Planning setpoints for contact force transitions in regrasp tasks of 3D objects

Patrick Grosch∗ Raúl Suárez∗ Raffaella Carloni∗∗ Claudio Melchiorri∗∗

Abstract: This paper presents a simple and fast solution to the problem of finding the time variation of \(n\) contact forces that keep an object under equilibrium while one of the \(n\) contact forces is removed/added from/to the grasp. The object is under a constant perturbation force, like for instance its own weight. It is assumed no acceleration of the object during the regrasp operation, as well as the knowledge of the starting and ending grasp configurations. The procedure returns the set points of the \(n\) contact forces for a feed-forward control system of a manipulator device in a regrasping action. The procedure was implemented and an illustrative numerical example is included in the paper.

1. INTRODUCTION

The search for flexible end-effectors and the development of grasping and manipulation strategies according to different criteria has become a growing research area during the last two decades [2, 3, 7, 10]. One of the issues within this research field lies is the regrasping of an object, i.e. the variation of the contact points on the grasped object while some grasp properties are kept. This particular task implies finding the initial/final grasp contact points, determining the finger movements, and computing the proper forces to be applied by the fingers when a contact is removed or a new contact is established in order to keep the equilibrium conditions and to satisfy the dynamic constraints of the system [12, 11]. Regrapping operations are typically needed when the pick-up grasp configuration is not compatible with the actions to be done with the object or with the object placement itself, for instance due to physical constraints in the environment, due to the non-holonomic constraints of the finger contacts, or due to the limits in the articulation ranges of the grasping device.

Different approaches have been presented in the regrasping problem. A detailed description including a discussion about the use of two manipulators can be found in [6]. Some relevant works are those of Tournassoud et al. [12], who proposed a system based on polyhedral models for manipulators equipped with parallel jaw grippers, and Kerr et al. [5] who used a multi-finger hand (this type of end-effectors are expensive and rarely found in industrial manipulators, but are useful in non-repetitive tasks in unstructured environments due to their high dexterity). Recent works in regrasp [1, 8, 9] are focused on algorithms to determine the sequence of grasps configurations to go from an initial state to a desired final state, but they did not deal with the forces needed to perform the regrasp, which is the central point of this paper. The computation of optimal grasping forces in a given grasp configuration is presented in [4].

After this brief introduction the paper is organized as follows. In Section 2 the problem to be solved is described and formalized, followed by a particularization of the problem for a planar objects in Section 3. In Section 4 the problem for planar objects is analyzed, the behavior of the system dynamics is characterized, and a graphical tool used to find the solution of the problem is introduced. The proposed solution for planar objects is described in Section 5. In Section 6 the solution for planar object is generalized for 3D objects. An example is presented in Section 7 to illustrate the proposed approach. Finally, the last section of the paper gives some conclusions and describes ongoing and future works.

2. PROBLEM STATEMENT

The problem to be solved can be summarized as follows: Given a \(n\) contact point grasp of a 3D object that balances an external perturbation force (it may be the own object weight), we want to remove one of the contacts while keeping, during the action, the balance of the external force, or, as inverse situation, given a \(n-1\) contact point...
grasp add a \( n \)th contact point such that the additional finger helps in the balance of the external perturbation. Then, the problem to be solved is the determination of the time variation of the force set point functions for the contact forces that allows the \( n \)th contact to be removed/added without loosing the force equilibrium during the process.

This problem is found in regrasping manipulation of objects, when a finger is removed from one contact point on the object surface to be place in another one. In this particular case the stated problem appears twice, first when retreating the finger and second when replacing it on the desire new contact point.

3. PARTICULARIZATION OF THE PROBLEM FOR PLANAR OBJECTS

Before solving the general problem, an initial study is done for the case of a planar object in transition from three contact point to two contact point grasp. The following basic nomenclature will be used throughout the paper.

\( S_A, S_B \) : two grasp states in equilibrium (forces applied at the contact points balance any external force)

\( f_{ext} \) : external force acting on the object (it may be the object own weight).

\( P_i \) : contact point \( i \) on the object boundary.

\( r_i \) : location of \( P_i \) with respect to CM.

\( f_i \) : force applied on \( P_i \).

\( C_i \) : friction cone at \( P_i \) (set of possible forces \( f_i \) applicable at \( P_i \)).

\( \theta_i \) : angle between \( f_i \) and the object normal direction at \( P_i \).

\( \tau_i \) : torque around CM produced by \( f_i \) applied on \( P_i \).

\( \pi_i \) : friction cone at \( P_i \) (set of possible forces \( f_i \) applicable at \( P_i \)).

\( \Pi_0 \) : force plane in the wrench space (i.e. null torque plane).

\( \Pi_i \) : plane in the wrench space containing all \( \pi_i \) generated at \( P_i \).

\( S_i \) : subset of \( \Pi_i \) containing \( \pi_i \) generated at \( P_i \) due to forces \( f_i \) inside \( C_i \).

\( S \Pi_i \) : representation of \( S_i \) with all the force heads on the cone origin.

Let \( S_A \) be a grasp with three contact points \( P_i, i = 1, 2, 3 \), on the object boundary (Figure 1a) and \( S_B \) be another grasp with only two contact points, which are points \( P_1 \) and \( P_2 \) from \( S_A \) (Figure 1b). It is assumed that in \( S_A \) and \( S_B \) the finger forces \( f_i \) applied at \( P_i \) balance an external perturbation force \( f_{ext} \), i.e. the summations of the forces and moments applied on the object are null.

The problem to be solved can now be stated as the search of the time variation of the finger forces \( f_1(t) \) and \( f_2(t) \) that balance \( f_{ext} \) while \( f_3(t) \) varies from its value in \( S_A \) to zero in \( S_B \) or vice versa. \( f_1(t) \), \( f_2(t) \) and \( f_3(t) \) are the setpoints values for the finger control system during the manipulation action.

4. PROBLEM ANALYSIS

4.1 Torques generated by contact forces

A force \( f_i \) applied at \( P_i \) produces, with respect to the object center of mass CM, a torque \( \tau_i = f_i \times r_i \), where \( r_i \) describes the position of \( P_i \) with respect to CM. Consider a line \( L_i \) parallel to \( r_i \) (see Figure 2). Any \( f_i \) applied at \( P_i \) such that the vector \( f_i \), represented with the tail at \( P_i \) has its head on \( L_i \) produces the same torque \( \tau_i \), thus we refer to the lines \( L_i \) as iso-torque lines. The value of \( \tau_i \) associated to a given \( L_i \) is the product of \([|r_i|]\) (which is constant for a given point \( P_i \)) times the distance \( d_i \) between \( L_i \) and \( P_i \). Since \( L_i \) and \( P_i \) linearly varies with respect to \( d_i \). This linearity means that, in the wrench space, all the wrenches \( \pi_i = (f_i, \tau_i) \) (i.e. the wrenches produced by a force \( f_i \) applied at \( P_i \)) define a plane \( \Pi_i \) (see Figure 3). Since \( P_i \) is a contact point on the object boundary, \( f_i \) cannot have any direction, it is constrained to lie inside the friction cone \( C_i \), and therefore only a subset of \( \Pi_i \), called \( S \Pi_i \), can be actually generated. \( S \Pi_i \) is the projection along the \( \tau \)-axis of \( C_i \) over \( \Pi_i \) (Figure 3).
4.2 Wrench loops

The system equilibrium under wrenches $\mathbf{w}_i$ in the 3D space due to forces $\mathbf{f}_i$ applied on $P_i$, would be graphically analyzed and characterized. The equilibrium condition is that $\sum \mathbf{w}_i + \mathbf{w}_{ext} = 0$, being $\mathbf{w}_{ext}$ the wrench produced by $\mathbf{f}_{ext}$; note that if $\mathbf{f}_{ext}$ is the object weight then $\mathbf{w}_{ext} = (\mathbf{f}_{ext}, 0)$. Graphically, this condition can be seen as a closed loop path in the 3D wrench space drawing all the vectors $\mathbf{w}_i$ and $\mathbf{w}_{ext}$ with the tail attached to the head of another one. From now on, this loop will be called “wrench loop”, and the set of all the possible wrench loops produced by the possible wrenches generated at the contact points will be called “Generic Wrench Loop” (GWL). Since the grasp states $S_A$ and $S_B$ are assumed to be in equilibrium, the GWL is always non null, and it can be graphically constructed as follows (remind that $\mathbf{w}_i$ can be represented as free vectors so they can be translated in the wrench space with no loss of significance).

1. Consider first the vector representing the external wrench $\mathbf{w}_{ext} = (\mathbf{f}_{ext}, \mathbf{f}_{ext}, 0)$ (the vector with the tail at the origin in Figure 4).
2. The second vector to be considered is the wrench $\mathbf{w}_1$ due to $\mathbf{f}_1$ applied on $P_1$. Since $\mathbf{f}_1 \in C_1$ then $\mathbf{w}_1 \in S_{H1}$, thus the entire $S_{H1}$ is represented with its vertex on the head of $\mathbf{f}_{ext}$ (Figure 4).
3. The third vector to be considered in the path loop is the wrench $\mathbf{w}_2$ due to $\mathbf{f}_2$ applied on $P_2$. As in the previous step, $\mathbf{f}_2 \in C_2$ then $\mathbf{w}_2 \in S_{HI2}$, and the entire $S_{HI2}$ can be represented with its vertex on the tail of $\mathbf{f}_{ext}$ (i.e. the origin of the wrench space)(Figure 4); this links the tail of the vectors $\mathbf{w}_2$ with the tail of $\mathbf{f}_{ext}$, in order to properly link the wrench vectors (i.e. make the head of $\mathbf{w}_2$ matching the tail of $\mathbf{f}_{ext}$), the vectors in $S_{HI2}$ are graphically represented with their heads on the vertex of $S_{HI2}$, defining in this way the cone $S_{HI2}$ symmetrical of $S_{HI2}$ with respect to the vertex, as it is illustrated in Figure 5 (for clarity purpose, from now on the plane $\Pi_0$ is not represented in the figures).

$LS_B = S_{HI1} \cap S_{HI2}$ is the set of points that define all the combinations of $\mathbf{w}_1$ and $\mathbf{w}_2$ that balance $\mathbf{f}_{ext}$ (see the enlargement in Figure 5), i.e. they indicate the combinations of forces $\mathbf{f}_1$ and $\mathbf{f}_2$ applied at $P_1$ and $P_2$ that balance $\mathbf{f}_{ext}$ and therefore a valid set of forces to reach the equilibrium in $S_B$. We refer to $LS_B$ as the equilibrium loci for $S_B$. Note that $\mathbf{w}_2$ could be considered in the Step 2 and then $S_{H1}$ would be considered in this step.

4. Finally, the vector $\mathbf{w}_3$ due to the $\mathbf{f}_3$ applied at $P_3$ is added. Assuming that the value of $\mathbf{w}_1$ is known (it is a point inside $S_{HI1}$), $S_{HI1}$ can be represented with its vertex on the head of the given value of $\mathbf{w}_1$ inside $S_{HI1}$. Doing this, $LS_A = S_{HI1} \cap S_{HI2}$ is the set of points that define all the combinations of $\mathbf{w}_2$ and $\mathbf{w}_3$ that balance $\mathbf{f}_{ext}$ for the given $\mathbf{w}_1$, generating a wrench loop and allowing therefore the equilibrium of $S_A$ (see Figure 6).

5. PROPOSED SOLUTION

The graphical representation of GWL is used now to determine the temporal evolution of $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$, to change from $S_A$ to $S_B$. Since the sets $S_{HI}$ are convex, the simplest way to change the wrenches $\mathbf{w}_i$ from their value in $S_A$ to their value in $S_B$ assuring that $\mathbf{w}_i \in S_{HI}$ is to make them follow a straight line, while keeping a closed wrench
loop. Thus, consider that \( \mathbf{w}_1 \) varies on a straight segment \( \text{Path}_1 \in S_{\Pi 1} \) and \( \mathbf{w}_2 \) on a straight segment \( \text{Path}_2 \in S_{\Pi 2} \).

The plane defined by \( \text{Path}_1 \) and \( \text{Path}_2 \), constrains \( \mathbf{w}_3 \) to lie on the intersection of this plane with \( S_{\Pi 3} \) while the vertex of \( S_{\Pi 3} \) slide on \( \text{Path}_1 \) when \( \mathbf{w}_1 \) change from \( S_A \) to \( S_B \). This intersection defines \( \text{Path}_3 \), which fix a constant direction for \( \mathbf{w}_3 \) while its module decreases from the initial value in \( S_A \) to zero in \( S_B \). In order to keep a closed wrench loop (i.e. the equilibrium), the three paths can be followed changing in a synchronized way the magnitudes of \( \mathbf{w}_1, \mathbf{w}_2 \) and \( \mathbf{w}_3 \), this makes the triangle defined by the three paths to decrease from the initial state \( S_A \) up to disappear in \( S_B \) keeping the same shape. Figure 7 shows an example of the vectors \( \mathbf{w}_1, \mathbf{w}_2 \) and \( \mathbf{w}_3 \) in an intermediate state (white vectors) while changing from \( S_A \) to \( S_B \), the final vectors \( \mathbf{w}_1 \) and \( \mathbf{w}_2 \) in \( S_B \) (white dashed line vectors), and the \( \text{Path}_1, \text{Path}_2 \) and \( \text{Path}_3 \).

Using the supraindex \( A \) and \( B \) to indicated the values of \( \mathbf{w}_i \) in states \( S_A \) and \( S_B \) respectively, and letting \( T(t) \) be a function that smoothly varies in time between one and zero, the time variations of \( \mathbf{w}_i \) according to this behavior can be expressed as,

\[
\mathbf{w}_1(t) = \mathbf{w}_1B + (\mathbf{w}_1^A - \mathbf{w}_1^B) \ T(t) \tag{1}
\]

\[
\mathbf{w}_2(t) = \mathbf{w}_2B + (\mathbf{w}_2^A - \mathbf{w}_2^B) \ T(t) \tag{2}
\]

\[
\mathbf{w}_3(t) = \mathbf{w}_3^B \ T(t) \tag{3}
\]

Note that \( \mathbf{w}_1 \) and \( \mathbf{w}_2 \) move, respectively, along the straight segments \( \text{Path}_1 \) and \( \text{Path}_2 \) as linear functions of \( T(t) \) while \( \mathbf{w}_3 \) decreases to zero keeping always the same direction.

### 6. GENERALIZATION TO 3D OBJECTS

The proposed solution can be generalized for the case of 3D objects considering 3-dimensional forces in

\[
f_i(t) = f_i^B + (f_i^A - f_i^B) \ T(t) \quad i = 1..n \tag{4}
\]

or considering 6-dimensional wrenches in

\[
\mathbf{w}_i(t) = \left( \frac{f_i(t)}{r_i \times f_i(t)} \right) = \mathbf{w}_i^B + (\mathbf{w}_i^A - \mathbf{w}_i^B) \ T(t) \tag{5}
\]

which are equivalent (applying the distributive property of the cross product). In order to prove that the solution given by equations (4) and (5) can be used as a general solution for \( n \) contact points on a 3D object it must be proved that: first, it satisfies the general equilibrium condition

\[
\sum_{i=1}^{n} \mathbf{w}_i(t) + \left( \frac{f_{\text{ext}}}{0} \right) = 0 \quad \text{for} \quad t_A \leq t \leq t_B \tag{6}
\]

and second, that \( f_i(t), i = 1..n \), lie inside the friction cone at the respective contact points.

To prove the first statement the equilibrium conditions at the initial grasp state \( S_A \) and the final grasp state \( S_B \) are used. These conditions can be written as

\[
\sum_{i=1}^{n} \mathbf{w}_i(t_A) + \left( \frac{f_{\text{ext}}}{0} \right) = \sum_{i=1}^{n} \mathbf{w}_i^A + \left( \frac{f_{\text{ext}}}{0} \right) = 0 \tag{7}
\]

and

\[
\sum_{i=1}^{n} \mathbf{w}_i(t_B) + \left( \frac{f_{\text{ext}}}{0} \right) = \sum_{i=1}^{n} \mathbf{w}_i^B + \left( \frac{f_{\text{ext}}}{0} \right) = 0 \tag{8}
\]

which can be rewritten as

\[
\sum_{i=1}^{n} \mathbf{w}_i^A = \sum_{i=1}^{n} \mathbf{w}_i^B = - \mathbf{f}_{\text{ext}} \tag{9}
\]

Replacing \( \mathbf{w}_i(t) \) in equation (6) by the expression given in equation (5),

\[
\sum_{i=1}^{n} \left[ \mathbf{w}_i^B + (\mathbf{w}_i^A - \mathbf{w}_i^B) \ T(t) \right] + \left( \frac{f_{\text{ext}}}{0} \right) = 0 \tag{10}
\]

rearranging equation (10)

\[
\sum_{i=1}^{n} \mathbf{w}_i^B + \left( \sum_{i=1}^{n} \mathbf{w}_i^A - \sum_{i=1}^{n} \mathbf{w}_i^B \right) T(t) + \left( \frac{f_{\text{ext}}}{0} \right) = 0 \tag{11}
\]

and replacing the summations in equation (11) using equation (9)
that gives zero \( \forall t \), which is the first prove needed.

For the second proof consider that \( \mathbf{f}_i(t) \) is a vector function defining in the force space points on the straight line defined by \( \mathbf{f}_i^A \) and \( \mathbf{f}_i^B \); now, since \( \mathbf{f}_i^A \) and \( \mathbf{f}_i^B \) belong to the friction cone and the friction cone is a convex space, all the points defined by \( \mathbf{f}_i(t) \) lie inside the friction cone.

7. EXAMPLE

The proposed approach has been implemented and we describe here an example in 2D to illustrate how it works. The problem to be solved is the force transition for the object and the states \( S_A \) and \( S_B \) shown in Figure 1.

Given the external force \( \mathbf{f}_{ext} = [-1.5 - 3.5] \), and the contact points \( P_1 = [-4 -4] \), \( P_2 = [4 -5] \) and \( P_3 = [0 8] \), the applied forces that produce equilibrium at \( S_A \) and \( S_B \) are:

\[
\begin{align*}
\mathbf{f}_1^A &= [3.7897 3.0034] \\
\mathbf{f}_1^B &= [-3.0096 4.4156] \\
\mathbf{f}_2^A &= [0.7199 - 3.9190] \\
\mathbf{f}_2^B &= [1.8557 2.4555] \\
\mathbf{f}_3^A &= [-0.3557 1.0445]
\end{align*}
\]

With these forces and contact points the following wrenches are generated:

\[
\begin{align*}
\mathbf{w}_1^A &= [3.7897 3.0034 3.1448] \\
\mathbf{w}_1^B &= [-3.0096 4.4156 2.6145] \\
\mathbf{w}_2^A &= [0.7199 - 3.9190 - 5.7593] \\
\mathbf{w}_2^B &= [1.8557 2.4555 - 2.3930] \\
\mathbf{w}_3^A &= [-0.3557 1.0445 2.3930]
\end{align*}
\]

In order to produce a smooth transition at the beginning and at the end of the finger remove action the function \( T(t) \) was defined by a spline with five control points (Figure 8), which assures \( dT(t)/dt = 0 \) at the initial time \((t = 0)\) and at the desired final time \((t = 4)\).

Using equations (1), (2) and (3), the functions \( \mathbf{w}_1(t), \mathbf{w}_2(t) \) and \( \mathbf{w}_3(t) \) that allow the object equilibrium were obtained; the results are graphically shown in Figure 9 that shows the variation in the magnitude of \( \mathbf{f}_i(t) \), \( i = 1, 2, 3 \), and Figure 10 that shows the variation in the angles \( \theta_i \) between the object normal direction and \( \mathbf{f}_i(t) \). Note that the direction of \( \mathbf{f}_3(t) \) is constant while its module decreases to zero, and that the directions of \( \mathbf{f}_1(t) \) and \( \mathbf{f}_2(t) \) remains all the time inside the friction cone limits. Figure 11 shows the physical object with the forces \( \mathbf{f}_i \) in an intermediate situation between the states \( S_A \) and \( S_B \) and the Path1, Path3 and Path1 being followed.

As an additional verification of the system equilibrium, it was checked whether \( \mathbf{f}_{ext}^T \mathbf{G} \mathbf{f}_g^T = 0 \) is satisfied, being \( \mathbf{G} \) the grasp matrix and \( \mathbf{f}_g = [\mathbf{f}_1^P, \mathbf{f}_2^P, \mathbf{f}_3^P]^T \) with \( \mathbf{f}_i^P \) the force \( \mathbf{f}_i \) expressed in a coordinate system fixed at \( P_i \); the condition was satisfied \( \forall t \).

8. CONCLUSIONS AND FUTURE WORKS

A fast non iterative solution to the problem of finding the force variations that keep the object equilibrium when a finger is removed from a \( n \) contact point grasp (or added to a \( n - 1 \) contact point grasp) has been proposed and implemented. The approach is simple and efficient.

The ongoing work includes the determination of a procedure to change from a grasp with \( n \) contacts to another grasp with \( n \) different contacts (doing in this way a full regrasp of the object), and to keep the object equilibrium automatically solving intermediate consecutive grasps \( S_j \) that differ in only one contact point.
and changing the object orientation when necessary using wrist movements. The whole procedure would generate position and force set points for the control system of the grasping device. Given the initial and final grasp with \( n \) contact points (fingers) the approach includes the following subproblems:

1. Automatic determination of a sequence of grasp states that balance the external force (object weight), alternately considering grasps with \( n-1 \) and \( n \) fingers (i.e. repositioning one finger at a time), which is equivalent to automatically and alternately determine for each step the grasp states \( S_A \) and \( S_B \) in this paper (other than the given initial and final states in the sequence). The search can be done using a "Generic Wrench Loop" (GWL) that describes the forces of the fingers that do not change and selecting a proper point on the corresponding region \( LS \) (equivalent to the region \( LS_B \) in Figure 5).

2. Automatic determination, if necessary, of wrist movements to change the object orientation and the force variations to keep the equilibrium when these movements are performed. Again, this can be done using the GWL representation.

Besides, some dynamic considerations could be addressed in future works, as well as some strategies to assure robustness in front of different sources of uncertainty.

REFERENCES


