Predictive control of hybrid electric vehicles in fixed-route service

Abstract: The efficiency of energy management strategies for hybrid electric vehicles depends significantly on the accuracy of the prediction of the load power which has to be provided by the hybrid drive. For vehicles in fixed-route service, measurements gained during vehicle operation can be used for the design of a predictor for the online calculation of the expected speed profile which is directly related to the load power profile. The paper describes, how the relevant information contained in measurements of the vehicle speed and position is processed and compressed in order to obtain a prediction algorithm that can cope with the limited memory space and computing power of a vehicle controller. The characteristics of the resulting algorithm are illustrated with real-life data.

Keywords: Energy management systems, prediction methods, hybrid vehicles, predictive control, data models

1. INTRODUCTION

The paper deals with the real-time prediction of the load power demand of a hybrid electric vehicle to be used for an efficient energy management. Since the performance of the energy management control strategy depends significantly on the accuracy of the load power prediction, the investigation of prediction approaches is an ongoing topic of research (Johannesson [2005], Finkeldei and Back [2004]).

In Bartholomaeus et al. [2007], the energy management problem for a hybrid fuel cell drive train was split into the prediction $P_L(.)$ of the load power $P_L(.)$, which is drawn by the drive motor, and the calculation of the optimal setpoint function $u^*(.)$ based on the current state $x(t)$ of the hybrid drive (see Figure 1). As a main result of that paper, the optimal control problem to be solved by the optimizer was formulated in such a way that real-time ability of the resulting algorithm was obtained. In order to complete the work towards a real-time energy management, the present paper describes how a prediction of the load power can be obtained for the specific case of a vehicle operating in fixed-route service.

As depicted in Figure 1, the input data for the real-time calculation of the prediction $P_L(.)$ of the load power are the values of the vehicle position $s(t)$ and the vehicle speed $\dot{s} = v(t)$ at the current time $t$. In order to determine the map $(s, \dot{s}) \mapsto P_L(.)$ a sufficiently large number of measurements of associated trajectories of vehicle position, vehicle speed and electrical load power over time have to be taken and processed in advance.

The paper describes an intermediate but crucial step towards that goal. Instead of determining the prediction $P_L(t|t) \ (t \geq t)$ of the load power over time we describe the calculation of the prediction $v(s|\tilde{s}) \ (s \geq \tilde{s})$ of the vehicle speed over position. Given that future speed profile $v(.|\tilde{s})$, well-known physical relations can be used to derive the prediction $P_L(.)$ of the load power for a given vehicle and altitude profile. Hence, the prediction of the vehicle motion $v(.|\tilde{s})$ already includes the key task of the predictor, which is the calculation of the future vehicle motion under the condition of an unknown driver behaviour and disturbances from the environment, e.g., from other vehicles.

The outline of the paper is as follows: In Section 2, the preprocessing of the measurements is described, which includes first the description of the vehicle route by using GPS measured trajectories of the vehicle (position over time) and second the merging of position and speed measurements in order to increase the accuracy of the noisy sensor data. In Section 3, the compression of the measurements into a smaller data set, which describes only the information relevant to the prediction of the speed profile, is presented. Then, the developed algorithm for the calculation of the prediction $v(.|\tilde{s})$ is described. In Section 4, that algorithm is applied to real-life data and the Summary gives an outlook on the future work concerning the improvement of the prediction algorithm.
2. PREPROCESSING OF THE MEASURED DATA

For vehicles operating in fixed-route service, the data necessary to derive the prediction of the speed profile \( v(t) \) can practically be obtained by monitoring the motion of the vehicle, e.g., by using a GPS device, which is able to measure the \((x, y)\)-position and the speed of the vehicle independently. By driving the same route repeatedly, the \( r \) samples

\[
(t_i, x(t_i), y(t_i), v(t_i)) \quad (i = 1, \ldots, I)
\]

are obtained, where \( t_i \) are the points of measurement time, \( x(t_i) \) and \( y(t_i) \) are the horizontal 2D-coordinates (latitude and longitude converted to meters) of the vehicle, and \( v(t_i) \) is the speed of the vehicle. The task of this Section is first to calculate the arc length of the driven route from the \((x, y)\)-position data and second to utilize the property that the measurement errors between the \((x, y)\)-data and the \( v \)-data are uncorrelated in order to increase the accuracy of the measured data.

2.1 Description of the vehicle route and calculation of the arc length

From the measurements given in (1), a spline function \( S : \tau \mapsto (x(\tau), y(\tau)) \) is determined which describes the route of the vehicle as a function of the argument \( \tau \in \mathbb{R} \). For that purpose, the spline function is calculated as a least squares approximation (de Boor [2001]) of the measured \((x, y)\)-position data with the knots sequence appropriately chosen by a heuristic rule.

The least squares spline approximation requires all \((x, y)\)-data points to be ordered by ascending values of the independent variable \( \tau \). Otherwise it can be shown by simple examples that the resulting spline "cuts the corner" of the originally driven route, which implies a poor approximation. Since the value of the argument \( \tau \) related to a given data point is a priori unknown, the data points cannot be ordered as required. Therefore, a two step procedure is applied in order to describe the route of the vehicle and the arc length of the vehicle trajectory.

(1) Normalize each of the given samples (1) in time leading to

\[
(t_i - t_1, x(t_i), y(t_i)) \quad (j = 1, \ldots, r)
\]

and merge the resulting normalized samples into a single sequence of length \( K = I_1 I_2 \cdots I_r \),

\[
(t_k, x(t_k), y(t_k))_k \quad (k = 1, \ldots, K),
\]

in such a way that \( t_{k+1} \geq t_k \) holds. Calculate the spline function \( S_1 \) that approximates the data points \((x(t_k), y(t_k)), k = 1, \ldots, K, \) in the least squares sense.

(2) For each data point \((x(t_k), y(t_k))\) calculate the argument \( \tau_k \) in such a way that

\[
\|S_1(\tau_k) - (x(t_k), y(t_k))\|_2
\]

is minimal.\(^2\)

(3) Order the triples \((\tau_k, x(t_k), y(t_k))\) by nondecreasing values of \( \tau_k \), i.e., find a permutation \( \Pi \) such that \( \tau_{\Pi(k+1)} \geq \tau_{\Pi(k)} \). Calculate the spline function \( S_2 \) that approximates the data points \((x(t_{\Pi(k)}), y(t_{\Pi(k)})), k = 1, \ldots, K, \) in the least squares sense.

(4) Given a fixed sample index \( j \in \{1, \ldots, r\} \). For each pair \((x(t_i), y(t_i))_j \quad (i = 1, \ldots, I)\) the argument \( \zeta_i \) is determined in such a way that

\[
\|S_2(\zeta_i) - (x(t_i), y(t_i))_j\|_2
\]

is minimal. Then, the arc length related to the point \((x(t_i), y(t_i))_j\) is calculated by

\[
s_i = \int_0^{\zeta_i} \|d/df [S_2(\zeta)]\|_2 d\zeta.
\]

In the following, the \((x, y)\)-information is replaced by the arc length. That means, instead of (1) the sequences

\[
(t_i, s_i, v_i) \quad (i = 1, \ldots, I)
\]

\[
(t_i, s_i, v_i)_r \quad (i = 1, \ldots, I_r),
\]

with the abbreviation \( v_i = v(t_i) \), are used.

2.2 Sensor data fusion

We consider a single sample \((t_i, s_i, v_i)_j \), i.e., the index \( j \) is fixed. Since the measurements of the \((x, y)\)- and the \( v \)- data are done by independent sensors channels, the data are not consistent in general, i.e., the (numerical) integration of the values \( v_i \) of speed at time \( t_i \) \((i = 1, \ldots, I)\) does not provide the arc length given by the related values \( s_i \). That inconsistence can be exploited to adjust the measured data in order to suppress the measurement noise. Since the step size in time is very small in practise (typically a small fraction of a second), we use the simple integration rule

\[
\hat{s}_{i+1} = (t_{i+1} - t_i) \hat{v}_i + s_i
\]

to describe the expected relation between the measurement of speed and the calculated arc length. The resulting task is to ensure equation (3) while the difference between the adjusted measurements, \( \hat{s}_i \) and \( \hat{v}_i \), and the original measurements, \( s_i \) and \( v_i \), is as small as possible. Mathematically, this can be expressed by the optimization problem

\[
\|s - \hat{s}\|_2 + \lambda \|v - \hat{v}\|_2 \rightarrow \min_{s, \hat{s}}
\]

with respect to the constraint (3) where

\[
s = \begin{bmatrix} s_1 \\ \vdots \\ s_{I_1} \end{bmatrix}, \quad \hat{s} = \begin{bmatrix} \hat{s}_1 \\ \vdots \\ \hat{s}_{I_1} \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_{I_1} \end{bmatrix}, \quad \hat{v} = \begin{bmatrix} \hat{v}_1 \\ \vdots \\ \hat{v}_{I_1} \end{bmatrix}.
\]

The scalar weight \( \lambda > 0 \) is used to balance the correction of the speed and the arc length data. Task (4) can be solved numerically by least squares algorithms.

By applying that procedure to each of the samples in (2), the corrected samples

\[
(t_i, \hat{s}_i, \hat{v}_i) \quad (i = 1, \ldots, I)
\]

\[
(t_i, \hat{s}_i, \hat{v}_i)_r \quad (i = 1, \ldots, I_r)
\]

are obtained, which are used as the basis for the speed profile prediction algorithm.
3. PREDICTION OF THE SPEED PROFILE

In this section, the preprocessed samples \( (5) \) of a vehicle operating in fixed-route service will be used in order to predict the speed profile of the vehicle. First, the prediction problem will be formulated and the principle of the solution approach will be presented. Due to the limited memory space in vehicle control units, for the practical implementation of the online algorithm the information contained in the samples will then be described by means of a set of characteristic functions, which is significantly smaller than the set of samples. Finally, based on that representation of the samples, the prediction algorithm is formulated in such a way that it is suitable for implementation into a vehicle controller.

In order to simplify the algorithm, a common grid on the arc length of the vehicle trajectory is used. For each sample, the function \( f_j \) is defined to be the linear interpolant of the pairs \((s_1, v_1), \ldots, (s_k, v_k)\). Given an equally spaced grid of points \( s_l = l \cdot \Delta s, l = 1, \ldots, m \), on the arc length, the values \( v_j = f_j(s_l) \) of \( f_j \) on that grid are augmented into the vector

\[
v^{(j)} = [v_1^{(j)} \ldots v_k^{(j)}]^T.
\]

Similarly, the values \( s_l \) are augmented into the vector

\[
s = [s_1 \ldots s_m]^T.
\]

For reasons of simplicity, the vectors \( v^{(j)} \) will be named samples from now on.

The prediction problem to be solved is formulated as follows. Given the samples \( v^{(j)} \) \((j = 1, \ldots, r)\) and given current speed measurements \( v_1, \ldots, v_k \) at arc length points \( s_1, \ldots, s_k \), where \( k < m \), from the current speed measurements, the vector

\[
v = [v_1 \ldots v_k]^T
\]

defined, and, the related subvector of the \( j \)th sample \( v^{(j)} \) is denoted by

\[
v^{(j)}_{1:k} = [v_1^{(j)} \ldots v_k^{(j)}]^T.
\]

PREDICTION ALGORITHM (A)

For a given measurement of the current speed profile \( v \), chose a sample \( j \) that satisfies

\[
d := \| v - v^{(j)}_{1:k} \| < \delta_1
\]

and

\[
\{ i : \| v^{(i)} - v^{(j)} \| < \delta_2 \}
\]

is maximal. \( (8) \)

Then use \( v^{(j)}_{k+1}, \ldots, v^{(j)}_m \) as the prediction \( v_{k+1}, \ldots, v_m \) of the vehicle speed at \( s_{k+1}, \ldots, s_m \). Here \(|S|\) denotes the number of elements in the set \( S \), and, the variables \( \delta_1 \) and \( \delta_2 \) are given positive constants.

That algorithm finds a sample, which is for \( s_1, \ldots, s_k \) consistent with the information of the current speed measurements \( v_1, \ldots, v_k \) (as formulated by \((7)\)), and, which is a representative sample (as required by \((8)\)). As an interpretation, it is plausible to argue that a sample which is close to the currently measured speed profile \( v_1, \ldots, v_k \) at points \( s_1, \ldots, s_k \) , i.e., a sample that provides a sufficiently small value of \( d \), should also give good information about the future progression of the speed at the points \( s_{k+1}, \ldots, s_m \). In general, there will be more than one sample that is classified as being close to \( v_1, \ldots, v_k \). Hence a sample should be chosen, that has as many samples in its neighborhood as possible, and therefore, describes a typical trajectory within the set of all samples leading to small values of \( d \). The choice of the parameters \( \delta_1 \) and \( \delta_2 \) as well as the type of the norm will be specified within the description of the implementation of the prediction algorithm below.

In order to develop an algorithm that can be implemented in a vehicle controller with limited memory space available, a data compression step is needed. For that purpose the matrix

\[
A = [v^{(1)} \ldots v^{(r)}]
\]

is defined. Here, \( r \gg m \) is assumed, i.e., the number of samples is much larger than the number of grid points on the arc length. Let

\[
A = U \Sigma V
\]

be the singular value decomposition of \( A \) where the columns of the matrix

\[
U = [u_1 \ldots u_m]
\]

are the left singular vectors. Practically, the number of dominating singular values, denoted by \( n \), is much smaller than \( m \). From standard linear algebra results, it is known that each sample can be approximated by a linear combination of the first \( n \) left singular vectors

\[
v^{(j)} \approx [u_1 \ldots u_n] \begin{bmatrix} \alpha^{(j)}_1 \\ \vdots \\ \alpha^{(j)}_n \end{bmatrix},
\]

where \( \alpha^{(j)}_1, \ldots, \alpha^{(j)}_n \) are appropriate real numbers.

By using the approximation \((11)\), each sample \( v^{(j)} \in \mathbb{R}^m \) can be replaced by the related (shorter) vector

\[
\alpha^{(j)} := [\alpha^{(j)}_1 \ldots \alpha^{(j)}_n]^T.
\]

The vector \( \alpha^{(j)} \) will be named parameter vector in the sequel.

A further reduction of the required memory space can be obtained by discretization of the parameter space, i.e., defining a set of grid points and rounding of the parameter vectors to the grid points. By counting the number of parameter vectors rounded to a given grid point, the frequency function \( h \) of the rounded parameter vectors is obtained. Practically, the support of \( h \), denoted by \( B \), is a finite set of grid points, and moreover, it has turned out that \( B \) can be chosen to be constant, even if the number of samples will increase during vehicle operation. Since the inequality

\[
|B| \leq \sum_{\alpha \in B} h(\alpha) = r
\]

holds, a significant data reduction is obtained from \( m \cdot r \) real numbers stored in the matrix \( A \) (that still increases with the number \( r \) of samples) to a total number of \( |B| \) integers \( h(\alpha) \) (\( \alpha \in B \)).

Based on the representation of the samples by the frequency function \( h \) in the domain \( B \) and on the matrix \( U \), the prediction algorithm (A) can be put into a form that is suitable for implementation. Denote \( U_1 \) and \( U_2 \) the
submatrices of $U$ that consist of the first $k$ rows and the last $m - k$ rows of the matrix $U$, respectively.

**Prediction Algorithm (B)**

(1) With the set $M$ defined by

$$M := \{ \alpha \in \mathcal{B} : \| v - U_1 \alpha \|_2 < \delta_1 \}$$

(14)

chose the smallest $\delta_1 > 0$ such that for the number

$$r_M := \sum_{\alpha \in M} h(\alpha)$$

(15)

of samples leading to parameter vectors in $M$ the inequality

$$r_M > \zeta \cdot \sqrt{r}$$

(16)

holds, where $\zeta > 0$ is a given constant.

(2) Use the (not necessarily unique) parameter vector defined by

$$\alpha^* := \arg \max_{\alpha \in M} h(\alpha)$$

(17)

for the calculation of the prediction

$$[v_{k+1} \ldots v_m] = (U_2 \alpha^*)^T.$$  

(18)

The set $M$ contains all parameter vectors $\alpha \in \mathcal{B}$ that lead to a vector $U_1 \alpha$ (consisting of speed values at $s_1, \ldots, s_k$) which differs less than $\delta_1$ from the currently measured speed profile $v$. Since the euclidian norm is used, the set $M$ is an ellipsoid in the parameter space. On the one hand, the distance bound $\delta_1$ should be as small as possible in order to use only those samples for the prediction which are nearly identical to the currently observed speed profile $v$. On the other hand, the prediction should rely on a number of samples that is "large enough", which requires that $\delta_1$ is not too small. Moreover, due to the finite total number $r$ of samples, for small $\delta_1$ the set $M$ might be empty. The algorithm above handles this tradeoff by using a heuristic approach, which has been shown in simulations to be a practicable solution. It is based on the idea to chose $\delta_1$ in such a way that the number $r_M$ increases with $r$ but the ratio $\frac{r_M}{r}$ tends to zero for increasing $r$. The use of the square root in (16) is only one way to satisfy these requirements. The value of $\zeta$ allows the control of the ratio $\frac{r_M}{r}$ when the algorithm is put into operation for the first time based on a given initial number of samples.

With (17) the parameter vector $\alpha^*$ is chosen to be the most frequent one of all parameter vectors in $\mathcal{B}$. This means that the speed values $v_{k+1}, \ldots, v_m$ predicted by (18) are calculated based on a sample which is most frequent amongst all samples that are similar to the observed speed values $v_1, \ldots, v_k$ on the arc length points $s_1, \ldots, s_k$.

The prediction algorithm (B) is one option of implementation of the prediction algorithm (A). The rounding of the parameter vectors to the grid points and the counting of the frequency of each rounded parameter vector can be considered as a realization of the idea lying behind (8) since the rounding to the grid points can be interpreted as the application of a weighted maximum norm in (8). The choice of a sample that is close to the currently measured speed values $v_1, \ldots, v_k$ in (7) is realized by (14) in conjunction with (17).

4. SIMULATION EXAMPLE

The example is based on a data set of the form (1) obtained by measurements in a tram vehicle operating in fixed-route service in public transport in the city of Dresden in Germany. Since the route of the tram vehicle is partially shared with other vehicles, the movement of the tram is highly disturbed and the samples have very different characteristics. In Figure 2 the measured speed profiles over the arc length between two regular stops are shown. There are $r = 131$ samples available, the total length of the route is $360$ m. The additional stops at $s = 50$ m and $s = 200 \ldots 220$ m are caused by traffic lights.

Fig. 2. Samples of the vehicle speed profile between two regular stops

In order to illustrate the results of the prediction algorithm described in Section 3, one of the samples is taken out of the set (1) of measurements, respectively. That sample is used for the validation of the prediction algorithm, i.e., it is considered as the speed profile to be predicted.

The remaining samples are preprocessed as described in Section 2. Then, on a common grid on the arc length defined by $s_j = l \cdot \Delta s$, $l = 1, \ldots, m$, $\Delta s = 2$ m, $m = 180$, the speed profiles (6) are calculated. The matrix $A$ is built up as in (9), and, via singular value decomposition, the matrix $U$ is obtained. For illustration purposes, the matrix $U$ is restricted to two columns, i.e., $n = 2$ holds in (11) and the parameter space is only two-dimensional. The grid in the parameter space is equally spaced over the range shown in Figure 3 with a step width of 1. The parameter vectors $\mathbf{a}^{(j)}$ related to the sample $v^{(j)}$ are calculated according to (11) and based on that, the frequency function $h$ and its support $\mathcal{B}$ is determined. For the given samples, the distribution of the parameter vectors $\mathbf{a}^{(j)} = [a_1^{(j)} a_2^{(j)}]^T$ rounded to the grid points of the parameter space is depicted in Figure 3. After these preliminary steps, the prediction algorithm (B) from Section 3 can be applied.

The results obtained with that algorithm are depicted in Figures 4 to 9. In Figure 4 the solid line shows the speed profile of the vehicle between the two regular stops at $s = 0$ m and $s = 360$ m. This is the sample taken out of the set of all samples as mentioned above. The current step is $k = 10$, i.e., the current arc length is $s_k = 20$ m, which is marked by a circle. That means that the speed at the arc length points $s_1 = 2$ m, $\ldots, s_k = 20$ m is already known and the speed at $s_{k+1} = 22$ m, $\ldots, s_m = 360$ m has to be predicted. In Figure 5, the set $M$, which is calculated with $\zeta = 1$ throughout this example, is depicted by dots.
The parameter vector $\alpha^*$ determined by (17) is marked by a triangle symbol. Additionally, the vector

$$\hat{\alpha} := \arg\min_{\alpha \in \mathbb{R}^2} \| [v_1 \ldots v_m]^T - U^T \alpha \|_2,$$

(19)

which is the optimal parameter vector (not rounded to the grid) in case that the speed profile were known a priori, is marked by a cross symbol. As can be seen in Figure 5, the information available at $k = 10$ is not sufficient to get the vector $\alpha^*$ close to the vector $\hat{\alpha}$. The resulting speed profile prediction is shown in Figure 4. Up to $s_k = 20$ m the dashed-dotted line depicts the approximation of the measured speed profile given by

$$[v_1 \ldots v_k]^T = U_1 \alpha^*.$$ 

From $s_k = 22$ m to $s_k = 360$ m the dashed-dotted line shows the prediction of the speed profile calculated by

$$[v_{k+1} \ldots v_m]^T = U_2 \alpha^*.$$ 

In Figures 6 and 7 and in Figures 8 and 9 the results for $k = 30$ and $k = 90$ are shown, respectively. It can be seen that the more information is gained with increasing $k$, the closer the parameter vector $\alpha^*$ comes to $\hat{\alpha}$, and, the more accurate the prediction at the arc length points $s_{k+1}, \ldots, s_m$ is. The prediction calculated at $k = 90$ does not change noticeable for $k > 90$ so that the real speed profile can be foreseen at about half of the total distance between the two stops.

It should be mentioned that for illustration purposes, the sample to be predicted was chosen in such a way that the optimal value of the objective function in (19) is small, i.e., the chosen sample is suitable to be approximated by only two left singular vectors. For the considered example of a tram operating in fixed-route service, the number

$n$ of left singular vectors necessary to parametrize all samples is larger than $n = 2$, depending on the required accuracy $n$ is in the range from 5 to 10. Moreover, simple considerations show, that the advantage of the proposed
prediction algorithm over just using the mean of the speed profiles strongly depends on the characteristics of the speed profiles, which means, that an accurate prediction presupposes that there is a correlation between the past and the future in each of the speed profiles. In other cases of operation, the degree of compliance with that restriction should be clarified prior to the application of the proposed algorithm.

5. SUMMARY

In this paper, an approach towards the real-time prediction of the speed profile of a vehicle operating in fixed-route service is presented in order to use this information for the predictive control of the vehicle hybrid drive train. The proposed algorithm utilizes previously taken measurements, e.g., by a GPS device, of the vehicle motion during fixed-route travel. That history information together with measurements along the currently driven part of the route is processed in order to predict the future speed profile of the vehicle. From the predicted speed profile, the calculation of the load power demand by using a model of the drive train and the vehicle dynamics is straightforward.

The main idea of the approach is the selection of a sample out of the entire set of training samples in the history database in such a way that this sample gives maximum information about the future progression of the current vehicle speed. In order to obtain a prediction algorithm that is implementable into a vehicle controller several steps were performed. First, the samples were preprocessed for noise suppression and data compression. Then by the prediction algorithm, a speed profile is chosen out of the samples in the history database in such a way that it is close to the currently measured speed profile and representative for speed prediction. The practicability of that approach is illustrated by using real-life data sets obtained in a tram vehicle.

The future tasks include the comparison of the obtained results to the outputs of prediction models that use transition probabilities for the description of the possible future progression of the speed, e.g., Markov chains. Since these methods require more extensive data sets which are not yet available, this is subject to future investigations.

In this paper the prediction of the speed profile between two regular stops of the vehicle in fixed-route service is considered. Another promising way seems to be the use of a receding prediction horizon strategy. Here, an important topic is the time-weighting of the past speed measurements for the current ride and the appropriate choice of the length of the prediction horizon.

Finally, it is planned to apply the approach to other vehicles and operational conditions, e.g., to trams with exclusive right of way, buses in urban fixed-route service and also to vehicles in individual service on known routes.

REFERENCES