1$ and the constraints $[A^*(\rho, j) \ b^*(\rho, j)]$ define surveillance levels. If the water levels downstream and upstream increase further $\rho \geq 2$ and the constraints $[A^*(\rho, j) \ b^*(\rho, j)]$ define alarm levels. By a further increase of the water levels $\rho = 3$ and the constraints define flood levels. If the flood level constraints get violated, flooding occurs. Note that the conditions on the water levels defining the switching of the value of $\rho$ are provided by the local water administration based on their experience and specific desires.

Now assume $x_p(k+i)$ the $p$th component of the state $x$ at time step $k+i$ and $n_x$ the state dimension and the first $n_w$
state components corresponding to the water levels. For a given \( j \) the constraints
\[
A^\ast_j (\rho) b^\ast_j (\rho)
\]
constrain the states \( x_p(k + i), \) for \( i = 1, \ldots, N \) and \( p \in S_j \subset \{1, \ldots, n_w\} \).
Remark that the indices in \( S \) only correspond to water levels of the state vector. So for a given \( j \) each \( [A^\ast_j (\rho) b^\ast_j (\rho)] \) bounds only a subset of all the future water levels. Remark that water levels related to constraints \( [A^\ast_j (\rho) b^\ast_j (\rho)] \) with a lower value for \( j \) are more important and therefore have a higher priority. Also note that if \( j_1 \neq j_2 \) then \( S_{j_1} \cap S_{j_2} = \emptyset \).

Cost Function

The cost function can be split into two parts
\[
J(x, u) = J(x(u)) + J(u)
\]
with \( J(x(u)) \) the cost related to the states and \( J(u) \) the cost related to the inputs. On his turn \( J(x(u)) \) can also be split as
\[
J(x(u)) = J^1(x(u)) + J^2(x(u))
\]
with
\[
J^1(x(u)) = \sum_{p \in S_j} \sum_{i=1}^{N} q_{\rho, p}^{\ast} (x_p(k + i) - x_p^0)^2
\]
\[
J^2(x(u)) = \sum_{p \in S_j} \sum_{i=1}^{N} q_{\rho, p}^{\ast} (x_p(k + i) - x_p^0)^2
\]
for \( j \in \{0, \ldots, n_p\} \). \( x_p^0 \) corresponds to the reference level for the corresponding state component \( p \). Remark that the cost function depends on the values of \( \rho \) and \( j \) and that \( \rho \) and \( j \) are the same variable as used in the explanation of the variable constraints. Also note that \( J^1(x(u)) \) contains the cost of the state components constrained by \( [A^\ast (\rho) b^\ast (\rho)] \) and \( J^2(x(u)) \) the cost of the unconstrained state components. Initially when the variable \( j = n_p \) the cost function \( J^2(x(u)) = 0 \). In case the constraints are too restrictive and lead to an unfeasible QP, the variable \( j \) is updated from \( j_{old} \) to \( j_{new} \) meaning that less important state components get unconstrained. The expression of the cost of these less important state components then migrates from \( J^1(x(u)) \) to \( J^2(x(u)) \), so the constrained components aren’t weighted anymore in \( J^1(x(u)) \). Also because the less important components get unconstrained, their corresponding weights increase meaning that for a certain component \( p \), \( q_{\rho, p}^{\ast} (\rho, j_{new}) > q_{\rho, p}^{\ast} (\rho, j_{old}) \).

Strategy

Solving the QP (5) is equivalent to solving the following QP:
\[
\min_{u} J(x, u)
\]
subject to the linear constraints (9). Under normal conditions this QP has a feasible solution. However, during heavy rainfalls, the QP can turn out to be unfeasible. In this situation constraints with a lower priority are removed from the constraint set which is equivalent to decreasing the variable \( j \) in (11). The variable \( j \) keeps getting decreased until the QP has a feasible solution. An overview of the constraint and cost function strategy is given in the following algorithm:

**Algorithm 1.**

1. Determine the value of \( \rho \) based on the current water levels.
2. Set the value of \( j \).
3. Based on the values for \( \rho \) and \( j \) compose the constraints \( [A(\rho, j) b(\rho, j)] \).
4. Solve the following QP
\[
\min_{u} J(x, u)
\]
subject to the constraints \( [A(\rho, j) b(\rho, j)] \).
5. • If the QP is feasible, then the optimal inputs are obtained and the inputs corresponding to the current time step are applied to the system.
6. • If the QP is unfeasible, go to the next step in the algorithm.
7. Set the value of \( j \) to
\[
j := j - 1
\]
8. Update the cost function taking the new value of \( j \) into account.
9. Update the cost function taking the new value of \( j \) into account and ensuring for the unconstrained state components that \( q_{\rho, p}^{\ast} (\rho, j) > q_{\rho, p}^{\ast} (\rho, j + 1) \) and go back to the step where the QP is solved.

Remark that this strategy ensures a feasible solution because in the worst-case \( j = 0 \) and only input constraints are imposed which can always be satisfied. In case of infeasibility the strategy ensures the more important constraints to be satisfied (if possible) and the less important ones to be violated as less as possible.

4. EXPERIMENT

The main objective of this experiment is to compare the current three position controller with the MPC controller. A three position controller is a controller that consists of some very simple logical rules based on the current water levels of the system in order to decide the control action to be implemented. Both controllers are compared by a simulation based on the flood event of 1998. In the following some important details of the simulation are discussed:

1. In practice the gates of the river system change each 15 minutes. This time is close to the upper limit of the computational time of the QP optimization.
2. Another very important remark is that in this experiment it is assumed that the nonlinear model is perfectly known, that the rain predictions are perfectly known and that the current state of the system is exactly known at each time step. In practice, however, this is never the case. But as pointed out in section 5 future work will focus in taking these uncertainties into account.
(3) In figure 2 a schematical overview is depicted of the small river system controlled in this work. The output of the system are 3 water levels with the following flood levels:

- hopw ≤ 23.2 m
- hsw ≤ 23.2 m
- hafw ≤ 22.75 m

(4) Two constraints are related to the gates and which are considered as hard constraints, meaning they should always hold. The first gate constraint limits all the gate movements to 0.1 m/hour. The second gate constraints the upper and lower limits of the gate position and in this work they are the same for the three gates, namely the gate position should be between 20 m and 23 m.

(5) An important control objective is to steer hopw as close as possible to 21.5 m during normal operation. Concerning the filling of the reservoir it is preferred to keep its water level beneath 23 m. If more capacity is needed the reservoir can be filled up until 23.2 m before it floods. However, filling it up higher than 23 m leads to the flooding of farm land obliging the local water administration to give the farmers a financial recompensation.

(6) The rainfall data used in this experiment is based on the historical flood event of 1998. In the Demer basin this was the most severe flood event of the last century. The results of the experiment are depicted in figures 4 and 5. The figures show that during normal operation the MPC controller steers hopw much closer to its reference level. With the MPC controller there is almost no flooding. During the second rainfall peak only hafw violates its flood level for a short period of time. With the three position controller the flooding during the second rainfall peak is extremely big and all the water levels violate their flood level. The MPC controller clearly outperforms the three position controller.

5. CONCLUSION AND FUTURE WORKS

In this study a conceptual model for describing water levels along the Demer was developed. The resulting model was of the reservoir type. This model was used in order to compare the performance of the current three position controller with that of a model predictive controller. In order to make this comparison the historical flood event of 1998 was simulated. This simulation showed the MPC outperformed the three position controller.

Future work will focus on controlling the complete model of the Demer river instead of a small part of it. The complete model has much more states and is more nonlinear which will raise challenges concerning computational speed as well as stability of the QP’s to be solved. Also uncertainties in the model and the rainfall predictions will be taken into account.

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