Vehicle Parameter Estimation and Stability Enhancement

Hassan Shraim ∗Mustapha Ouladsine ∗ Leonid Fridman ∗∗

∗ Laboratory of sciences of informations and of systems, LSIS UMR
6168 University of Paul Czanne, Aix-Marseille III Av escadrille de Normandie Niemen 13397 Marseille Cedex 20
∗∗ Department of Control, Division of Electrical Engineering,Engineering Faculty, National Autonomous University of Mexico(UNAM), Ciudad Universitaria, 04510,D.F, Mexico

Abstract: In this paper, high order sliding mode observers are used to estimate tires longitudinal forces, vehicle side slip angle and velocities. Two types of high order sliding mode observers are used. A third order one is used to estimate the longitudinal forces, while for the estimation of the vehicle side slip and velocities, an observer based on the super twisting algorithm is proposed. Validations with the simulator VE-DYNA pointed out the good performance and the robustness of the proposed observers. Controller design for the braking is accomplished using a reduced state space model representing the movement of the vehicle center of gravity in the (X,Y) plane. Driver’s reactions are taken into account. The performance of the closed loop system is carried out by means of simulation tests.

1. INTRODUCTION

In the recent years, important research has been undertaken to investigate the safe driving conditions in both normal and in critical situations. Safe driving requires the driver to react extremely quickly in dangerous situations, which is generally very difficult unless for experts, that result the instability of the system. Consequently, the improvement of the vehicle dynamics by active chassis control is necessary for such catastrophic situations. Increasingly, commercial vehicles are being fitted with micro processor based systems to enhance the safety and to improve driving comfort.

Considerable attention has been given to the development of the control systems over the past few years, authors have investigated and developed different methods and different strategies for enhancing the stability and the handling of the vehicle such as, the design of the active automatic steering You et al. [1999] and the wheel ABS control Petersen [2003], Unsa et al. [1999], or the concept of a four-wheel steering system (4WS) which has been introduced to enhance vehicle handling.

In terms of vehicle safety, and in order to develop a control law for the vehicle chassis, accurate and precise tools such as sensors should be implemented on the vehicle, to give a correct image of its comportment. Difficulties in measuring all vehicle states and forces, due to high costs of some sensors, or the non existence of others, make the design and the construction of observers necessary. In the field of automotive engineering, the estimation of vehicle side slip angle and wheel interaction forces with the ground are very important, because of their influence on the stability of the vehicle. Many researchers have studied and estimated vehicle side slip angle, using a bicycle model as in Stephant [2004], or by using an observer with adaptation of a quality function as in Von Vietinghoff et al. [2005] which requires a certain linearised form of the model. Moreover, an extended Kalman filter is used for the estimation of wheel forces Samadi et al. [2001].

The problem of observation has been actively developed within Variable Structure Theory using sliding mode approach. Sliding mode observers (see for example Edwards et al. [1998], V. Utkin et al. [1999] and Poznyak [2003]) are widely used due to their attractive features:

- Insensitivity (more than robustness) with respect to unknown inputs;
- Possibilities to use the values of the equivalent output injection for the unknown inputs identification (V. Utkin et al. [1999]);
- Finite time convergence to the reduced order manifold.

In Levant [2003] robust exact differentiators were designed ensuring finite time convergence to the real values for derivatives, as an application of super-twisting algorithm. New generation of observers based on the high order sliding mode differentiators are recently developed (see Barbot et al. [2004] and Bejarano et al. [2007]). Those observers:

- provide a convergence to the exact values of states variables;
- allow the exact identification the unknown inputs without filtration;
- guarantee the best possible accuracy of the state estimation w.r.t. to the sampling steps and deterministic noises.

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In this paper we propose a robust sliding mode controller together with robust high order sliding mode observers for estimation of the states, forces and parameters, and for the stabilization of the vehicle on a reference trajectory in order to ensure safety in the critical situations.

In the estimation part of this paper, the model is decoupled into two parts in order to insure the observability. First, the longitudinal forces are estimated by a high order sliding mode observer based on the dynamical equations of the wheels. These forces are then injected, and supposed as known inputs, in the reduced model of the vehicle center of gravity. Based on this reduced model, the vehicle side slip angle and velocity are estimated by a sliding mode observer based on the super twisting algorithm.

The paper is organized as follows: In section 2, estimation of longitudinal forces is proposed using high order observers. In this section two cases are considered depending on the wheel measurements. In Section 3, the estimation of vehicle side slip angle and vehicle velocity is shown. In section 4, a controller design is presented. In section 5 simulations are shown and finally a conclusion is presented.

In this study it is supposed that we can measure:

- angular positions of the wheels (in the case that they are not measured and the angular velocities are measured we have also proposed a solution)
- front wheel angle;
- yaw rate

and it is required to estimate:

- angular velocity of the wheels;
- contact forces;
- vehicle velocity;
- vehicle side slip angle.

It is important to note that the torques applied at the wheels are estimated and then injected in the model as known input.

2. ESTIMATION OF LONGITUDINAL FORCES

In order to identify the longitudinal forces, we use the following model for the dynamics of the wheels (Figure 1):

\[ I_{ri} \dot{\Omega}_i = -r_{li} F x_i + \text{torque}_{i} ; i = 1:4 \]  

(1)

In fact, the design of an observer for the estimation of the longitudinal forces is related to the available measurements. For this reason we have two possibilities.

2.1 If the wheel angular position is available

If the wheels angular positions are available then, the dynamical equation of the wheel can be written in the following form:

\[ \dot{x}_1 = x_2 \]

\[ \dot{x}_2 = f(x_1, x_2, u) \]  

(2)

where \( x_1 \) and \( x_2 \) are respectively \( \theta_i \) (which is measured) and \( \Omega_i \) (to be observed) (appears implicitly in \( F x_i \)), and \( u \) is \( \text{torque}_i \). In fact this torque may be measured as shown in Rajamani et al. [2006], and it can also be estimated by estimating the motor and the braking torque, the motor torque may be estimated as in Khiar et al. [2006], while the braking torque is estimated by measuring the hydraulic pressure applied at each wheel (existing on the most of the vehicles).

A Third Order sliding mode Observer  

The proposed third order sliding mode observer has the following form:

\[ \dot{\theta}_i = \dot{\Omega}_i + \lambda_0 |\theta_i - \dot{\theta}_i|^{1/2} \text{sign}(\theta_i - \dot{\theta}_i) \]  

\[ \dot{\Omega}_i = \text{torque}_i \]  

\[ z_1 = \frac{v_0}{I_{ri}} + z_1 \]  

(3)

where \( \dot{\theta}_i \) and \( \dot{\Omega}_i \) are the state estimations of the angular positions and the angular velocities of the four wheels respectively, \( z_1 \) is the correction factor based on the super-twisting algorithm having the following forms:

\[ z_1 = \lambda_1 |\Omega_i - v_o|^{1/2} \text{sign}(\Omega_i - v_o) + Z_1 \]  

\[ \dot{Z}_1 = \lambda_2 \text{sign}(Z_1 - v_1) \]  

(4)

with \( \lambda_2 = 3\sqrt{\lambda_0} \), \( \lambda_1 = 1.5\sqrt{\lambda_0} \) et \( \lambda_0 = 2f^+ \). The sliding occurs in (32) in finite time (see Levant [2003]). In particular, this means that the longitudinal forces can be estimated in a finite time without the need for low pass filter of discontinuous output injections.

2.2 If the wheel angular velocity is available

If the angular position is not available, then we propose a sliding mode observer based on the super twisting algorithm. This observer has the following form:

\[ \dot{\Omega}_i = \frac{\text{torque}_i}{I_{ri}} + \frac{|\Omega_i - \dot{\Omega}_i|^{1/2} \text{sign}(\Omega_i - \dot{\Omega}_i) + Z_1}{v_0} \]  

\[ \dot{Z}_1 = \Delta \text{sign}(\Omega_i - \dot{\Omega}_i) \]  

(5)

2.3 Estimation of the side slip angle and the velocity of the vehicle

In this part, a sliding mode observer based on the super-twisting algorithm is used to estimate vehicle velocity and side slip angle. The model of the vehicle is a non-linear model and it can be written as follows:

\[ \dot{v}_{\text{COG}} = \frac{1}{M} \left( \cos(\beta) \sum F_L - \sin(\beta) \sum F_S \right) \]  

\[ \dot{\beta} = \frac{1}{M v_{\text{COG}}} \left( \cos(\beta) \sum F_S - \sin(\beta) \sum F_L - \dot{\psi} \right) \]  

(6)

(7)

With

\[ \psi = \frac{1}{M v_{\text{COG}}} \left( \cos(\beta) \sum F_S - \sin(\beta) \sum F_L - \dot{\psi} \right) \]
\[
\sum F_L = F_{\text{wind}} + \cos(\delta_f)(F_{x1} + F_{x2}) + \cos(\delta_r)(F_{x3} + F_{x4}) - \sin(\delta_f)(F_{y1} + F_{y2}) - \sin(\delta_r)(F_{y3} + F_{y4})
\]

And

\[
\sum F_S = \sin(\delta_f)(F_{x1} + F_{x2}) + \sin(\delta_r)(F_{x3} + F_{x4}) + 
\cos(\delta_f)(F_{y1} + F_{y2}) + \cos(\delta_r)(F_{y3} + F_{y4})
\]

\[
I_Z \ddot{\psi} = t_f \{\cos(\delta_f)(F_{x2} - F_{x1}) + \sin(\delta_f)(F_{y1} - F_{y2})\} + 
I_1 \{\sin(\delta_f)(F_{x2} + F_{x1}) + \cos(\delta_f)(F_{y1} + F_{y2})\} + 
I_2 \{\sin(\delta_r)(F_{x3} + F_{x4}) - \cos(\delta_r)(F_{y3} + F_{y4})\} + 
t_r \{\cos(\delta_r)(F_{x4} - F_{x3}) + \sin(\delta_r)(F_{y4} - F_{y3})\}
\]

This model can be rewritten as a state space model having the following form:

\[
\dot{x} = f(x, u) = A(x) + B(x)u
\]

where

\[
x = \begin{bmatrix} v_{\text{COG}} & \beta & \dot{\psi} \end{bmatrix}
\]

the input:

\[
u = [F_{x1} \ F_{x2} \ F_{x3} \ F_{x4}]
\]

and the measurement vector

\[
y = [\dot{\psi}]
\]

Before the design of the sliding mode observer for the model of equation 9, the observability of the model must be investigated and tested. The observability definition is local and uses the Lie derivative. It is a function of the state trajectory and the inputs to the model. For the system described by equation 9 the jacobian of the observability function has a full rank and hence the system is observable.

So, the proposed sliding mode observer based on the hierarchical super twisting algorithm is:

\[
\begin{cases}
\dot{x} = f(\dot{v}_{\text{COG}}, \beta, \dot{\psi}, u) + Z_2 \\
Z_2 = \Delta_1 [(\dot{\psi} - \dot{\psi}^1)^T \ \text{sign}(\dot{\psi} - \dot{\psi}^1) + Z_1 \\
\dot{Z}_1 = \Delta \ \text{sign}(\dot{\psi} - \dot{\psi})
\end{cases}
\]

Where \(\Delta\) and \(\Delta_1\) are the gains of the sliding mode observer.

**SIMULATIONS RESULTS** The estimated variables, \(\dot{v}_{\text{COG}}\) and \(\beta\) are compared to that provided by VE-DYNA. The operation conditions are given by a variation in \(\delta_f\) figure 10 and \(\text{torque}_{i}\) figure 11, which constitute a significant driving situation. \(F_{x1}\), are estimated from the third order sliding mode observer presented in the previous section and injected in this model. In figures 12, 13, 14 and 15, we see the observed \(\dot{v}_{\text{COG}}, \beta\) and \(\dot{\psi}\) and those provided by VE-DYNA. The rapid convergence pointed out the good performance of the proposed observer.

3. CONTROLLER DESIGN

In this section a sliding mode control strategy is presented. For the design process, the model presented by equation (9) is used. As we have described, the state vector is \(x = [\dot{v}_{\text{COG}}, \beta, \dot{\psi}]^T\) and the input \(u = [F_{x1}, F_{x2}, F_{x3}, F_{x4}]^T\) (the steering angle is supposed given by the driver). As the only available actuators are the brakes, then only braking torques can be generated by the controller Uwe et al. [2005]. These braking torques are directly related to the hydraulic pressure applied at each wheel Uwe et al. [2005]:

\[
\text{torque}_{controller} = -r_{ik}k_{bip}p_{BR_i} \quad i = 1:4
\]

To design a sliding mode controller, a sliding surface is proposed as:

\[
s = x - x_{ref}
\]
With \( x_{\text{ref}} = [v_{\text{COG,ref}}, \beta_{\text{ref}}, \psi_{\text{ref}}]^T \) represents the reference states which are derived as shown in Uwe et al. [2005].

The control objective is to derive the state vector \( x \) to the reference state vector \( x_{\text{ref}} \). In order to ensure the stability, let us suppose that Lyapunov candidate is given by \( V \). Utkin [1992]:

\[
V = \frac{1}{2} s^T s > 0
\]

its derivative can be written as:

\[
\dot{V} = s^T \dot{s}
\]

where:

\[
\dot{s} = \dot{x} - \dot{x}_{\text{ref}}
\]

by substituting (9) into (18) we get:

\[
\dot{s} = A(x) + B(x)u - \dot{x}_{\text{ref}}
\]

Then we have:

\[
\dot{V} = s^T (A(x) + B(x)u - \dot{x}_{\text{ref}})
\]

let us suppose the following control law:

\[
u = -K \text{sign}(B(x)^T s)
\]

substituting \( u \) in the above equation, we get: \( \dot{V} = s^T (A(x) + B(x)K \text{sign}(B(x)^T s) - \dot{x}_{\text{ref}}) \).

In order to satisfy the conditions of stability in the Lyapunov sense, \( \dot{V} \) should be negative, that means:

\[
s^T (A(x) + B(x)K \text{sign}(B(x)^T s) - \dot{x}_{\text{ref}}) < 0
\]

since \( K \) is scalar, we can write:

and then \( k > \frac{s^T (A(x) - \dot{x}_{\text{ref}})}{|s^T (B(x))|} \)

a necessary condition for the existence of the control law is that \( s^T B(x) \neq 0 \). Where \( \text{sign}(s) \) is a sign function which equals to 1 when \( s > 0 \) and \(-1 \) if \( s < 0 \). The chattering of the function \( \text{sign}(s) \) may be reduced by \( \text{sat}(\frac{s}{\Phi}) \), where \( \Phi \) is a design parameter denoting the boundary layer thickness Slotine et al [1991].

4. SIMULATION RESULTS

In this section, computer simulations are carried out to verify the effectiveness of the proposed observers and controller. Simulation is made, in which driver’s inputs are given by the simulator VE-DYNA. The driver wants to move on a ‘chicane’ (double lane) trajectory described by driver’s steering angle figure 8 and wheel torques figure 9. The inputs of the driver are considered and the reference trajectory, to know if we are in the safe region is found as in Uwe et al. [2005]. Based on the driver’s inputs and the calculated reference trajectory, we found that in some situations the vehicle goes its safe region. The controller reacts as shown in figure 10. In this figure 10, we see three curves, the reference yaw rate, the yaw rate for the system without the controller and that with the sliding mode controller, it is seen how the controller pushes the controlled yaw rate to its reference value. In figure 11, three curves also are shown for the side slip angle which are: the reference side slip angle, the side slip angle without the controller and that with the sliding mode control. In figure 12, the response of the controller is shown, it is seen that in the normal cases where the side slip angle and the yaw rate are in their safety regions, the controller gives zero, other wise, when they exceed their limits, the controller tries to regulate this problem giving different torques on the different wheels.

5. CONCLUSIONS

Sliding mode observers are proposed in this work to estimate vehicle parameters and states which are not easily measured. These observers have shown a short time of convergence and robustness in the automotive applications that we have proposed. The validation of the proposed observers is realized by comparing the observers output with the outputs of the simulator VE-DYNA, reasonable and acceptable results have been shown. In the second
part of this work, sliding mode controller is designed. This controller shows its strong and rapid reactions on the braking systems in the critical situations where we need the controller to work.

REFERENCES


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J. Stephant. Contribution l'étude et la validation expérimentale d’observateurs appliqués la dynamique du véhicule Thesis presented to have the doctor degree from the UTC, University of Technology Compigne, 2004.


**Table 1. Nomenclature.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_i$</td>
<td>angular velocity of the wheel</td>
</tr>
<tr>
<td>$M$</td>
<td>total mass of the vehicle</td>
</tr>
<tr>
<td>$r_i$</td>
<td>radius of the wheel $i$</td>
</tr>
<tr>
<td>$COG$</td>
<td>center of gravity of the vehicle</td>
</tr>
<tr>
<td>$r_{1i}$</td>
<td>dynamical radius of the wheel $i$</td>
</tr>
<tr>
<td>$F_{x_i}$</td>
<td>longitudinal force applied at the wheel $i$</td>
</tr>
<tr>
<td>$F_{y_i}$</td>
<td>lateral force applied at the wheel $i$</td>
</tr>
<tr>
<td>$\text{torque}_i$</td>
<td>Engine torque-Braking torque</td>
</tr>
<tr>
<td>$I_Z$</td>
<td>moment of inertia around the $Z$ axis</td>
</tr>
<tr>
<td>$\psi$</td>
<td>yaw angle</td>
</tr>
<tr>
<td>$\dot{\psi}$</td>
<td>yaw velocity</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>front steering angle</td>
</tr>
<tr>
<td>$\delta_r$</td>
<td>rear steering angle</td>
</tr>
<tr>
<td>$V_x$</td>
<td>longitudinal velocity of the center of gravity</td>
</tr>
<tr>
<td>$V_y$</td>
<td>lateral velocity of the center of gravity</td>
</tr>
<tr>
<td>$I_{r_i}$</td>
<td>moment of inertia of the wheel $i$</td>
</tr>
<tr>
<td>$v_{COG}$</td>
<td>total velocity of the center of gravity</td>
</tr>
<tr>
<td>$L_1$</td>
<td>distance between $COG$ and the front axis</td>
</tr>
<tr>
<td>$L_2$</td>
<td>distance between $COG$ and the rear axis</td>
</tr>
<tr>
<td>$h_{COG}$</td>
<td>height of $COG$</td>
</tr>
<tr>
<td>$t_f$</td>
<td>front half gauge</td>
</tr>
<tr>
<td>$t_r$</td>
<td>rear half gauge</td>
</tr>
<tr>
<td>$l$</td>
<td>$t_f + t_r$</td>
</tr>
<tr>
<td>$F_{xwind}$</td>
<td>air resistance in the longitudinal direction</td>
</tr>
<tr>
<td>$F_{ywind}$</td>
<td>air resistance in the lateral direction</td>
</tr>
<tr>
<td>$A_L$</td>
<td>front vehicle Area</td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
</tr>
<tr>
<td>$C_{aer}$</td>
<td>coefficient of aerodynamic drag</td>
</tr>
<tr>
<td>$\beta$</td>
<td>side slip angle at the $COG$</td>
</tr>
<tr>
<td>$p_{BR_i}$</td>
<td>braking pressure at the wheel $i$</td>
</tr>
<tr>
<td>$k_{bi}$</td>
<td>brake coefficient of the wheel $i$</td>
</tr>
</tbody>
</table>

**Table 2. Parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>1296 Kg</td>
<td>$L_1$</td>
<td>0.97 m</td>
</tr>
<tr>
<td>$L_2$</td>
<td>1.53 m</td>
<td>$r_{1i}$</td>
<td>0.28 m</td>
</tr>
<tr>
<td>$I_Z$</td>
<td>1750</td>
<td>$H$</td>
<td>0.52 m</td>
</tr>
<tr>
<td>$I_{r_i}$</td>
<td>0.9 Kg.m$^2$</td>
<td>$t_f$</td>
<td>0.7 m</td>
</tr>
<tr>
<td>$t_r$</td>
<td>0.75 m</td>
<td>$l_o$</td>
<td>-0.03 m</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.12 m</td>
<td>$\rho$</td>
<td>1.25 Kg/m$^3$</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>50000 N/rad</td>
<td>$A_L$</td>
<td>2.25 m$^2$</td>
</tr>
</tbody>
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