Compliant coordination and control of multiple vehicles with discrete-time periodic communications

João Almeida ∗ Carlos Silvestre ∗ António Pascoal ∗

∗ Institute for Systems and Robotics, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal (e-mails: {jalmeida,cjs,antonio}@isr.ist.utl.pt)

Abstract: This paper addresses the problem of coordinated path-following of networked autonomous vehicles with discrete-time periodic communications. The objective is to steer a group of autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern. For a class of vehicles, we show how Lyapunov based techniques, graph theory, and results from networked control systems can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. Compliant vehicle coordination is achieved by adjusting the speed of each vehicle along its path according to information exchanged periodically on the positions of a subset of the other vehicles, as determined by the communications topology adopted. The closed loop system, obtained by putting together the path-following and coordination strategies, takes an interconnected feedback form where both systems are input-to-state stable (ISS) with respect to the outputs of each other. Stability and convergence of the overall system are guaranteed for an adequate choice of gains.

1. INTRODUCTION

Multi-vehicle systems have received much attention by the research community over the last few years. Applications include such diverse scenarios as seafloor surveying, traffic monitoring, and forest fire detection, among others. In all scenarios, coordination among the vehicles is essential in order to accomplish the mission goals in an efficient and robust manner. In most cases, coordination is achieved through the exchange of information among vehicles, implying that control mechanisms must be designed taking into account such practical constraints as limited bandwidth and intermittent communication failures.

As a contribution to the study of these issues, this paper addresses the problem of coordinated path-following (CPF, see, e.g. Ghabcheloo et al. [2006a], Ghabcheloo et al. [2006b], Ihle et al. [2006]) with discrete-time periodic communications, whereby a set of vehicles is required to follow pre-defined spatial paths while keeping a desired inter-vehicle formation pattern in time. This problem arises, for example, in the operation of multiple autonomous underwater vehicles for fast acoustic coverage of the seabed. By imposing constraints on the inter-vehicle formation pattern, the efficacy of the task can be largely improved. Also, due to the environment involved (water) the bandwidth for communications is severely reduced, allowing only for communications in short and small bursts.

We solve the CPF problem for a class of fully-actuated autonomous vehicles moving in two-dimensional space. Nevertheless, the results presented here can be extended to the three-dimensional case. The solution adopted is based on Lyapunov stability theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory, where we consider communication topologies with unidirectional or directed links: one vehicle sends information to its neighbors but does not necessarily receive information back. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path (see, e.g., Encarnação and Pascoal [2001], Skjetne et al. [2005]). The speed of each vehicle is then adjusted so that the whole group keeps a desired formation pattern. A supporting communications network provides the vehicles with the medium over which to exchange the information required to synchronize them. Because of bandwidth constraints, the information exchange between vehicles takes place at discrete time instants which occur with a fixed frequency. We assume that the transmission delay can be neglected and that there are no packet collisions when the vehicles communicate simultaneously. Due to the absence of information in the intervals between transmission times, the control action of each vehicle runs in open loop, based on a simple model that predicts the evolution of its neighbors (as in Model-Based Networked Control Systems, Montestruque and Antsaklis [2003]). At transmission times, each vehicle sends information through the network that is used to achieve coordination and to update the models.

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This paper continues the work started in Almeida et al. [2007] by introducing a feedback signal from the coordination control subsystem to the path-following controller, thus yielding smoother convergence of the vehicles to the paths. The system that is obtained by putting together the path-following and vehicle coordination strategies takes an interconnected form, where both systems are input-to-state stable (ISS) with respect to the outputs of each other. With the control structure adopted, path-following (in space) and inter-vehicle coordination (in time) become essentially decoupled. In fact, both control subsystems are designed separately, ignoring the signals interconnecting them, and then stability and convergence of the closed loop system are analyzed and guaranteed using the small-gain theorem (see, e.g., Isidori [1999]).

The paper is organized as follows. Section 2 describes the dynamic model of the autonomous vehicles considered. In Section 3, key concepts from graph theory are introduced that are necessary to formally state the coordinated path-following problem in Section 4. Section 5 illustrates the general structure of the proposed CPF controller that is formed by a path-following controller presented in Section 6, and a coordination controller operating with periodic communications developed in Section 7. Section 8 gives an illustrative example where simulation results are presented. Finally, Section 9 contains the conclusion and directions of future research.

The proofs of all lemmas and theorems have been omitted and can be found in Almeida [2007].

2. VEHICLE MODELING

The kinematic and dynamical equations of a large class of vehicles are summarized next (see, e.g., Fossen [1994]). To this effect, consider an autonomous vehicle modeled as a rigid body subject to external forces and torques moving in a two dimensional space, as, for example, a surface vessel moving at sea. Let \( \mathcal{I} \) be an inertial coordinate frame and \( \mathcal{B} \) a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The generalized position of the vehicle is \( \eta = (x, y, \psi) \), where \( (x, y) \) are the coordinates of the origin of \( \mathcal{B} \) in \( \mathcal{I} \) and \( \psi \) is the orientation of the vehicle (yaw angle) that parameterizes the body-to-inertial coordinate transformation matrix

\[
J := J(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

Denoting by \( \nu := (u, v, r) \) the generalized velocity of the vehicle relative to \( \mathcal{I} \) expressed in \( \mathcal{B} \), the following kinematic relations apply:

\[
\dot{\eta} = J \nu, \quad \dot{J} = rJS,
\]

where \( S \) is a skew-symmetric matrix. We consider fully-actuated vehicles with dynamic equations of motion of the form

\[
M \ddot{\nu} = f(\nu, \eta) + \tau,
\]

where \( M \in \mathbb{R}^{3 \times 3} \) denotes a constant symmetric positive definite mass matrix, \( \tau = (\tau_u, \tau_v, \tau_r) \) is the generalized control input, and \( f(\nu, \eta) \) represents all the remaining equivalent forces and torques acting on the body, related to Coriolis, centripetal and hydrodynamic damping effects.

For marine vessels, \( M \) also includes the so-called hydrodynamic added-mass \( M_A \), i.e., \( M = M_{RB} + M_A \), where \( M_{RB} \) is the rigid-body mass matrix.

3. GRAPH THEORY

This section contains some key concepts and results in graph theory that play an important role in what follows. See Godsil and Royle [2001] for an in-depth presentation of this subject. A directed graph or digraph \( G = (\mathcal{V}, E) \) consists of a finite set \( \mathcal{V} = \{1, 2, \ldots, n\} \) of \( n \) vertices and a finite set \( E \) of \( m \) ordered pairs of vertices \((i, j) \in E \) named arcs. Given an arc \((i, j) \in E \), its first and second elements are called the tail and head of the arc, respectively. The out-degree of a vertex \( i \) is the number of arcs with \( i \) as its tail. If \((i, j) \in E \) then we say that \( i \) is adjacent to \( j \). A path from \( i \) to \( j \) is a sequence of distinct vertices starting with \( i \) and ending with \( j \) such that consecutive vertices are adjacent. If there is a path in \( G \) from vertex \( i \) to vertex \( j \), then \( j \) is said to be reachable from \( i \). A vertex \( i \) is globally reachable if it is reachable from every other vertex in \( G \). The adjacency matrix of a digraph, denoted \( A \), is a square matrix with rows and columns indexed by the vertices, such that the \( i, j \)-entry of \( A \) is 1 if \((i, j) \in E \) and 0 otherwise. The out-degree matrix \( D \) of a digraph is a diagonal matrix where the \( i, i \)-entry is equal to the out-degree of vertex \( i \).

The Laplacian of a digraph is defined as \( L = D - A \). By definition, the Laplacian satisfies \( L1 = 0 \) with \( 1 = [1, \ldots, 1]^\top \), therefore 0 is eigenvalue of \( L \) with 1 being its associated right eigenvector. In Lin et al. [2005], it is shown that a digraph has at least one globally reachable vertex if and only if zero is a simple eigenvalue of \( L \). This implies that rank \( L = n - 1 \) for a graph with one globally reachable vertex. Hence, there exist matrices \( F \in \mathbb{R}^{n \times (n-1)} \) and \( G \in \mathbb{R}^{(n-1) \times n} \) such that \( L = FG \), where rank \( F = \) rank \( G = n - 1 \). The matrix \( G \) also satisfies \( G1 = 0 \).

4. PROBLEM STATEMENT

We now consider the problem of coordinated path-following (CPF). In the most general setup, we are given a set of \( n \geq 2 \) autonomous vehicles and a set of \( n \) spatial paths \( \eta_{xi} = (x_{di}, y_{di}, \psi_{di}) \) for \( i = 1, 2, \ldots, n \). Each path \( \eta_{di}(\gamma_i) \) is parameterized by a continuous variable \( \gamma_i \) and it is required that vehicle \( i \) follow path \( \eta_{di} \). Following Ghabcheloo et al. [2006a], the CPF problem is separated in two subproblems: a path-following problem for each vehicle, where we require the vehicle to follow a predefined desired spatial path, and a coordination problem for all vehicles addressing the constraints imposed by the topology of the communications network. The problem of path-following is formally stated next.

Path-following problem: Let \( \eta_{di}(\gamma_i) \in \mathbb{R}^3 \) be a desired smooth path parameterized by a continuous variable \( \gamma_i \in \mathbb{R} \) and \( v_{di} \in \mathbb{R} \) a desired, constant speed assignment for vehicle \( i \). For each vehicle with equations of motion given by (1) and (2), design a feedback control law for \( \tau_i \) such that all closed-loop signals are bounded, and, as \( t \to +\infty \), the position of the vehicle converges to the desired path, i.e., \( \|\eta_i(t) - \eta_{di}(\gamma_i(t))\| \to 0 \).
For the coordination problem, we start by introducing a measure of the degree of coordination of a fleet of vehicles. As in Ghabcheloo et al. [2006b], this is done by reparameterizing each path $\eta_{i}(\tau_{i})$ in terms of a conveniently defined variable $\xi_{i}$ such that coordination is said to be achieved along the paths if and only if $\xi_{i} = \xi_{2} = \ldots = \xi_{n}$. At this point, we formally define the “along-path” distances between vehicle $i$ and $j$ as $\xi_{i,j} = \xi_{i} - \xi_{j}$. Then, coordination is achieved if and only if $\xi_{i,j} = 0$ for all $i, j \in \{1, 2, \ldots, n\}$. Let the reparameterization of the path be represented by $\gamma_{i} = \gamma_{i}(\xi_{i})$ and define $R_{i}(\xi_{i}) := \partial \gamma_{i} / \partial \xi_{i}$, which is assumed to be positive and bounded for all $\xi_{i}$. The dynamics of $\xi_{i}$ and $\gamma_{i}$ are related by

$$\dot{\gamma}_{i} = R_{i}(\xi_{i})\dot{\xi}_{i}. \quad (3)$$

This paper considers the case where $R_{i}(\xi_{i})$ is constant. While restrictive, it still allows us to consider paths where there is no need for reparameterization ($\xi_{i} = \gamma_{i}$) or it amounts to a constant scaling.

Suppose one vehicle, henceforth referred to as vehicle $L$, is elected as the “leader” and let the corresponding path $\eta_{L}$ be parameterized by $\gamma_{L} = \xi_{L}$. For this vehicle, $R_{L} = 1$. Let $\nu_{L}$ be the desired constant speed assigned to the leader in advance, that is $\xi_{L} = \nu_{L}$ in steady-state, known to all vehicles. From (3), it follows that the desired “along-path” speeds for the vehicles are $\nu_{di} := R_{i}\nu_{L}$. It is important to point out that $L$ can always be taken as a “virtual” vehicle that is added to the set of “real” vehicles as an expedient to simplify the coordination strategy.

So far, the problem of coordination has been reduced to that of “aligning” the coordination states $\xi_{i}$. To go from an alignment to a more complex spatial configuration, we introduce appropriate offsets in the desired positions of the vehicles relative to the mean point of the formation as defined with respect to the paths. To this effect, let $\xi := [\xi_{i}]_{n \times 1}$ and define the formation mean point and offsets as $\xi := \frac{1}{n}1^{T}\xi$ and $\delta := \xi - \xi_{1}$, respectively. Notice that $1^{T}\delta = 0$. Let $\phi \in \mathbb{R}^{n}$ represent a desired constant formation pattern that verifies $1^{T}\phi = 0$. The problem of coordination with pattern tracking is reduced to that of making $\phi \rightarrow 0$ as $t \rightarrow +\infty$.

From a graph theoretical point of view, each vehicle is represented by a vertex and a communication link between two vehicles is represented by an arc. The communication links are assumed unidirectional, thereby inducing a directed graph. We consider time-invariant communication topologies and assume that the induced graph has at least one globally reachable vertex. The flow of information in an arc is directed from its head to its tail. The set of neighbors of vertex $i$ is represented by $\mathcal{N}_{i}$ and contains all vertices $j$ such that $(i, j) \in \mathcal{E}$. In other words, it is the group of vehicles from which vehicle $i$ receives information.

For design purposes, we will take each $\dot{\xi}_{i}$ as a control input of the coordination dynamics (3). In order to satisfy the constraints imposed by the communication network, the control law for vehicle $i$ must be decentralized, i.e., it may only depend on local states and/or on information exchange with its neighbors as specified by $\mathcal{N}_{i}$. The coordination problem is formulated as:

$$X_{j} : j \in \mathcal{N}_{i}$$
The CC subsystem will be shown to be ISS with respect to state $x_{Ci}$ (which represents the state of the CC subsystem of vehicle $i$) and input $\zeta_i$.

A sampled-data based approach to the problem of coordinated path-following was proposed in Ihle et al. [2006], where the variables parameterizing the paths ($\gamma_i$) evolve in a discrete fashion, and therefore the coordination control problem is posed in discrete-time. However, the authors only considered communication topologies with bidirectional links.

6. PATH-FOLLOWING

Central to the development of CPF strategies is the derivation of appropriate path-following control laws for each vehicle. In this section, we present a path-following controller for an autonomous vehicle described by the equations of motions introduced in Section 2, that follows the lines of those developed in Encarnação and Pascoal [2001], Skjetne et al. [2005]. The controller is local to each vehicle so the index $i$ will be omitted for the sake of clarity.

Define the position error in the body-fixed frame as $z_1 := J^T (\eta_1 - \eta_d)$, and let $\zeta := \gamma_1 - v_d$ denote the “along-path” speed tracking error. The goal of the path-following controller is to drive $z_1$ and $\zeta$ to zero. Applying backstepping design procedures (see, e.g, Krstić et al. [1995]), the Lyapunov function

$$ V := \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_d^T z_d + \frac{1}{2} \zeta^2, $$

(5)

yields the feedback control laws

$$ \tau = -z_1 - K_2 z_d - f + M (\alpha^T + \alpha^T \gamma), $$

(6)

and

$$ \dot{\zeta} = -w \mu - k_c \zeta, $$

(7)

where $z_2 := v - \mu$, $\mu := - (\eta_d^T)^T J z_1$,

$$ \alpha := J^T \eta_d^T + K_1 z_1 + \alpha_2 := K_1 J^T \eta_d^T + J^T \eta_d^T v_d, $$

and

$$ \alpha_1 := -K_1 (v - r S z_1) - r S J^T \eta_d^T v_d + J^T \eta_d^T \dot{v}_d, $$

with $\eta_d^p := \partial \eta_d / \partial p$ for $p = 1, 2$. The time derivative of (5) along the solutions of (1) and (2), using (6) and (7) as feedback laws, is

$$ \dot{V} = -z_1^T K_1 z_1 - z_2^T K_2 z_d - k_c \zeta^2 + \mu \dot{v}. $$

The closed loop system with $\dot{\zeta}$ as input has the following important property.

Lemma 1. The PF subsystem described is ISS with respect to state $x_P := [z_1^T, z_d^T, \zeta]^T \in \mathbb{R}^3$ and input $\dot{v}$, that is,

$$ \|x_P(t)\| \leq k_P e^{-\lambda_P t} \|x_P(t_0)\| + \sigma_P \sup_{s \in [t_0, t]} |\dot{v}(s)| $$

for some positive constants $k_P, \lambda_P$, and $\sigma_P$. Moreover, $\sigma_P$ can be made arbitrarily small by increasing the PF gains $K_1, K_2$ and $k_c$.

7. CPF WITH PERIODIC COMMUNICATION

We now offer a solution to the coordination problem. Rewriting (4) in terms of $\dot{\zeta}_i$ and using a control law for $\dot{v}_i$ adapted from Ghabcheloo et al. [2006b], we obtain a controller that solves the coordination problem under continuous communication:

$$ \dot{\zeta}_i = v_L + R_i^{-1} (\dot{v}_i + w_i \zeta_i), $$

(8)

$$ \dot{v}_i = -k_c_i \sum_{j \in N_i} (\xi_j - \phi_i - \xi_i + \phi_j), $$

(9)

where $k_{ci_i} > 0$ is an adjustable control gain and $\phi_i$ are the components of desired formation pattern represented by $\phi$. Notice that the information required by vehicle $i$ about its neighbors is $\chi_j := \xi_j - \phi_i$, that we refer to as information state, and not the coordination state $\xi_i$ itself. The control law (8) can be rewritten as

$$ \dot{\chi}_i = v_L + R_i^{-1} (\dot{v}_i + w_i \zeta_i), $$

(10)

$$ \dot{v}_i = -k_c_i d_i \chi_i + k_c_i \sum_{j \in N_i} \chi_j, $$

(11)

where $d_i$ is the number of neighbors of vehicle $i$ (out-degree of vertex $i$), $k_{ci_i} = 0$ if $d_i = 0$. When using periodic communications, vehicle $i$ does not receive $\chi_j : j \in N_i$ between update times, so it needs to model their evolution in that interval. Let $\chi_j$ represent local “replicas” of each $\chi_j$ as seen by vehicle $i$, that we refer to as predictor states. Analyzing (10), we see that if a steady-state condition is achieved, then $\dot{\chi}_i = v_L$ for all $i$. This suggests that the dynamics of $\chi_j$ can be predicted as $\dot{\chi}_j = v_L$, thus yielding the controller

$$ \dot{\chi}_i = v_L + R_i^{-1} (\dot{v}_i + w_i \zeta_i), $$

(10)

$$ \dot{v}_i = -k_c_i d_i \chi_i + k_c_i \sum_{j \in N_i} \chi_j, $$

(11)

However, this is not sufficient to achieve coordination due to initial conditions that do not match the desired formation pattern. To overcome this problem, a reset is made to the predictor states when information is exchanged. We therefore add the following condition to the controller:

$$ \chi_j (t_k) = \chi_j (t_k^-), $$

(14)

where the notation $x(t^+)$ stands for the left limit or limit from below, i.e., $x(t^+) := \lim_{t \to s^-} x(s)$. Because all $\chi_j$ are initialized with the same value $\chi_j (t_0)$, and because we assume there is absolute synchronization with respect to update times, vehicles that model the same predictor state have equal values, i.e, $\chi_{i1} = \chi_{i2}$ for all $i_1, i_2$, and $j$. Therefore, we do not need to refer to $\chi_{i1}^j$ and $\chi_{i2}^j$ as different states, we simply refer to them as $\chi_j$.

Defining $\dot{\chi} := \chi - \phi = [\chi]_{n \times 1}$, $\zeta := [\zeta]_{n \times 1}$, the diagonal matrices $K_c := \text{diag}(k_c i_{n \times n})$, $W := \text{diag}(w_i n \times n)$, and $C := \text{diag}(R_i i_{n \times n})$, equations (12)-(14) can be written in vector form as

$$ \dot{\chi} = v_L C + (K_c D A \dot{X} + W \zeta), $$

(15a)

$$ \dot{\chi} = v_L C, $$

(15b)

$$ \chi(t_k) = \chi(t_k^-). $$

(15c)

The coordination error is defined as

$$ \theta := G(\zeta - \phi) = G \chi \in \mathbb{R}^{n-1}, $$

where $G$ is obtained from the decomposition of the Laplacian discussed in Section 3. Since $G1 = 0$ and using the definitions of formation mean point and offsets of Section 4, we have

$$ G(\zeta - \phi) = G(\delta + \chi 1 - \phi) = G(\delta - \phi). $$
Because $1^\top \delta = 1^\top \phi = 0$, $\delta - \phi$ is normal to the null space of $G$. We conclude that $\theta = 0$ if and only if $(\delta - \phi) = 0$. Let

$$\bar{\chi} := \chi - \hat{\chi}$$

represent the predictor state error. If $\bar{\chi} = 0$, then the information states are coherent, i.e., the predictor states equal the actual states. Considering (15), the error dynamics for $\theta$ and $\bar{\chi}$ are given by

$$\begin{aligned}
\dot{\chi} &= -C_k F_\theta - C_k A \bar{\chi} + C W \zeta, \\
\theta &= -G C_k F_\theta - G C_k A \bar{\chi} + C G W \zeta,
\end{aligned}$$

where we used the fact that $L \chi = F G \chi = F \theta$. Defining the aggregated state variable $x_C := (\theta, \bar{\chi})$, the above error dynamics can be written as

$$\begin{aligned}
\dot{x}_C &= \Lambda x_C + B \zeta, \\
x_C(t) &= (\theta(t^-), 0), t = t_k
\end{aligned}$$

(16)

where

$$\begin{aligned}
\Lambda &= \begin{bmatrix}
-G C_k F & -G C_k A \\
-C_k F & -C_k A
\end{bmatrix}, \\
B &= \begin{bmatrix}
C G W \\
C W
\end{bmatrix}.
\end{aligned}$$

(17)

(18)

The dynamic system (16) is a linear impulsive system which is ISS if a certain condition is verified.

**Lemma 2.** The system described by (16) is ISS with respect to state $x_C$ and input $\zeta$, if

$$\Phi := \begin{bmatrix}
I & -1 \\
0 & 0
\end{bmatrix} e^{\Lambda h}$$

(19)

is a convergent matrix (all its eigenvalues are strictly inside the unit circle).

The following lemma gives conditions under which matrix $\Phi$ is guaranteed to be a convergent matrix.

**Lemma 3.** For any communication graph with at least one globally reachable vertex, matrix $\Phi$ defined in (19) is a convergent matrix.

The next theorem states that the control structure proposed in Section 5, with the control laws developed in Sections 6 and 7, solves the CPF problem presented in Section 4. Its proof is a straightforward application of the small-gain theorem (see, e.g., Isidori [1999]).

**Theorem 4.** The overall system formed by the interconnection of the $n$ combined PF/CC systems is globally asymptotically stable.

### 8. ILLUSTRATIVE EXAMPLE

We consider a group of three identical vehicles whose kinematic and dynamic equations of motion can be written as in (1)-(2), with

$$M = \text{diag}(m_u, m_u, I),$$

and

$$f(\nu, \eta) = \begin{bmatrix}
X_u + |X_u| u |u| & -m_u r \\
0 & Y_v + Y_{|v|} |v| & 0 \\
0 & 0 & N_r + N_{|r|} |r|
\end{bmatrix} \nu.$$

In the simulations presented, the physical parameters are

- $X_u = -1 \text{ kg/s}$,
- $Y_v = -200 \text{ kg}$,
- $m_u = 500 \text{ kg}$,
- $X_{|u|} = -25 \text{ kg/m}$,
- $N_r = -0.5 \text{ kgm}^2/\text{s}$,
- $m_v = 1000 \text{ kg}$,
- $Y_v = -10 \text{ kg/s}$,
- $N_{|r|} = -1500 \text{ kmg}^2$,
- $I_c = 700 \text{ km}^2$.

The communication graph is illustrated in Fig. 2. The communication period is set to $h = 1 \text{s}$.

**Fig. 2.** Graph representing the communications network.

To illustrate the compliance of the control system, that is, to show why the existence of feedback from the PF subsystem to the CC subsystem is important, we consider a simulation where the vehicles must follow straight lines parallel to the $x$-axis, while the propulsion system of vehicle-2 has a force limitation of 20 N along the $x$-direction (that is, $|r_{\eta_2}(t)| \leq 20 \text{ N}$ for all $t \geq 0$). This keeps vehicle-2 from acquiring the necessary speed to hold the desired formation pattern. The initial conditions of each vehicle are $\eta_1(t_0) = (-5 \text{ m}, -5 \text{ m}, \pi/3 \text{ rad})$, $\eta_2(t_0) = (-5 \text{ m}, 15 \text{ m}, -\pi/4 \text{ rad})$, and $\eta_3(t_0) = (5 \text{ m}, 17 \text{ m}, 2\pi/3 \text{ rad})$, $v_1(t_0) = v_3(t_0) = 0 \text{ m/s}$, and $r_1(t_0) = 0 \text{ rad/s}$ for $i = 1, 2, 3$. The initial condition for $\gamma$ is chosen so that for each vehicle $i$, $\gamma_i$ yields the closest point on the respective path, which gives $\gamma_i(t_0) = (-5, -5, 5) \text{ [m]}$. The reference speed is set to $\nu_g = 1 \text{s}^{-1}$. The vehicles are required to keep an in-line formation pattern, that is, $\phi_i = (0, 0, 0)$. Figure 3(a) illustrates the initial state of the vehicles.

Next, we consider two cases.

1. The gains are $K_1 = 0.15 I_3$, $K_2 = 200 I_3$, $K_3 = 1$, and $W \geq 0$. In this case, there is no feedback from the PF subsystem to the CC subsystem (situation studied in Almeida et al. [2007]). Therefore, vehicle-2 is left behind and its position error increases continuously as shown in Fig. 4. Figure 3(c) illustrate the final state of the vehicles.

2. The gains are $K_1 = 0.15 I_3$, $K_2 = 200 I_3$, $K_3 = 1$, and $W = I_3$. Due to the existence of feedback, the vehicles reach a compromise between converging to the paths and achieving coordination as can be seen in Fig. 5, where both the PF and CC errors do not tend to zero but are bounded. Figure 3(d) illustrate the final state of the vehicles.

For comparison purposes, Figure 3(b) illustrates the vehicles’ final state for the ideal scenario where vehicle-2 has no force restrictions, using the gains of Case (2).

### 9. CONCLUSION

We addressed the problem of coordinated path-following for a group of autonomous vehicles using periodic communications. We proposed and analyzed a decentralized control structure formed by an interconnection of two subsystems named path-following and coordination control. Stability and convergence of the overall system are guaranteed for any fixed update period, as long as the graph induced by the communication network has at least one globally reachable vertex.

Future work includes the study of the implementability of the proposed control structure to address coordination problems involving time delays, communication failures, and asynchronous updates.

**REFERENCES**


