Implementation of Optimal Decisions in the Presence of Uncertainty

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Abstract: In this work, the implementation of optimal and robust decisions in the presence of various uncertainties comprising the model parameters, external conditions and the closed loop behavior of basic controllers is presented. In order to compute optimal and reliable decisions, a chance constrained optimization problem is formulated. The efficient solution approach is based on the relaxation of the original stochastic problem formulation to a standard NLP problem. By this means, nominal optimal solutions are relocated in order to guarantee both feasibility and process operation as close to the true optimum as possible. The solution implicates the minimization of additional costs which result from conservative strategies so as to compensate for uncertainty. The experimental verification of the developed approach is carried out on a distillation pilot plant for the separation of an azeotropic mixture.

1. INTRODUCTION

Several approaches have been suggested to formulate and solve optimization problems under uncertainty, differing in how uncertainty is handled. Conventional deterministic optimization strategies treat uncertain variables by their expected values and the resulting nominal optimal decisions are not robust. The common procedure in industrial practice is to select an extremely conservative strategy. This implicates that the operation costs will be much higher than required and the strategy applied is far from the optimum. Accordingly it is desirable to include uncertainty explicitly in the problem formulation.

In general, the uncertainty nature and its significance determine the formulations of the objective function and the constraints. To treat the objective function in an optimization or control problem under uncertainty, minimizing the expected value and the variance of the objective function is usually adopted (Sahinidis [2004], Flemming et al. [2007]):

\[ \min E[f(x, u, \xi)] + \omega D[f(x, u, \xi)] \quad (1) \]

Here, \( x, u \) and \( \xi \) are state, decision and random vectors, respectively. \( E \) and \( D \) are the operators of expectation and variance, \( \omega \) is a weighting factor between the two terms.

Nagy and Braatz [2003] derive an analytical expression that provides an estimate of the distribution of the states and outputs as a function of time. For this purpose, the description of the probability distribution of the uncertain parameters is carried out by a bounded set and the worst case deviation is computed using a series expansion of the model equations. In Govatsmark and Skogestad [2005] the robustness of the operation of a plant is realized choosing the right control structure. By this means, the variation of the objective function value is minimized accounting for uncertainties.

The true process optimum for operation of chemical processes often lies on the boundary of the feasible region defined by active constraints. Thus, the process optimum and the set-points can be infeasible ignoring uncertainty in the parameters, the measurement and the implementation of optimal decisions. Here, the compliance with constraints on process outputs is more challenging than those on process inputs. Moreover, if uncertainty can not be compensated by feedback, the use of conservative margins, called back-off, is necessary to ensure feasibility (Srinivasan et al. [2003]). The back-off describes the distance between the nominal optimal solution and the correction adding a conservative distance. For constrained process states which are measured online, a direct back-off can be implemented and considered in the optimization problem (Barz et al. [2006], Govatsmark and Skogestad [2005], Visser et al. [2000]). Moreover, under the assumption that the set of active constraints is known a priori, an online measurement based optimization strategy can be implemented (Srinivasan et al. [2003]), which tracks the necessary conditions of optimality (NCO-tracking). In this way not only infeasibility but also a highly suboptimal operation can be avoided.

However, many variables in the engineering practice can not be measured on-line. These variables often represent the product quality and, thus, their control is desired. The same problem exists with predictive control applications. Here optimal and robust trajectories regarding the compliance of output constraints have to be calculated (Visser et al. [2000]). One approach for the consideration of uncertainties in constraints on output variables is to formulate chance or probabilistic constraints (Arellano-Garcia et al. [2007], Wendt et al. [2002]). Thus, it is required to evaluate the probability of violating these constraints at the decided operation point. Besides, they have to be satisfied with a user defined probability level.

\[ \Pr \{h_i(x, u, \xi) \leq 0, \quad i = 1, \cdots, m \} \geq \alpha \quad (2) \]
\[ \Pr_i \{h_i(x, u, \xi) \leq 0 \} \geq \alpha_i, \quad i = 1, \cdots, m \quad (3) \]
The value $\alpha \in [0,1]$ represents the probability level. Since $\alpha$ can be defined by the user, it is possible to select different levels and make a compromise between optimality of costs and the risk of constraint violation. Equation (2) denotes the simultaneous formulation (joint chance constraints) with $\alpha$ being a defined probability of holding all constraints at once, while (3) corresponds to the formulation of single probabilities ensuring each inequality separately. In this form, different confidence levels $\alpha_i$ can be assigned to different outputs based on their requirements.

For linear systems with uncertain parameters characterized by Gaussian PDF, different solutions have been proposed. Visser et al. [2000] presents a cascade scheme for the computation of a robust set-point trajectory where margins from constraints are introduced. Loeblein and Perkins [1999] and Schwarm and Nikolau [1999] use the information of the uncertainty directly in a model predictive control algorithm in order to guarantee feasibility concerning output constraints. In engineering praxis the process model $gM$ and thus the relations between uncertain parameters and output variables are highly nonlinear.

$$gM(x, u, \xi) = 0 \quad (4)$$

While the inputs have known stochastic properties, the output probability distribution is unknown. This issue is emphasized in Figure 1 for the vapour-liquid-equilibrium of a binary azeotropic mixture (acetonitrile-water). It is demonstrated that for variations in temperature $T$ in form of a Gaussian distribution, the output variable concentration $x_{Ac}$ shows a strongly non-symmetric behaviour which is not Gaussian. One possible way is to evaluate the integral over the uncertain output variable space by efficient sampling methods and an approximated integration Wendt et al. [2002]. An alternatively efficient approach is discussed in this paper.

1.1 Implementation of Robust Optimal Decisions in an hierarchical automation scheme

The development and implementation of optimal control strategies is normally achieved through a hierarchical system of layers (Govatsmark and Skogestad [2005]). In the optimization upper layer, decisions about the optimal process states with respect to various objectives are taken. The results are then sent in form of set-points to the regulatory control layer where the optimal strategies are implemented keeping the system state at the optimal operating point (Fig. 2). Overstepping existing constraints makes the operation infeasible, which not only means a loss of quality but also a safety risk. However, the inherent plant disturbance spectrum leads inevitably to controller performance deviations from their optimal set-points. To guarantee robust implementation of optimal decisions for all process states, the controller deviations and model uncertainties are required to be part of the control system i.e. to integrate implementation errors and stochastic parameters. By this means, effects of disturbances and model uncertainties can be compensated and robust set-points for the regulatory control layer are then obtained. Accordingly, the challenge of plant operation optimization lies in reducing the conservative distance to the constraints and pushing the plant to its limits without exceeding critical limitations in the presence of uncertainty.

2. CHANCE CONSTRAINED SOLUTION APPROACH

For this purpose, in this work, chance constrained optimization is proposed, i.e. the objective function (e.g. costs) is improved and the constraints are then to be satisfied with a predefined confidence level (chance constraints). Thus, the resulting robust decisions ensure the probability of satisfying constraints, i.e. the reliability of being feasible. The essential challenge lies then in the computation of the probabilities of complying with the constraints, and their gradients, which means a multivariate integration over the unknown PDF of the constrained output variables. In order to relax the stochastic optimization problem to a standard NLP problem two steps are required Wendt et al. [2002]. First, the constraints of the problem formulation are reduced to the inequality constraints only. This is realized following the sequential solution approach and solving the equality constraints (mostly model equations $gM$) in an extra simulation layer. Second, the chance constrained region of the outputs is mapped to a bounded region of the uncertain input variables with known PDF. The method relies upon the sufficient condition of a monotonic relationship between the constrained output variables $y_1 \in Y_1$ with $y_1 = h_i(x, u, \xi)$ and at least one uncertain input variable, here $\xi_s \in \Xi_s$ where $\Xi_s$ is a subspace of $\Xi$. So, for instance if $\xi_s \uparrow \Rightarrow y_1 \uparrow$, then:

$$\Pr \{y_1 \leq y_1^{max} \} = \int_{-\infty}^{y_1^{max}} \tilde{\rho}(y_1) dy_1 \quad (5)$$

![Fig. 1. Effect of controller deviations in Temperature (known input) on the concentration (unknown process state: Prediction based on a simple VLE-Model for acetonitrile/water.)](image)

![Fig. 2. Hierarchical process control scheme with basic regulatory control and optimization layer.](image)
\[ Pr \{ \xi_s \leq \xi_{s}^{\max} \} = \int_{-\infty}^{\xi_{s}^{\max}} \rho(\xi_s) d\xi_s \]

And for the multivariate case:

\[ Pr \{ y_i \leq y_{i}^{\max} \} = Pr \{ \xi_s \leq \xi_{s}^{\max} \} \quad (6) \]

\[ = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} \rho(\xi_1, \ldots, \xi_{s-1}, \xi_s, \xi_{s-1}^{\max}, \ldots, \xi_1^{\max}) d\xi_1 d\xi_2 \ldots d\xi_s \]

where \( \rho(\xi) \) is now the joint distribution function of \( \xi \).

The input boundary \( \xi_{s}^{\max} \) is computed through reverse projection based on the output value of \( y_{i}^{\max} \) in the simulation layer.

\[ \xi_{s}^{\max} = gM^{-1}(\xi_1, \ldots, \xi_{s-1}, y_{i}^{\max}, u) \quad (7) \]

The boundaries of the infinite integrals in (6) are chosen as a function of the standard deviation \( \sigma \), to \([-3\sigma, +3\sigma]\).

In principle, the solution approach can be used to solve problems under uncertainties with any kind of joint correlated multivariate distribution function, provided that the density function is available or can be approximated. The extension for the joint probabilities (2) is discussed in (Arellano-Garcia et al. [2006], Wendt et al. [2002]).

2.1 Illustrating example

A simple example with one uncertain parameter \( \xi \) is considered in problem (8). Whereas the objective function \( J \) is only a function of the decision variables \( u_1, u_2 \), the constraint dependent variables \( y_1 \) and \( y_2 \) are affected also by the uncertain parameter \( \xi \). The uncertainty \( \xi \) possesses a strong nonlinear influence on the constraints \( P_{y_1}, P_{y_2} \). Thus, a chance constrained optimization problem is formulated with the additional parameters \( \alpha_1, \alpha_2 \) defining the probability level of satisfying each constraint \( P_{y_1} \) and \( P_{y_2} \).

\[
\begin{align*}
\text{min} & \quad J(u_1, u_2) \\
\text{s.t.} & \quad 0.75 \leq u_1 \leq 1.75, \quad 1 \leq u_2 \leq 3.5 \\
& \quad P_{y_1} := Pr\{y_1 (u_1, u_2, \xi) \leq 0\} \geq \alpha_1 \\
& \quad P_{y_2} := Pr\{y_2 (u_1, u_2, \xi) \leq 0\} \geq \alpha_2 \\
& \quad \xi \sim N(\mu = 0, \sigma = 2/7)
\end{align*}
\]

The required monotonic relationship between both stochastic outputs and the uncertain parameter is fulfilled within the feasible region. In this example, a positive monotony between \( \xi \) and \( y_1, y_2 \) is given. Since all relations are given in an explicit form, the effect of \( \xi \) on the original constraints \( y_1 \) and \( y_2 \) can be plotted (Fig. 3). They are marked as straight and dashed lines for different values \( \xi = [-3\sigma : \sigma : +3\sigma] \). From Figure 3 can be seen that while the uncertain parameter \( \xi \) changes the exponential trait of the constraint \( y_1 \), for the quadratic constraint \( y_2 \) a term of third degree becomes active and the feasible area can become non convex for certain values of \( \xi \). The objective function is displayed by (dotted) isolines of \( J(u_1, u_2) \) and can be interpreted as heights with respect to the \( u_1-u_2 \)-plane. Before solving the chance constrained optimization problem, the stochastic variable \( \xi \) is replaced by its expected value, here \( \mu = 0 \). Doing so, the nominal optimal decisions can be obtained solving an deterministic optimization problem with the constraints:

\[ y_1(u_1, u_2, \xi = 0) \leq 0 \]

\[ y_2(u_1, u_2, \xi = 0) \leq 0 \]

For the nominal solution both output constraints are active and the minimum of \( J \) at \( u = [0.7, 2] \) can not be achieved (see Fig. 3). Moreover, if the stochastic nature of \( \xi \) is considered, a feasible operation can not be guaranteed.

The solution of the chance constrained problem formulation is given for different probability level \( \alpha_i = [0.1, 99.9]\% \). It should be noted that \( \alpha_1 \) and \( \alpha_2 \) are varied here simultaneously. However, it is also possible to define different values \( \alpha \) to consider one constraint as more relevant than the other (e.g. for safety reasons). It should though noted that like in the nominal case both constraints are active for a wide range of \( \alpha_i = [2\%, 98\%] \). For \( \alpha_i \geq 98\% \) only \( P_{y_2} \), for \( \alpha_i \leq 2\% \) only \( P_{y_1} \) remains active. In addition to the nonlinear relation between \( u, \xi \) and \( y \), the probability integration of the unknown PDF causes also a strong nonlinear effect on \( P_{y_1} \) and \( P_{y_2} \). Thus, the solution exhibits discontinuity around \( \alpha_i \approx 98\% \). Following this, the realized back-off is the distance be-
3. EXPERIMENTAL CASE STUDY: OPERATION OF A HIGH PRESSURE DISTILLATION COLUMN

3.1 The pilot plant and its basic regulatory control layer

The distillation pilot plant is composed of 28 bubble cup trays and is embedded in a coupled two-pressure column system for the complete separation of a binary heterogeneous azeotropic mixture (acetonitrile-water). The column is operated at a pressure of $p_{\text{top}} = 2\, \text{bar}$ and the feed is supplied into the column with an acetonitrile concentration below the azeotropic point. The high boiling point component (water) is removed from the bottom whereas at the column top maximal separation is defined by a distillate concentration equal to the concentration of the azeotropic point, which is a function of the column top pressure $x_{Ac} = f(p_{\text{top}})$. Feasible operation of the column is defined by the bottom and top product specifications which can only be measured off-line by a gas-chromatograph. Thus, no direct feed-back control can be applied. Figure 4 shows the individual high-pressure column and the control loops corresponding to the regulatory control layer. Whereas the top specification is controlled in a pure feed-forward mode, setting the reflux ratio, the bottom product concentration is indirectly controlled by the stabilization of the temperature profile in the stripping section. This is done by a temperature control loop controlling the sensitive temperature on a selected tray (set-point: $T^{\text{SP}} = 110^\circ\text{C}$).

3.2 The optimization problem

High top and bottom product purities can only be achieved at a high expense of reflux and reboiler heat being their maximal value $x_{\text{top}}^{\text{Ac}} \rightarrow x_{\text{top}}^{\text{Ac}}(2\, \text{bar})$ and $x_{\text{bottom}}^{\text{Ac}} \rightarrow 0\, \text{mole}\%$. The optimal operation is characterized by a minimal heat input satisfying the product specifications. Considering only continuous decisions, the nominal optimal operating point is defined by the active constraints of bottom and top product specifications. The formulated optimization problem is given in (10). For this specific case the minimization of the reboiler duty $\dot{Q}$ can be replaced by the reflux ratio $r$. Doing so, the objective $J$ is independent from the uncertainties $\xi$, which are considered later. As a result, the objective function can be treated as in a standard NLP problem (see also Barz et al. [2006], Flemming et al. [2007]). No expected value or variance have to be calculated.

$$
\min_{r, Y^{\text{feed}}_l, Y^{\text{top}}_k} J = \dot{Q} \Rightarrow J = r
$$

s.t. model equations $gM$

$$
x_{\text{top}}^{\text{Ac}} \geq 63.2\, \text{mole}\% \\
x_{\text{bottom}}^{\text{Ac}} \leq 0.5\, \text{mole}\%
$$

$$
\begin{cases}
\forall l \in D^{\text{feed}}
F_{\text{feed}}^l = 15\, \text{litr} / \text{h} \\
n \in D^{\text{feed}} \setminus \{l\}
\end{cases}
$$

$$
Y^{\text{feed}}_l \in \{\text{True, False}\}, l \in D^{\text{feed}}
$$

$$
\begin{cases}
\forall k \in D^{\text{sens}}
Y^{\text{sens}}_k = \{\text{True, False}\}, k \in D^{\text{sens}}
\end{cases}
$$

Taking into account the regulatory control layer implemented at the pilot plant, the optimal set-point for the reflux ratio $r$ is calculated. Furthermore the optimal tray $k \in D^{\text{sens}}$ for the temperature control loop in the stripping section has to be determined. Fixing the temperature on the respective tray, the reboiler duty $\dot{Q}$ is set by the basic regulatory controller. Additionally, it can be chosen between 10 different feed trays $l \in D^{\text{feed}}$, in order to supply the feed at the optimal feed plate into the column. Following this, the optimization problem (10) involves the continuous decision variable reflux ratio $r$ as well as discrete decisions for the optimal feed location $Y^{\text{feed}}_l$ and the tray for the temperature control $Y^{\text{sens}}_k$. In order to prevent a simultaneous use of more than one feed tray or temperature tray, respectively, disjunctive conditions are formulated in (10). Stein et al. [2004] presented an approach for the elimination of the discrete decision variables by adding a set of continuous variables and constraints that represent the discrete decision space of the optimization problem. By this means, a standard SQP algorithm can be applied solving a sequence of NLP problems, where the additional constraints are steadily tightened in order to force the relaxed variables to their discrete solution.

In this work, the circle conditions applied in Stein et al. [2004] are used for the reformulation of the disjunctive feed conditions. However, it should be noted that it is not possible to relax $Y^{\text{sens}}_k$ as well. This is because the control loop for the temperature control is either active or not. A superposition of partially activated control loops is neither in practise nor in a simulation feasible.

For the solution of (10) a steady state rigorous tray by tray column model based on mass and energy balances with a detailed description for the pressure loss is used (Barz et al. [2006]). The physical properties of the mixture

![Diagram of the pilot plant and control loops](image-url)

Fig. 4. Basic regulatory control of the column.
are calculated using equations from CHEMCAD whereas for the phase equilibrium the Wilson model is used. This results in a large-scale DAE system with 237 states.

3.3 Nominal optimal solution and impact of uncertainties

The solution of the nominal optimization problem (10) gives a minimal reflux \( r \approx 0.4 \). The location for the temperature control loop is set to the first tray \( Y_{\text{feed}}^{\text{sens}} = \{\text{True}\} \) for \( k = 1 \) and the feed is introduced in the stripping section at \( Y_{\text{feed}} = \{\text{True}\} \) for \( l = 8 \). This shows that the column is rather over-dimensioned and reflux and reboiler duty, respectively, can be minimized using as much trays in the rectifying section as possible. While the top product concentration lays directly on the specification limit \( x_{\text{top}}^{\text{opt}} \approx 63.2 \text{ mole}\% \), the bottom product concentration is higher than required, with \( x_{\text{bottom}}^{\text{opt}} \approx 0.3 \text{ mole}\% \). This is due to the discrete decision of the tray number for the temperature control loop used for the indirect bottom concentration control.

However, in the presence of uncertainties deviations from the optimal process states are caused. For the quantitative description, all uncertain variables are replaced by normal-distributed model parameter \( \xi \sim N(\mu, \sigma) \). In practice, besides uncertainties in model parameters, also controller deviations and external uncertainties have to be considered. Controller deviations around the setpoint, also called implementation errors (Govatsmark and Skogestad [2005]), were obtained measuring directly the closed loop variance for disturbances. Analysing industrial data (Wozny and Jeromin [1991]) fluctuations of the feed conditions were defined. It should be noted that both feed concentration and flow can vary over a broad range during few hours. This causes the continual adjustment of the manipulated variables in the basic regulatory control layer and disturbs other related control loops. Uncertainties in model parameters were obtained using Maximum Likelihood Parameter Estimation (MLE) for different steady operating points. We found out that the Murphy tray efficiency, which is defined for stripping \( \eta_S \) and rectifying \( \eta_R \) section separately, assumes different values depending on the liquid load of the column. Moreover, the pressure resistance coefficient for the dry pressure loss \( c_w \) had to be adjusted in order to fit the experimental data properly. This highlights structural uncertainties and means a limited scope of application for the model. Thus, we identify the corresponding parameters as uncertain too. Table 1 shows all considered uncertainties characterized by their mean \( \mu \) and the confidence interval of \( 6\sigma \). As can be seen the reflux temperature \( \Delta T_{\text{reflux}} \), the performance of the pressure and temperature control loop \( p_{\text{top}}, T_{\text{sens}} \) and the controlled reflux ratio \( r \) have been considered also as stochastic parameters.

![Fig. 5. Effects of individual uncertain parameters on the top product concentration.](image)

### Table 1. Model uncertainties

<table>
<thead>
<tr>
<th>uncertain parameters</th>
<th>( \mu )</th>
<th>( \pm 3\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>pressure resistance coeff.</td>
<td>( c_w )</td>
<td>1.5</td>
</tr>
<tr>
<td>subcooling condensat</td>
<td>( \Delta T_{\text{reflux}}(\circ C) )</td>
<td>70.0</td>
</tr>
<tr>
<td>controlled reflux ratio</td>
<td>( r )</td>
<td>( r_{\text{opt}} )</td>
</tr>
<tr>
<td>controlled temperature</td>
<td>( T_{\text{sens}}(\circ C) )</td>
<td>110.0</td>
</tr>
<tr>
<td>controlled top pressure</td>
<td>( p_{\text{top}}(\text{bar}) )</td>
<td>2.000</td>
</tr>
<tr>
<td>Murphy tray efficiency</td>
<td>( \eta_S, \eta_R )</td>
<td>0.45</td>
</tr>
<tr>
<td>feed flow</td>
<td>( F_{\text{feed}}(\text{ltr/h}) )</td>
<td>15.0</td>
</tr>
<tr>
<td>feed concentration</td>
<td>( x_{\text{Ac}}^{\text{feed}}(\text{mole}%) )</td>
<td>45.0</td>
</tr>
</tbody>
</table>

control scheme. Furthermore, the characteristic nonlinear behaviour of the product concentration is apparent. Based on the analysis of the results for bottom and top concentration, four key uncertainties were identified showing a maximal effect on the constrained states, namely: \( \xi = [\eta_S, \eta_R, x_{\text{Ac}}^{\text{feed}}, T_{\text{sens}}]T \).

The selection of the probability level \( \alpha \) means to directly decide on how conservative or aggressive the solution should be (see also Fig. 3). Here, the required probability of holding the constraints is set to \( \alpha = 95\% \). Furthermore, the tray efficiencies \( \eta_S, \eta_R \) are treated as one uncertain variable \( \eta \). By this means, the dimension of \( \xi = [\eta, x_{\text{Ac}}^{\text{feed}}, T_{\text{sens}}] \) and, thus, the computational effort for solving the chance constrained optimization problem is reduced (Wendt et al. [2002]). The solution of the robust optimization problem gives an higher reflux \( r \approx 0.74 \) in order to compensate for the uncertainties. The robust and optimal location for the temperature control loop is shifted to the second tray \( Y_k^{\text{sens}} = \{\text{True}\} \), \( k = 2 \) whilst the optimal feed tray remains the same \( Y_{l,\text{feed}} = \{\text{True}\} \), \( l = 8 \). As expected both product concentrations are generally above the required specification.
3.5 Implementation and Validation

In order to demonstrate the application and to validate the computed results both nominal and robust decisions were implemented at the pilot plant. Here, additionally to the considered uncertainties, other process parameters did not match their assumed values. The biggest bias was observed for the feed concentration \( x_{\text{Ac}}^{\text{feed}} \approx 53\,\text{mole\%} \) instead of 45 mole\% (a variation of 15\%), for the feed flow \( F^{\text{feed}} \) and the reflux ratio \( r \) with 7 and 2.5\%, respectively. Both the computed optimal and robust concentration profiles and the analysed samples are shown in Figure 6. The robust optimal operation shows for both the analytical and the experimental values a wider concentration profile, which results from the more conservative operation. Hence, an additional heat amount of \( Q \approx 6.3\,\text{KW} \) (instead of \( Q \approx 5.6\,\text{KW} \)) was measured.

![Diagram of concentration profiles](image)

Fig. 6. Analytically vs. experimentally determined concentrations for nominal and robust optimal decisions.

Table 2 shows the results for the simulated and measured product concentrations. The experimental data confirms the analytical results and shows clearly the relevance of the proposed approach.

<table>
<thead>
<tr>
<th>specification</th>
<th>nominal operation</th>
<th>robust operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{\text{Ac}}^{\text{top}} \geq 63.2 )</td>
<td>63.2 ± 0.5</td>
<td>63.6 ± 0.5</td>
</tr>
<tr>
<td>( x_{\text{Ac}}^{\text{bottom}} \geq 0.5 )</td>
<td>0.3 ± 0.5</td>
<td>0.05 ± 0.5</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

A chance constrained optimization approach has been discussed and experimental verified, which can be applied to constrained process operation under uncertainty allowing the user to make a direct decision about the strategy to be applied w.r.t. reliability or costs. Optimal decisions are then obtained by computing the probabilities of satisfying the constraints. The resulting operating points are as close as possible to the constrained output variables so that the economical benefit can be maximized.

REFERENCES


