Boundary Stabilization of Marine Structure

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Abstract:
This note addresses the stabilization problem of a marine structure (i.e. cable/riser), connected to a surface vessel at one end and to a thruster unit at the other. Here, only lateral motion is considered. Based on boundary measurements, stabilizing control laws are designed. The controllers consist only on feedback from boundary measurements. The costs are thus minimized and the spillover instabilities are avoided. Simulation results are included.

1. INTRODUCTION

This note addresses the stabilization problem of a system consisting of a marine structure (i.e. cable/riser) connected to a surface vessel at the top end and to a thruster unit (e.g. robot system, ROV, mass modul, etc.) at the bottom end (Figure 1). The function of the thruster unit may be several, e.g. to perform maintenance and repair on underwater installations; while the marine structure is needed to provide power, control signals and other necessary signals for operating the thruster unit. Due to the motion of the surface vessel and fluid forces (i.e. wave and current forces), the marine structure undergoes deformations, which lead to reduced performance of the thruster unit. Thus, robust and high performance controllers for the thruster system are needed.

The dynamics of marine cable/riser have been studied by numerous authors, among others [2],[6],[7],[8],[9],[19],[20],[21],[22],[25],[26] and references therein. In [8],[20],[25],[26] modelling and analyzing of marine cable are studied. In [5],[11],[16] boundary control of elastic cable/beam are studied. Aamo and Fossen [1] considered modelling and control of mooring lines. Jensen et al. [13] study modelling and control of offshore marine pipeline, where the model of the system is based on the standard robot equation. In [17],[22],[27] modelling and control of towed marine cables are studied. As opposed to [17], the control design in [22],[27] are based on discretized models of the cables.

For discretized ordinary differential equation models of flexible system there exists many control design tools (see e.g. [3],[4],[14],[22],[27] and the references therein). A substantial difficulty in the design of these controllers is the choice of the discretization order. Reduction of the infinite dimensional continuum model to a finite dimensional (\(n^{th}\) order) discrete model means that certain motions (\(\infty - n\)) are neglected. Typically, modal analysis motivates the model reduction. With sufficient system damping, higher order modes can be neglected if the controller rolls off (i.e. the controller gain drops sharply) at high frequency. Choice of \(n\) too small results in spillover instability that occurs when the controller, designed for the finite dimensional model, senses and actuates higher order modes, driving them unstable (see e.g. [4]). Reduction of the control gain to eliminate spillover often results in poor performance. Choice of \(n\) too large results in a high order compensator that can be difficult and costly to implement. So to avoid the spillover instabilities and complexity associated with discretized and distributed controllers, the control design for flexible mechanical systems should be based on the distributed parameter models, which will be considered in this note. This note is an extension of [18], and is inspired by the work of Lindegaard et al. [15],[23], where acceleration feedback in dynamic position system (DP) was first introduced.

Fig. 1. Marine structure with marine vessels.

2. SYSTEM MODEL

Consider the system in Figure 1. The marine structure of length \(L > 0\) is connected to a surface vessel of mass \(M_{RB} > 0\) at one end and to a thruster unit of mass \(m_{RB} > 0\) at the other. Here, only lateral motion is considered. The mathematical model of the system is adopted from [12],[24].

Let \(\Omega = ]0,L[, \Omega = [0,L]\) and \(\eta(z,t)\) denote the deflection of the marine structure at the point \(z \in \Omega\) and time \(t \geq 0\). The equations of motion of the system are given as,

\[
\begin{align*}
\frac{\partial}{\partial t} \frac{\partial \eta}{\partial t} + \beta \frac{\partial \eta}{\partial t} - \gamma \frac{\partial \eta}{\partial z} &= \frac{\partial}{\partial z} \left( \frac{\partial \eta}{\partial z} \right) + \beta \frac{\partial \eta}{\partial z} + \gamma \frac{\partial \eta}{\partial \Omega} + \frac{\partial F}{\partial \Omega} \\
\end{align*}
\]
\[ p \dot{h} = -(EI_{zz})_{zz} + (T_{zz})_z - d \dot{h} + c_0 \omega + c_2 (U - \dot{h}) | \dot{h} + U - \dot{h} |, \ z \in \Omega \]

**for** \( t > 0 \). Assume that the marine structure is connected to the vessels by means of ball-joints. This results in small angles of deflection and zero bending. Hence, the remaining boundary conditions of (1)-(3) are

\[ E I_{zz,l} = E I_{zz,2l} = 0 \]

**for** \( t \geq 0 \). Here, \( E I(z) \) and \( T(z) \) denote the stiffness and tension of the structure at \( z \in \bar{\Omega} \), respectively, \( d(z) \) represents the structural damping at \( z \in \Omega \), \( \omega(z,t) \) is the lateral wave velocity at \( z \in \Omega \) and time \( t \geq 0 \), \( U(z,t) \) denotes the lateral current velocity at \( z \in \bar{\Omega} \) and time \( t \geq 0 \), respectively, \( c_0, c_2, C_1, C_2, C_3, C_4 > 0 \) are the hydrodynamic coefficients, \( X_{wind} \) and \( X_{waves} \) represent the generalized forces acting on vessels due to the wind and waves, \( X_{disturb} \) and \( X_{disturb} \) denote the generalized forces acting on the vessels due to unmodelled disturbances, \( t_0, t_L : R^+ \rightarrow R \) denote the thruster forces generated by the surface vessel and the thruster unit, respectively, and

\[ \rho = \rho_R + \rho_A [C_m(z) - 1] \frac{\pi D_A^2}{4} \]

\[ M = M_{RH} + M_A (\omega | \dot{h}) \]

\[ m = m_{RH} + m_A (\omega | \dot{L}) \]

\[ c_0(z) = \frac{\rho_0 \pi D_A^2}{4} C_m(z) > 0, \ z \in \Omega \]

\[ c_2(z) = \frac{\rho_2 D_0}{2} C_d(z) > 0, \ z \in \Omega \]

\( \rho_R \) is the mass per unit length of the structure, \( \rho_A \) is the mass density of the ambient water, \( C_m, C_d \) and \( D_A \) denote the mass coefficient, drag coefficient and diameter of the structure, respectively, \( M_A \) denotes the added mass of the surface vessel, and \( m_A \) is the added mass of the thruster unit.

The generalized forces due to the wind and the waves, \( X_{wind} \) and \( X_{waves} \), are modelled as [12],

\[ X_{wind} = \frac{1}{2} C_X \rho_A A_T V^2 \]

\[ X_{waves}(s) = \frac{K_s}{s^2 + 2 \omega_s \omega_0 s + \omega_0^2} w_1 + d_1 \]

where \( C_X \) is the empirical force coefficient, \( \rho_A \) is the mass density of the air, \( A_T \) is the transverse projected area of the surface vessel, \( V_T \) denotes the relative wind speed (i.e. \( V_T = V_{wind} - \bar{v}_0 \), where \( V_{wind} \) is the speed of the wind), \( K_s > 0 \) is the wave constant, \( \omega_0 > 0 \) is the damping coefficient, \( \omega_s \) is the encounter frequency, \( s \) denotes the Laplace variable, \( d_1 \) represents the wave drift force modelled as slowly-varying bias term

\[ d_1 = w_2, \ t > 0 \]

and \( w_1, w_2 \) are Gaussian white noise processes. The encounter frequency \( \omega_s \) is generally given as

\[ \omega_s (V_{vessel}, \omega_0, \beta) = \omega_0 - \frac{\omega_0^2}{g} V_{vessel} \cos \beta \]

where \( \omega_0 \) is the dominating wave frequency, \( g \) is the acceleration of gravity, \( V_{vessel} \) is the total speed of the surface vessel, \( \beta \) is the angle between the heading and the direction of the wave. However, the wave frequency of a dynamically positioned vessel can be sufficiently described by \( \omega_e = \omega_0 \), since \( V_{vessel} \) is close to zero [12].

Similarly, \( X_{waves} \) is given as

\[ X_{waves}(s) = \frac{K_L s}{s^2 + 2 \omega_k s + \omega_0^2} \]

where \( K_L > 0 \) is the wave constant, \( \omega_k > 0 \) is the damping coefficient, \( \omega_L \) is the encounter frequency, and \( \omega_3 \) is Gaussian white noise process.

The lateral wave velocity \( \omega(z,t) \) below the water surface is given by [10],

\[ \omega(z,t) = \sum_{i=0}^\infty \omega_i W_i e^{-2 \xi_i \sin(\omega_i t)}, \ z \in \Omega, \ t \geq 0 \]

where \( W_i \) is the wave amplitude, \( \omega_i \) is the wave frequency, and \( \lambda_i \) is the wave length. See [10],[12] for further discussion on the topics above.

Let the initial conditions be given as

\[ \omega(z,0) = \sum_{i=0}^\infty \omega_i W_i e^{-2 \xi_i \sin(\omega_i t)}, \ z \in \Omega, \ t \geq 0 \]

where \( W_0 \) and \( V_0 \) are the initial position and velocity functions of the structure, respectively. Throughout this note, the subscript (.) denotes the partial derivative with respect to \( z \) and \( t \) respectively.

### 2.1 Assumptions

- **A.1** the wave and current velocities are much larger then the velocity of the structure [6], i.e.

\[ \omega + U + \dot{h} \approx \omega + U \], \( z \in \Omega \), \( t \geq 0 \)

- **A.2** the system parameters \( \rho, \omega, T \in L_2(\Omega) \) are finite and strictly positive, i.e.

\[ 0 < \rho_{min} \leq \rho(z) \leq \rho_{max} \rightarrow \infty, \ z \in \Omega \]

\[ 0 < \omega_{min} \leq \omega(z) \leq \omega_{max}, \ z \in \Omega \]

\[ 0 < T_{min} \leq T(z) \leq T_{max}, \ z \in \Omega \]

for constants \( \rho_{min}, \rho_{max}, \omega_{min}, \omega_{max}, T_{min}, T_{max} > 0 \).

Application of A.1 to (1)-(3) yields

\[ p \ddot{h} = -(EI_{zz})_{zz} + (T_{zz})_z - d \ddot{h} + c_0 \omega + c_2 (U - \dot{h}) | \dot{h} + U - \dot{h} |, \ z \in \Omega \]

**for** \( t > 0 \). With the boundary conditions (4) and initial conditions (11).
3. CONTROL FORMULATION

The objectives of the controllers are to control the position and velocity of the thruster unit and the surface vessel such that \( \{ \eta_q, \dot{\eta}_q, \eta_L, \dot{\eta}_L \} \to [0, 0, 0, 0] \) as \( t \to \infty \). Additionally, the designed controllers should also be able to attenuate the vibrations and oscillations in the system due to the sea loads, i.e. \( \{ |\eta(z,t)|, |\dot{\eta}(z,t)| \} < \infty, \forall z \in H \) and \( t \geq 0 \).

Inspired by the work of Lindegaard et al. [15],[23], where acceleration feedback in dynamic position system (DP) was first introduced, we propose the control laws

\[ \tau_0(t) = -K_p \dot{\eta}_0 - K_d \ddot{\eta}_0 - k_m \dot{\eta}_L - k_i \int_0^t \dot{\eta}(0, \xi) d\xi \]

(15)

\[ \tau_L(t) = -k_p \ddot{\eta}_L - k_d \dot{\eta}_L - k_m \dot{\eta}_L - k_i \int_0^t \dot{\eta}(L, \xi) d\xi \]

(16)

for \( t \geq 0 \), where \( K_p, K_d, k_m, k_i, K_p, K_d, K_m, K_i > 0 \) are controller gains. The integral action is included to yield the influence of the sea current and wind, while the inertia term is to increase the robustness of the system against disturbances (see Remark 1).

Before one can proceed, it is necessary to assume that the system (12)-(14) with the boundary conditions (4), the initial conditions (11), and the control laws (15)- (16) is well-posed, i.e. the closed loop system has a unique solution and the solution is sufficiently smooth both in time and space.

Now, let the external disturbances be zero, i.e. \( X_{\text{wind}}(t) = X_{\text{waves}}(t) = X_{\text{disturb}}(t) = X_{\text{waves}}(t) = X_{\text{disturb}}(t) = 0, \quad U(z,t) = \omega(z,t) = 0, \forall z \in \Omega, t \geq 0. \)

Define the state vector

\[ \mathbf{q}(t) = \left[ \int_0^t \eta_0 d\xi, \eta_0, \dot{\eta}_0, \eta_L, \dot{\eta}_L \right]^T \]

Consider now the storage functional

\[ V(\mathbf{q}) = \frac{1}{2} \int_\Omega (T\eta_0^2 + EI\eta_0^2 + \rho \dot{\mathbf{q}}^2) d\xi + \frac{1}{2} \int_\Omega \rho \dot{\mathbf{q}} \cdot \dot{\mathbf{q}} d\xi \]

+ \frac{1}{2} \gamma (M + K_m) \ddot{\eta}_0^2 + \frac{1}{2} \gamma K_p \eta_0^2 + \frac{1}{2} \gamma K_i \left( \int_0^t \eta_0 d\xi \right)^2

+ \gamma (M + K_m) \eta_0 \dot{\eta}_0 + K_i \eta_0 \int_0^t \dot{\eta}_0 d\xi

\]

(17)

where \( \gamma \) is the Lyapunov gain.

First, using the inequalities

\[ |\eta(z,t)| \leq |\eta(0,t)| + \left[ L \int_\Omega |\eta_L|^2 d\xi \right]^{1/2}, \quad z \in \Omega \]

\[ (a + b) \leq 2a^2 + 2b^2, \quad \forall a, b \in \mathbb{R} \]

yields

\[ |\eta(z,t)|^2 \leq 2 |\eta(0,t)|^2 + 2L \int_\Omega |\eta_L|^2 d\xi, \quad z \in \Omega \]

Thus,

\[ \int_\Omega |\eta_L|^2 d\xi \geq \frac{1}{2 L^2} \int_\Omega |\eta|^2 d\xi - \frac{1}{T^2} \int_\Omega |\eta(0,t)|^2 \]

(18)

Application of (18) to (17) gives

\[ V \geq \frac{1}{2} \int_\Omega \left( EI\eta_0^2 + \rho q^2 \right) d\xi + \gamma \int_\Omega \rho \dot{q} \cdot \dot{q} d\xi \]

+ \frac{T_{\text{min}}}{4L^2} \int_\Omega \eta^2 d\xi - \frac{T_{\text{min}}}{2L^2} \eta_0^2 \]

+ \frac{1}{2} \left( M + K_m \right) \dot{\eta}_0^2 + \frac{1}{2} K_p \eta_0^2 + \frac{1}{2} K_i \left( \int_0^t \eta_0 d\xi \right)^2

+ \gamma \left( M + K_m \right) \eta_0 \dot{\eta}_0 + K_i \eta_0 \int_0^t \dot{\eta}_0 d\xi

\]

(19)

Given \( k_i, k_m, K_i, K_m > 0 \). Choose the remaining gains as

\[ 0 < \gamma < \min \left\{ \frac{\gamma_{\text{min}}}{P_{\text{max}}}, \frac{T}{2L^2} \right\} \]

(20)

\[ k_d > 2\gamma (m + k_m) \]

(21)

\[ K_d > 2\gamma (M + K_m) \]

(22)

\[ k_p > \max \left\{ \gamma^2 (m + k_m) + \frac{k_i}{\gamma}, 2\gamma(k_d + C_1) \right\} \]

(23)

\[ K_p > \max \left\{ \gamma^2 (M + K_m) + \frac{K_i}{\gamma}, \frac{T_{\text{min}}}{2L^2}, 2\gamma(k_d + C_1) \right\} \]

(24)

It follows thus

\[ V(\mathbf{q}) > 0, \quad \forall \mathbf{q} \neq 0 \]

Note that there are several ways to select the Lyapunov gain and controller gains. The selection (20)-(24) is just one of the possibilities, and is based on the analysis below and the way the expression of \( V \) and the time derivative of \( V \) are written on (cf. eq. (19) and (27)).

Next, taking the time derivative of (17) along solution trajectories of (12)-(14) gives

\[ \dot{V} = - \int_\Omega [\dot{\mathbf{q}} - \gamma \dot{\mathbf{q}}] \dot{\mathbf{q}} \cdot \dot{\mathbf{q}} d\xi \]

\[ - \gamma \int_\Omega EI\dot{\eta}_0^2 d\xi - \gamma \int_\Omega \rho \dot{\mathbf{q}} \cdot \dot{\mathbf{q}} d\xi \]

\[ - \left[ K_d + C_3 - \gamma (M + K_m) \right] \ddot{\eta}_0^2 \]

\[ - \gamma [K_d + C_3] \eta_0 \dot{\eta}_0 - \gamma [K_p - K_l] \eta_L^2 \]

\[ - [k_d + C_3] \gamma \left[ \gamma + (m + k_m) \right] \eta_L^2 \]

\[ - \left[ k_d + C_3 \right] \eta_0 \left[ \eta_0 - \gamma C_2 \eta_0 \eta_L \right] \eta_0^2 \]

\[ - C_2 \eta_0 \left[ \dot{\eta}_L + \gamma C_2 \eta_0 \eta_L \right] \eta_0^2 \]

\[ - C_2 \eta_L \left[ \dot{\eta}_L - \gamma C_2 \eta_0 \eta_L \right] \eta_L^2 \]

(25)

where integration by parts has been successively applied. Application of (18) and the assumption A.2 to (25) yields
\[
\dot{V} \leq - \int_{\Omega} [d - \gamma \eta]^2 d\eta + \int_{\Omega} d\eta d\xi = - \gamma \int_{\Omega} E I n_{z z}^2 d\eta - \gamma \int_{\Omega} \frac{E I n_{z z}^2}{2} d\eta
\]

where

\[
\dot{V} \leq - \gamma \int_{\Omega} E I n_{z z}^2 d\eta - \gamma \int_{\Omega} \frac{E I n_{z z}^2}{2} d\eta
\]

Let

\[
p(z,t) = [\eta(z,t), \dot{\eta}(z,t)]^T, \quad z \in \Omega, \quad t \geq 0
\]

The right-hand side of (26) can be rewritten as

\[
\dot{V} \leq - \gamma \int_{\Omega} E I n_{z z}^2 d\eta - \gamma \int_{\Omega} \frac{E I n_{z z}^2}{2} d\eta
\]

where

\[
P = \begin{bmatrix}
\frac{\gamma T_{\text{min}}}{2} d - \gamma \rho \\
\frac{\gamma K_p}{2} d - \gamma \rho
\end{bmatrix}
\]

\[
P_0 = \begin{bmatrix}
\frac{\gamma K_p}{2} & \frac{\gamma (K_d + C_1)}{2} \\
\frac{\gamma (K_d + C_1)}{2} & \frac{\gamma K_p}{2} + \frac{\gamma \eta_0^2}{2} + \frac{\gamma C_2 \eta_0 \eta_1}{2}
\end{bmatrix}
\]

\[
P_L = \begin{bmatrix}
\frac{\gamma k_d}{2} & \frac{\gamma (k_d + C_1)}{2} \\
\frac{\gamma (k_d + C_1)}{2} & \frac{\gamma k_d}{2} + \frac{\gamma \eta_0^2}{2} + \frac{\gamma C_2 \eta_0 \eta_1}{2}
\end{bmatrix}
\]

Since the Lyapunov gain and the controller gains are chosen according to (20)-(24), it follows that \(P, P_0, P_L > 0\). Furthermore, it is straightforward to verify that the last two terms in (27) can be made negative semi-definite for sufficiently large design parameters \(k_p, k_d, K_p, K_d > 0\) and sufficiently small \(\gamma > 0\). Hence,

\[
\dot{V} \leq 0
\]

It follows from Lyapunov’s stability theorem that the equilibrium point \(q^*\) is stable and the solution \(q(t)\) is bounded for \(t \geq 0\), where

\[
U_{\infty}(z) = \lim_{t \to \infty} U(z,t) < \infty, \quad z \in \Omega
\]

\[
V_{\text{wind}, \infty} = \lim_{t \to \infty} V_{\text{wind}}(t) < \infty
\]

and \(\eta^*\) can be obtained by solving the equation

\[
(E I n_{z z}^2)_{zz} + (T n_{z z}^2) + c_2 U_{\infty} |U_{\infty}| = 0, \quad z \in \Omega
\]

with the boundary conditions

\[
\eta^*_0 = \eta^*_L = E I n_{z z}^2|_0 = E I n_{z z}^2|_L = 0
\]

Moreover, from the LaSalle’s theorem it follows that the equilibrium point \(q^*\) is globally uniformly asymptotically stable.

Remark 1. It should be noticed that beside increasing the masses from \(M\) to \(M + K_m\) and \(m + k_m\), respectively, the acceleration feedback also reduces the gain in front of the disturbances \(X_{\text{wind}} + X_{\text{waves}} + X_{\text{disturb}}\) and \(X_{\text{waves}} + X_{\text{disturb}}\) from \(1/M\) and \(1/m\) to \((1/(M + K_m))\) and \((1/(m + k_m))\), respectively. The system is thus less sensitive to external disturbances. The design can be further improved by introducing a frequency dependent virtual mass (see [12],[15],[23]), i.e. replacing \(K_m\) and \(k_m\) with transferfunctions \(H_m(s)\) and \(h_m(s)\) in (15)-(16), where \(s\) denotes the Laplace-variable. If \(H_m(s)\) and \(h_m(s)\) are chosen as low-pass filters,

\[
H_m(s) = \frac{K_m}{1 + T_m s}, \quad h_m(s) = \frac{k_m}{1 + T_m s}
\]

with the gains \(K_m, k_m > 0\) and time constants \(T_m, \tau_m > 0\), then the total masses are \(M + K_m\) and \(m + k_m\) at low frequencies (\(s \to 0\)), respectively, while at high frequencies (\(s \to \infty\)) the total masses \(M + K_m\) and \(m + k_m\) reduce to \(M\) and \(m\), respectively.

4. SIMULATION

To simulate the system (12)-(14) with the feedback control laws (15)-(16), the finite-element method with Hermitian basis functions has been applied. The marine structure was divided into 20 elements. For simplicity, the system parameters are set to be constant. The state variables were initially set to zero. The system parameters are summarized below. We let the unmodelled disturbances be zero, i.e. \(X_{\text{disturb}}(t) = X_{\text{disturb}}(t) = 0, \forall t \geq 0\). Simulation results are shown in Figure 2 - 5. The position of the vessels is shown in Figure 3, and the 2-norm of the state vector \([\eta_0, \eta_L, \dot{\eta}_L, \dot{\eta}_L]^T\) is shown in Figure 5. Obviously, \([\eta(z,t), \dot{\eta}(z,t)] < \infty, \forall z \in \Omega, t \geq 0\), and \([\eta_0, \dot{\eta}_0, \eta_L, \dot{\eta}_L] \to 0\) as \(t \to \infty\).

Marine structure:

\[
[p, L]^T = [1 \text{ kg/m}, 600 \text{ m}]^T
\]

\[
[c_0, c_2]^T = [80, 82]^T
\]

\[
[d, E, T]^T = [0.1 \text{ Ns/m}^2, 4.2 \cdot 10^6 \text{ Nm}^2, 1.1 \cdot 10^6 \text{ N}]^T
\]
Wind, wave and current:
\[
[i, \lambda, W]^T = [1, 10 \text{ m}, 0.1 \text{ m}]^T \\
[K_e, \zeta_e, \omega_e]^T = [0.07, 0.11, 0.64]^T \\
[K_L, \zeta_L, \omega_L]^T = [0.035, 0.11, 0.64]^T \\
[C_X, A_T]^T = [2.15, 10^3 \text{m}^2]^T \\
[\rho, V_{wind}]^T = [1.25 \text{kg/m}^3, 20 \text{ m/s}]^T \\
U(z) = \frac{U_0 - U_1}{L} z + U_0
\]

where \([U_0, U_1]^T = [1, 0.1]^T \text{ m/s}\)

Surface vessel:
\[
M = 9.6 \cdot 10^7 \text{ kg} \\
[C_0, C_1, C_2]^T = [4.8, 0.9, 1]^T \cdot 10^6
\]

Thruster unit:
\[
m = 30 \text{ kg} \\
[C_0, C_1, C_2]^T = [1.5, 100, 820]^T
\]

Controller gains:
\[
[K_p, K_d, K_i, K_m]^T = [1, 3.75, 0.085, 0.4] \cdot 10^7 \\
[k_p, k_d, k_i, k_m]^T = [1, 3.75, 0.085, 0.4] \cdot 10^4
\]

Fig. 2. Deflection of the marine structure \(\eta(z, t)\) at selected nodes.

5. CONCLUSIONS

The stabilization problem of a marine structure connected to a surface vessel at one end and a thruster unit at the other end is considered. The dynamics of the marine structure and the vessels are described by a partial differential equation and ordinary differential equations, respectively. The control laws consist only of feedback from boundary measurements. The measurement- and implementation-cost are thus minimized and spillover instabilities are avoided. The theoretical results are verified by simulation results, and they are in agreement.

Fig. 3. Position of the surface vessel [solid line] and the thruster unit [dashed line].

Fig. 4. Evolution of the state \(\eta(z, t)\).

Fig. 5. 2-norm of \([\eta(0, t), \dot{\eta}(0, t), \eta(L, t), \dot{\eta}(L, t)]^T\).
REFERENCES


