Cooperative control of underwater glider fleets by fault tolerant decentralized MPC

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Abstract: A fleet of Autonomous Underwater Vehicles (AUVs) moving together in a prescribed pattern can form an efficient data acquisition network. The problem is to control several AUVs such that after transients, they form a required formation and move along a desired trajectory. The capability to accomplish the mission even in case of faults, is a fundamental requirement for these kind of missions. A completely decentralized predictive control and FDI strategy is here proposed for allowing cooperation. Through an underwater communication channel, each vehicle broadcasts its position, its future behavior and its actuator/sensor fault situation. Based both on the local and the received information, each vehicle selects the desired formation to keep and plans its future actions. Simulation results are provided to validate the approach.

Keywords: Predictive Control, Decentralized Systems, Co-operative Control, Robot Navigation, Fault Tolerance.

1. INTRODUCTION

Exploration by means of unmanned multi-robot formations is gaining interests in different fields. The advent of satellite navigation and communication made possible the advent of small inexpensive autonomous underwater gliders that can observe the ocean in place of the humans and provide high quality and high density information. Marine exploration by means of autonomous glider fleets allows to save money and to safely perform dangerous missions. Autonomous underwater gliders use a buoyancy engine to move following a sawtooth path where satellite navigation and communication are carried out at the surface. The considered vehicles are highly nonlinear and the feasible control actions are constrained.

Different decentralized solutions to the formation control of multiple autonomous vehicles have been recently developed (Keviczky et al. [2006], Dunbar and Murray [2006], Fang and Antsaklis [2006]). In Balderud et al. [2006, 2007] operational robustness is addressed for applications where clusters of autonomous vehicles are deployed for achieving a shared common objective. In order to improve autonomy and performance of the decentralized control architecture, different cooperative solutions based on communication exchange have been proposed in literature. A decentralized robust Model Predictive Control (MPC) algorithm for multi-vehicle trajectory optimization is presented by Kuwata et al. [2006]. A decentralized MPC for formation control have been developed, analyzed and compared with the centralized solution by Vaccarini and Longhi [2007a,b].

One of more important requirements for autonomous underwater vehicle fleets concerns fault tolerance, i.e., the capability to accomplish the mission even in case of fault of one or more vehicles. The vehicle fleets should have the capability of maintaining a formation in the presence of fault of one or more members, such as the loss of a vehicle, with a high degree of autonomy. This can be achieved only through the development of a fault tolerant decentralized system, where each vehicle is equipped with a local Fault Detection and Isolation (FDI) system. The diagnostic and fault tolerant problems have been widely investigated, and there exist many publications on these subjects (e.g. Patton et al. [2000], Blanke [2003]). Surveys on FDI approaches are provided in Isermann [1997], Frank [1990] and Patton et al. [2000]. Most papers in this field have been concentrated on the single-vehicle case (see Monteriù et al. [2007], Zhuo-hua et al. [2005]). Fault tolerant problem for a single underwater vehicle has been investigated by many researchers (see Perrault and Nahon [1998], Podder and Sarkar [2001], Lingli [2001]). Fault tolerant for multi-vehicle formations is discussed in Antonelli and Chiaverini [2004], Daigle et al. [2007].

In this paper, the formation control of a system of multiple autonomous underwater vehicles is considered. The agents have to maintain a given formation while moving according to a given trajectory. Only the motion on the horizontal plane is considered for control purposes. The formation control problem is simultaneously solved by a set of autonomous control agents which make use of vehicle’s nonlinear models for imposing a defined relative displacement. Cooperation based on Decentralized Model Predictive Control (D-MPC) and an information exchange through an underwater local area network is proposed for improving the global control performance and for managing faulty situations. In such a way the cooperation itself is achieved by a fault tolerant decentralized solution. The developed solution enables vehicles to continue to complete given tasks by reorganizing their formation, when some members are in fault. In case of unrecoverable fault on a leader vehicle, each follower reconfigures the controller for formation rearrangement. The information flow changes with the formation pattern. Faults in the network connections are recovered by changing the leaders and rerouting the information flow. Simulation results show that this approach is robust and tolerant to loss of communication or loss of any vehicle of the fleet.
2. VEHICLES

2.1 Sensors and Actuators

An underwater glider is powered by an internal blader/ballast which is inflated to vary the buoyancy and providing vertical lift forces. A sliding mass is used for fine adjustments in pitch and roll. Although many different actuation systems are employed by the different manufacturers, all of them are equivalently described by an internal moving mass $m_b$ which correspond to a blader/ballast. By varying the position of $m_b$ with respect to the Centre of Buoyancy (CB), the pitch and roll movements of the vehicle are controlled. By adjusting $m_b$, buoyancy of the vehicle can be regulated in order to produce vertical displacements and, therefore, horizontal displacements produced by the wings.

Sensors measure depth, pitch, roll, and compass heading. Additional sensors can be added in order to improve the performances and to allow for fault recovery. For instance, compasses and inclinometers could be added for the attitude where gyros and accelerometers could improve the estimation of the velocities. The effect of underwater currents can be compensated through current estimators and relative controllers. Vehicle absolute position is determined at the surface through Global Positioning System (GPS).

2.2 Communication

The proposed solution is based on coordinated independent agents and on a Local Area Network used for coordinating them. Each agent implements a decentralized MPC policy on the basis of both local and external information acquired by the network.

When a glider dives it uses and merges sensors data with information coming from the other neighboring gliders. The underwater communication is performed by means of sonar modems that allows to reach vehicles within a defined radius. In this way, sensor fusion and decentralized localization techniques are applied for improving the localization of the whole formation. The underwater communication is also used to exchange the vehicle configurations and it allows to assume the fully accessible state hypothesis.

Technologies for underwater communication are already available and allows multiple access to the communication channel by using Code Division Multiple Access (CDMA) sonar modems. Since the available bandwidth of these modems is relatively tight, the information traffic have to be reduced to the minimum necessary. Some problems may also occur due to multiple paths and echoes, and this may produce temporary unavailability of the network that must be taken into account.

2.3 Formation Vector Model

Let consider a set of $N$ underwater gliders $\mathcal{V}^i$, $i = 1, \ldots, N$, that should accomplish to the considered formation keeping task: for each vehicle the position of the leader vehicle with respect to him should be kept equal to a desired value. Assuming that at time $t$ a low level controller imposes the desired surge, sway and yaw (angular) speeds $v^i(t)$, $s^i(t)$ and $w^i(t)$ on the horizontal plane by adjusting the position $r$ of mass $m$ and the value of buoyancy mass $m_b$. Assume to sample the continuous-time variables with sampling interval $T_s$ and define the sampled variables $v^i_k \triangleq T_s v^i(kT_s)$, $s^i_k \triangleq T_s s^i(kT_s)$, $w^i_k \triangleq T_s w^i(kT_s)$ that represents finite movements within each sampling interval $T_s$. These movements can also be seen as velocities normalized w.r.t the sampling interval $T_s$ and, in the following, they will be referred as velocities.

Due to physical limits, the positions of mass $m$ and the values of mass $m_b$ are constrained to stay within a given range. This implies that surge, sway and angular velocities of the vehicle $v^i_k$, $s^i_k$, $w^i_k$ are constrained and their limits depend on level controller and on dynamic behaviour of the vehicle. However, for the sake of simplicity, fixed constraints are assumed in the following:

\[
\begin{align*}
\bar{v} \leq v^i_k \leq \bar{v}, & \quad \bar{s} \leq s^i_k \leq \bar{s}, & \quad |w^i_k| \leq \bar{w}, \\
|\Delta v^i_k| \leq \Delta \bar{v}, & \quad |\Delta s^i_k| \leq \Delta \bar{s}, & \quad |\Delta w^i_k| \leq \Delta \bar{w}.
\end{align*}
\]

Defining rotation matrix $T(\alpha) \triangleq \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the absolute vehicle configuration on the horizontal plane $q^i_k \triangleq [x^i_k \ y^i_k \ \theta^i_k]^T$ is determined by integrating the control action $u^i_k \triangleq [v^i_k \ s^i_k \ w^i_k]^T$ by the following discrete-time kinematic model:

\[
q^i_{k+1} = q^i_k + T^{-1}(q^i_{k+1})u^i_k. 
\]

Referred to the frame fixed to vehicle $\mathcal{V}^i$, the relative displacement of vehicle $\mathcal{V}^j$, $d^{ij}_k \triangleq [x^{ij}_k \ y^{ij}_k \ \theta^{ij}_k]^T = T(q^i_{k+1})(q^j_{k+1}) - q^i_k$ by some manipulation (Vaccarini and Longhi [2007b]) gives the following discrete-time vector model:

\[
d^{ij}_{k+1} = A^i_k d^{ij}_k + B^i_k u^i_k + E^i_k u^j_k.
\]

3. FORMATION CONTROL PROBLEM

The formation control problem is here formulated as a cascaded leader-follower problem (see Figure 1) in which:

**Assumptions 1.**

- The reference trajectory $T^*$ is generated by a virtual reference vehicle $\mathcal{V}^0$ which moves according to the considered unicycle model.
- Each vehicle $\mathcal{V}^i$, $i \neq j$, follows one and only one leader $\mathcal{V}^j$; $\mathcal{V}^i$ follows virtual vehicle $\mathcal{V}^0$ which exactly tracks the reference trajectory $T^*$.
- Each vehicle $\mathcal{V}^i$ should keep the reference formation pattern $d^{ii}_j$ from its leader $\mathcal{V}^j$.

In this framework the multi-vehicle formation is a directed tree that can be formally expressed by the notation of the directed graphs. Let denote with the tuple $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ the digraph with nodes $\mathcal{V} \triangleq \{\mathcal{V}^1, \ldots, \mathcal{V}^m\}$ and edges $\mathcal{E} \triangleq \{e^1, \ldots, e^m\}$, in which each edge is an ordered pair of nodes $e^i = (\mathcal{V}^j, \mathcal{V}^i)$ establishing a link from $\mathcal{V}^j$ to $\mathcal{V}^i$. Each node corresponds to a real vehicle and the direction of the edge goes from the leader to the corresponding follower, in agreement with the information flow.
In a general leader-follower formation, it is possible to distinguish between leaders and followers, but the leader of the whole formation is unique. The formation can be divided into several layers, depending on the member’s level in the formation, as shown in Figure 1. In this way, any member in formation is a node in the hierarchy graph. The first layer has only one node, namely the father node, which represents the leader of the whole formation. Second layer could have one or different local nodes which are child nodes of the father node, and so on. The direct connections occur among father and local nodes, or local and child nodes. Note that there are not connections among nodes of the same layer, but connections are possible only from nodes of different layers. This analysis is valid also in the event of a different formation shape. In effect, all connections are logical, indicating that the formation is not fixed, and nodes in the same layer need not be parallel.

The formation can be completely described by the graph, its adjacency matrix and its incidence matrix. The adjacency matrix $\mathbf{A}$ of digraph $G$ has $n \times n$ entries $a_{ij} = 1 \ iff \ (V^i, V^j) \in E$ and $a_{ij} = 0$ otherwise. The incidence matrix $\mathbf{C}$ of digraph $G$ has $n \times m$ entries $c_{il} = 1 \ iff \ edge \ \mathcal{E}_l \ exits \ from \ node \ V^i$, $c_{il} = -1 \ iff \ edge \ \mathcal{E}_l \ enters \ in \ node \ V^i$ and $c_{il} = 0$ otherwise.

According to Vanni et al. [2007], the formation control problem is decomposed into an inner-loop dynamic task, which consists of making the vehicle’s velocity track a reference one, and an outer-loop kinematic task, which assigns the reference speed for tracking a desired trajectory.

### 3.1 Inner-loop dynamic controller

Inflates the bladder and moves the masses for achieving the desired speeds (surge, sway and angular speeds). The low level controller is assumed to drive the internal masses and inflate the bladder in order to track velocities $v, s, w$. With this assumption, the considered high level control problem becomes a path planning problem for the low level controller. The high level controller should define the optimal speeds $v, s, w$ that allows to keep the desired formation with the minimum possible efforts.

The control vector $\mathbf{u}_k = [v_k, s_k, w_k]^T$ is the reference vector for the lower level controller which moves $\dot{m}$ and changes $m_k$ in order to track these velocities.

### 3.2 Outer-loop kinematic controller (Decentralized MPC)

Given the desired vehicle to track, it plans the future movements over a fixed horizon, send them to the neighbors and applies the first sample.

In this subsection, a decentralized control strategy based on Networked MPC is introduced. Each vehicle $V^i$ is equipped with an independent control agent $A^i$ which collects local and remote information and iteratively performs a nonlinear optimization for computing the local control action. As previously stated, each vehicle $V^i$ tracks a leader $V^l$ with a defined displacement. The set of all displacements defines the formation (Figure 1).

Note that for allowing coordination among the vehicles, the following assumptions will be considered:

**Assumptions 2.**

- Each control agent $A^i$ communicates with its neighboring agents by a Local Area Network (LAN) only once within a sampling interval.
- The communication network introduces a delay $\tau = 1$.
- The agents are synchronous.
- Each control agent is able to measure the relative configurations of the neighbours.

The previously described localization system, composed by a GPS integrated with proprioceptive sensors, allows to determine the $q_k^i$ configuration of each vehicle $V^i$. As a result, the state $d_k^j$ can be considered fully accessible for the considered control problem, at least in this stage. In addition, using sonar underwater communication, the synchronization among the agents is achieved. The drifts of the different clocks are very slow and each agent $A^i$ synchronizes its own clock with the clock of its leader agent $A^l$. In fact, each leader broadcasts to its followers the predictions about its future behaviour together with its clock data.

The interaction vector $u_k^j$ is unknown for the local control agent $A^i$. By the Local Area Network the local agent acquires only predictions about the future behaviour $\{u^{k|k-1}_{k-1} \bar{u}^{k+1|k-1} \bar{u}^{k+1|k-1} \ldots\}$ generated by the other independent agents. Defining the product matrix of a set of indexed matrices as $\prod_{n=1}^m A_n \equiv A_1 \cdot A_2 \cdot \ldots \cdot A_m$, the $h$-ahead prediction computed at time $k$ of a vector (matrix) as $\bar{v}_k \equiv A^k h+k k \bar{v}_k$ and the change at time $k$ of vector $v_k$ as $\Delta v_k \equiv v_k - v_{k-1}$, by (3) the state predictions at time $k$, for all $h \geq 1$, are given by:

$$
\dot{\bar{d}}^{ji}_{k+1\ldots h} = \prod_{n=1}^h A^{ji}_{k+h-n|k} \bar{d}^{ji}_{k|k} + \sum_{l=1}^{h-1} \bar{A}^{ji}_{k+h-n|k} \cdot \left[ \bar{B}^{ji}_{k+l-1|k} \bar{u}^{k+l-1|k} + \bar{E}^{ji}_{k+l-1|k} \bar{u}^{k+l-1|k} \right] + \bar{B}^{ji}_{k+h-1|k} \bar{u}^{k+h-1|k} + \bar{E}^{ji}_{k+h-1|k} \bar{u}^{k+h-1|k},
$$

(5)
where the receding horizon strategy imposes that $u_{i}^{k} \triangleq \hat{u}_{i|k}$ and the hypothesis of fully accessible state implies that $d_{i|k}^{ji} = d_{i|k}^{j}$. In order to evaluate the performance of a follower $A_{i}$, a measure of the difference between the predicted formation vector $d_{i|k}^{ji}$ and the constant desired displacement $d_{i}^{j}$ is needed. The following scalar is chosen here as a measure of the performance for control agent $A_{i}$:

$$
\langle d_{i}^{j} - d_{i|k}^{ji} \rangle^{2} \triangleq \rho_{s}(\sigma_{k}^{ji} - \bar{x}_{i}^{ji})^{2} + \rho_{g}(y_{k}^{ji} - \bar{y}_{i}^{ji})^{2} + \rho_{b} \sin^{2} \frac{\theta_{i}^{ji} - \bar{\theta}_{i}^{ji}}{2}.
$$

Let define the cost function as:

$$
J_{k}^{i} = \sum_{h=1}^{p} \langle d_{k+h|k}^{ji} - \hat{d}_{i|k}^{ji} \rangle^{2} + \mu \langle \hat{u}_{k+h|k}^{i} \rangle^{2} + \sigma \langle \Delta \hat{u}_{k+h|k}^{i} \rangle^{2} + \eta \sum_{h=1}^{p-1} \langle |\hat{u}_{k+h|k}^{i} - \hat{u}_{k+h-1|k}^{i}| \rangle^{2},
$$

(7)

which penalizes control efforts $\hat{u}_{k+h|k}^{i}$, large changes on the control effort $\Delta \hat{u}_{k+h|k}^{i}$ and deviations from the previously broadcasted control profile $\hat{u}_{k+h|k}^{i}$.

**Definition 1.** (Decentralized MPC problem). Given desired state $d_{i}^{j}$ and positive scalars $\mu$, $\sigma$ and $\eta$, the Decentralized MPC problem for vehicle $V^{i}$ at sample time $k$ consists in solving the nonlinear optimization problem:

$$
\min_{\hat{u}_{k}^{i}} J_{k}^{i},
$$

subject to physical constraints (1) and predictive model constraint (5). The first sample $u_{k+h|k}^{i}$ of the optimal control profile $u_{k}^{i*}$ is defined by:

$$
u_{k}^{i*} = \begin{cases} u_{k}^{i*} & l = 0, \ldots, p - 1 - \tau, \\

\hat{u}_{k+p+1-\tau|k}^{i*} & l = p - \tau, \ldots, p - 1. \end{cases}
$$

This nonlinear constrained optimization problem is iteratively set up and solved at each sample time by minimization algorithms such as the fmincon Matlab® function.

It implements a Sequential Quadratic Programming algorithm that iteratively solves QP sub-problems by means of the active set strategy and a positive quasi-Newton approximation of the Hessian of the Lagrangian.

The following result holds.

**Proposition 1.** (Stability). Consider the set $V$ of all vehicles $V^{i}$, $i = \ldots, N$ with structure (3) and, for each vehicle $V^{i}$ with leader $V^{j}$, an independent agent $A_{i}$ minimizing cost function (7) under condition:

$$
\langle d_{k+p|k}^{j} - \hat{d}_{i|k}^{j} \rangle^{2} + \mu \langle \hat{u}_{k+p|k}^{i} \rangle^{2} + \sigma \langle \Delta \hat{u}_{k+p|k}^{i} \rangle^{2} \leq r_{k}^{i}, 
$$

(9)

where $r_{k}^{i}$ is known at time $k$ and defined as:

$$
r_{k}^{i} = \langle d_{k+p|k}^{j} - \hat{d}_{i|k}^{j} \rangle^{2} + \mu \langle \hat{u}_{k+p|k}^{i} \rangle^{2} + \sigma \langle \Delta \hat{u}_{k+p|k}^{i} \rangle^{2} - \sum_{h=1}^{p} \langle d_{k+h|k}^{j} - \hat{d}_{i|k}^{j} \rangle^{2} - \langle d_{k+h|k}^{j} - \hat{d}_{i|k}^{j} \rangle^{2} - \langle d_{k+h|k}^{j} - \hat{d}_{i|k}^{j} \rangle^{2},
$$

(10)

then the set $A$ of control agents $A_{i}$, $i = 1, \ldots, N$, guarantees the local stability (in the sense of Lyapunov) of the equilibrium point $d = [(d_{C,1}^{i})^{T}, \ldots, (d_{C,N}^{i})^{T}]^{T}$ for the whole closed-loop system.

The proof is not provided here for the sake of brevity. However, it is obtained by choosing as Lyapunov function candidate the optimal value of cost function and using the previous control profile completed with one more control action as upper bound for the Lyapunov function (Vaccarini and Longhi [2007b]).

Condition (9) defines a particular terminal constraint set for the state prediction $\hat{u}_{k+p|k}^{i}$ at the end of the horizon, computed with the previous optimal control sequence and the last current control action $\hat{u}_{k+p-1|k}^{i}$. This set is a sphere centred in $d_{i}^{j}$ with radius dependent on the last control action and movement and the term $r_{k}^{i}$. Therefore, the tuning parameters that influence the stability are the prediction horizon $p$ and the weights $\mu$ and $\sigma$. Weight $\eta$ does not enter in constraint (9) and can be used for improving the control performances of the followers by penalizing control performances of the leader.

4. FAULT TOLERANCE FOR UNDERWATER GLIDER FLEETS

Safety of a single vehicle of the fleet can be guaranteed providing it of a local FDI system. Moreover, in order to improve the probability of success of the whole formation in achieving the mission task, an higher level decentralized fault management system is needed. The functioning of the local FDI system and of the decentralized fault-tolerant scheme are illustrated in the next subsections.

4.1 Local FDI and Reconfiguration Control

Each vehicle of the fleet is equipped with a model-based fault diagnostic system which is able to detect and isolate a fault on one of its sensors or actuators within a bounded “delay”. Once a fault is detected and isolated, the low level dynamic controller is reconfigured using the redundant actuators/sensors. After a fixed time delay, if the local detected fault has not been resumed, the faulty vehicle leaves the formation following a prescribed escape maneuver, which brings it safely outside the formation. Before taking the escape route, the faulty vehicle switches off its communication module to prevent the vehicles that were using it as a reference to follow. The basic idea for the safe escape procedure is that the glider goes directly to the sea surface, either inflating air into the bladder or deflating the water to the ballast. This avoids possible collisions among the vehicles, and prevents that the vehicle remains to the sea-bottom.

4.2 Decentralized Fault-Tolerant Scheme

The decentralized FDI enables vehicles to continue to complete given tasks by reorganizing their formation, when some faults occur. Two kinds of faults are possible: “communication faults” and “vehicle faults” (Cheng and Wang [2004]).
Fault detection on "communication / vehicle"

All vehicles continue the mission using the MPC law

Faulty vehicle activates an escape maneuver

Replace faulty vehicle

Rearrange the formation with the remaining vehicles

Keep the formation in the original shape

Is faulty vehicle recovered within fixed time delay?

Faulty vehicle is not recovered

Faulty vehicle recovered

Fig. 2. Fault tolerance procedure.

When a communication fault occurs in a vehicle, the leader or the follower of that vehicle will lose their connectivity and no information is exchanged. The leader or follower of that vehicle will try to reconnect the faulty vehicle and, after a fixed time delay, if the vehicle does not reply, the other vehicles know that such vehicle is in communication fault. The maximum time delay is fixed in the prediction horizon \( p \), and until the faulty vehicle is not reconnected, the predictions on its future behaviour are obtained from the previous data. If vehicle \( \mathcal{V}^j \) doesn’t reply for a time delay \( 1 \leq \tau \leq p - 1 \), expression (8) is used by the follower in order to predict its future behaviour.

If a glider is in a vehicle fault, its local FDI system and control will try to recover the fault. In this situation, the vehicle broadcasts its current position and faulty information to the others vehicles of the fleet. In such situation, all other vehicles of the fleet know which vehicle is into vehicle fault.

Fault tolerance procedure

The fault-tolerance mechanism introduced here is essentially based on hierarchy graph theory of leader-follower formation. In case of unrecoverable fault on a leader vehicle, the local diagnostic system detects the fault and, when possible, it broadcasts its faulty situation. Recognizing the faulty situation in the multi-vehicle formation, each follower reconfigures the controller for formation reconfiguration. Faults in the network connections are recovered by changing the leaders and rerouting the information flow.

The fault tolerance procedure developed for a fleet of underwater vehicles is resumed in Figure 2. After a communication/vehicle fault has been detected, the FDI system of the faulty vehicle tries to recover the fault. If this is possible, the vehicles rearrange the fleet in the original shape. In case of unrecoverable fault, the faulty vehicle moves away from the formation through an escape maneuver, and the remaining vehicles substitute the faulty vehicle and rearrange the fleet obeying to some pre-assigned formation shape, depending by the number of remaining vehicles.

The substitute for faulty vehicle must be one of its child nodes in hierarchy graph of the fleet. Through two priority rules, it is determined the vehicle which takes the place of the faulty vehicle Cheng and Wang [2004]:

i. The number of total follower vehicle nodes is larger, while the priority is lower; ii. The pre-assigned sequence number in the hierarchy is smaller, while the priority is higher.

In this way, if by the first rule is impossible to determine the substitute, than this is univocally chosen through the second rule. Note that the rank of the first rule is higher than that of the second rule.

For each structure of the formation, the two hierarchy rules (i,ii) can be always summarized by a unique label assignment to the vertices of graph \( \mathcal{G} \) that becomes a vertex-labeled graph. Assigning the vehicle’s names as the hierarchy levels, vehicle \( \mathcal{V}^j \) has highest priority with respect to vehicle \( \mathcal{V}^i \) iff \( j < i \).

If vehicle \( \mathcal{V}^i \) is in unrecoverable fault situation, the replacement policy establishes that it is replaced by the subsequent child vehicle \( \mathcal{V}^j \) in the hierarchy and it is excluded from the formation. This change in the graph is an editing operation \( \varphi \) that maps vertices and edges between the two graphs by two functions. This is equivalent to the following transformations in the adjacency:

\[
\tilde{A} = E_{(j)} P_{(i,j)} A P_{(i,j)} E_{(j)}^T \quad (11)
\]

where \( P_{(i,j)} \) is the permutation matrix made by exchanging \( i \)-th and \( j \)-th rows of an \( n \times n \) identity matrix, and \( E_{(j)} \) is an elimination matrix made by removing \( j \)-th row from an \( n \times n \) identity matrix. Incidence matrix is transformed as:

\[
\tilde{C} = P_{(i,j)} C_{(i)} \quad (12)
\]

where \( I_{(j)} \) is an isolation matrix made by removing \( i \)-th column from an \( n \times n \) identity matrix.

5. SIMULATION RESULTS

The developed strategy is here tested on the formation control of an underwater glider fleet composed by \( N = 5 \) vehicles. Gaussian measurement noise have been added in order to make the simulations more realistic. A simulation of 150 samples is proposed here with tuning parameters \( \rho_x = 1, \rho_y = 10, \rho_w = 200, \mu = 0.5, \sigma = 1, \eta = 0.4, p = 5 \) and constraints:

\[
0.2m \leq v_k^i \leq 0.5m, \quad |\Delta v_k^i| \leq 0.1m, \quad (13a)
\]

\[
-0.1m \leq s_k^i \leq 0.1m, \quad |\Delta s_k^i| \leq 0.05m, \quad (13b)
\]

\[
-0.1rad \leq \psi_k^i \leq 0.1rad, \quad |\Delta \psi_k^i| \leq 0.05rad. \quad (13c)
\]

The obtained results are reported in Figure 3 for a randomly moving virtual vehicle \( \mathcal{V}^\varphi \). The performed trajectories and the vehicles are drawn with different colors and the fleet configurations have been frozen at five significant conditions. The fault occur exactly in the middle on vehicle \( \mathcal{V}^2 \) identified by a circle. The followers try to recover the vehicle by waiting for \( p \) samples. After that, \( \mathcal{V}^2 \) is excluded by the fleet and the formation is rearranged in a new one, following the presented hierarchy rules and shapes. In particular, after the single fault occur, vehicle \( \mathcal{V}^2 \) is replace by its child node \( \mathcal{V}^3 \) and the desired formation shape changes. Vehicle \( \mathcal{V}^\varphi \) is more penalized than the others because the bends in the desired path imposes to him a big speed and the fault requires the replacement of the faulty vehicle. However, results show that the control performances are satisfactory and the control strategy is effective even in presence of faults.
The use of MPC allows to take into account constraints, behaviour predictions among the agents and allows the vehicles to accomplish their tasks also in presence of temporary failures in the vehicles/network. When an unrecoverable fault occur, the proposed decentralized fault tolerant scheme is able to reconfigure the formation. The decentralization improves autonomy and reliability and MPC provides good control performances. Simulations have shown the effectiveness of the developed approach.

Further research is needed in order to generalize the approach by improving feasibility and considering non collision constraints among the agents. In order to complete the architecture, the dynamic controller of the glue have to be developed, for instance by an MPC inner-loop. For the real-time implementation of the control strategy, optimization of the control algorithm is required. Improvement of the overall proposed solution could be reached through the application of underwater vehicles with strong actuators/sensors redundancy. This aspect is still under investigation.

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