A design method of robust stabilizing simple multi-period repetitive controllers

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Abstract: A multi-period repetitive control system is a type of servomechanism for a periodic reference input. Even if a plant does not include time-delay, the transfer function from the periodic reference input to the output and that from the disturbance to the output of the multi-period repetitive control system generally have an infinite number of poles. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, Yamada et al. propose the concept of simple multi-period repetitive control systems such that the controller works as a stabilizing multi-period repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles. However, the method by Yamada et al. cannot apply for the plant with uncertainty. The purpose of this paper is to propose the concept of robust stabilizing simple multi-period repetitive controllers for the plant with uncertainty and to clarify the parametrization of all robust stabilizing simple multi-period repetitive controllers.

Keywords: periodic signal, multi-period repetitive controller, finite number of poles, parametrization

1. INTRODUCTION

In this paper, we investigate a problem to seek all robust stabilizing simple multi-period repetitive controllers for a plant with uncertainty. A repetitive control system is a type of servomechanism for a periodic reference input for the periodic signal without steady state error (Inoue et al., 1981; Omata et al., 1987; Watanabe and Yamatari, 1986; Hara et al., 1988; Yamamoto and Hara, 1988; Nakano et al., 1989; Ikeda and Takano, 1990; Yamamoto and Hara, 1992; Yamamoto and Hara, 1993; Katoh and Funahashi, 1996; Weiss, 1997). That is, the repetitive control system follows the periodic reference input without steady state error, even if there exists a periodic disturbance or the plant has uncertainty (Inoue et al., 1981; Omata et al., 1987; Watanabe and Yamatari, 1986; Hara et al., 1988; Yamamoto and Hara, 1988; Nakano et al., 1989; Ikeda and Takano, 1990; Yamamoto and Hara, 1992; Yamamoto, 1993; Katoh and Funahashi, 1996; Weiss, 1997). The repetitive control system was proposed for 'high accuracy control for magnet power supply of proton synchrotron in recurrent operation' (Inoue et al., 1981). Various papers on the repetitive control have been studied (Inoue et al., 1981; Omata et al., 1987; Watanabe and Yamatari, 1986; Hara et al., 1988; Yamamoto and Hara, 1988; Nakano et al., 1989; Ikeda and Takano, 1990; Yamamoto and Hara, 1992; Yamamoto, 1993; Katoh and Funahashi, 1996; Weiss, 1997).

On the other hand, there exists important control problem to find all stabilizing controllers named the parametrization problem (Youla et al., 1976; Kucera, 1979; Glaria and Goodwin, 1994; Vidyasagar, 1985). First, the parametrization of all stabilizing modified repetitive controllers that follow the periodic reference input with small steady state error even if there exists a periodic disturbance or the uncertainty of the plant was studied by (Hara and Yamamoto, 1986). In (Hara and Yamamoto, 1986), since the stability sufficient condition of repetitive control system is decided as $H_{\infty}$ norm problem, the parametrization for repetitive control system is given by solving into the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi gave the parametrization of all stabilizing repetitive controllers for minimum phase systems by solving exactly Bezout equation (Katoh and Funahashi, 1996). However, Katoh and Funahashi (Katoh and Funahashi, 1996) assumed the plant is asymptotically stable. This implies that they gave the parametrization of all causal repetitive controllers for an asymptotically stable and minimum phase plant. That is, they do not give the explicit parametrization for minimum phase systems (Katoh and Funahashi, 1996). In addition, in (Katoh and Funahashi, 1996) it is assumed that the relative degree of low-pass filter in the repetitive compensator is equal to that of the plant. Extending the results in (Katoh and Funahashi, 1996), Yamada and Okuyama gave the parametrization of all stabilizing repetitive controllers for minimum phase systems (Yamada and Okuyama, 2000). Yamada et al. gave the parametrization of all stabilizing repetitive controllers for the certain class of non-minimum phase systems (Yamada et al., 2002a). They obtained the parametrization of all repetitive controllers using fusion of the parallel compensation technique and the solution of Bezout equation. However, they gave the parametrization of all repetitive controllers for limited class of non-minimum phase systems. Yamada et al. gave the complete parametrization of all stabilizing modified repetitive controllers for non-minimum phase single-input/single-output systems (Yamada et al., 2002b). They obtained the...
parametrization of all repetitive controllers using fusion of the parallel compensation technique and the solution of Bezout equation. However, they gave the parametrization of all repetitive controllers for limited class of non-minimum phase systems. Yamada et al. gave the complete parametrization of all stabilizing modified repetitive controllers for non-minimum phase single-input/single-output systems (Yamada et al., 2002b). In this way, the parametrization of all stabilizing modified repetitive controllers has been considered.

However the modified repetitive controllers has bad disturbance attenuation characteristic (Gotou et al., 1987). In order to improve the disturbance attenuation characteristic of the modified repetitive controllers, the multi-period repetitive controllers was proposed by (Gotou et al., 1987). The parametrization of all stabilizing multi-period repetitive controllers for non-minimum phase systems, which is used to improve the disturbance attenuation characteristics of the modified repetitive controller, was solved in (Yamada et al., 2004). Since the method in (Yamada et al., 2004) cannot apply for the plant with uncertainty, Satoh, Yamada and Mei proposed the parametrization of all robust stabilizing multi-period repetitive controllers for the plant with uncertainty (Satoh et al., 2006). However, using the method in (Satoh et al., 2006), the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, even if the uncertainty does not exist. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input to the output and that from the disturbance to the output are desirable to have a finite number of poles.

In this paper, we propose the concept of robust stabilizing simple multi-period repetitive controllers for the plant with uncertainty and clarify the parametrization of all robust stabilizing simple multi-period repetitive control systems such that the controller works as a robust stabilizing multi-period repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles.

### 2. PROBLEM FORMULATION

Consider the unity feedback system in

\[
\begin{align*}
    \dot{y} &= G(s)u + d \\
    u &= C(s)(r - y)
\end{align*}
\]

where \( G(s) \in R(s) \) is the plant, \( C(s) \) is the controller, \( u \in R \) is the control input, \( d \in R \) is the disturbance, \( y \in R \) is the output and \( r \in R \) is the periodic reference input with period \( T \) satisfying

\[
r(t + T) = r(t) \quad (\forall t \geq 0).
\]

The nominal plant of \( G(s) \) is denoted by \( G_m(s) \in R(s) \). Both \( G(s) \) and \( G_m(s) \) are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of \( G(s) \) in the closed right half plane is equal to that of \( G_m(s) \). The relation between the plant \( G(s) \) and the nominal plant \( G_m(s) \) is written as

\[
G(s) = G_m(s)(1 + \Delta(s)).
\]

The set of \( \Delta(s) \) is all rational functions satisfying

\[
|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+),
\]

where \( W_T(s) \) is an asymptotically stable rational function.

The robust stability condition for the plant \( G(s) \) with uncertainty \( \Delta(s) \) satisfying (4) is given by

\[
\|T(s)W_T(s)\|_\infty < 1,
\]

where \( T(s) \) is the complementary sensitivity function given by

\[
T(s) = \frac{G_m(s)C(s)}{1 + G_m(s)C(s)}.
\]

According to (Gotou et al., 1987), the multi-period repetitive controller \( C(s) \) in (1) is written by the form in

\[
C(s) = C_0(s) + \sum_{i=1}^{N} C_i(s)e^{-sT_i},
\]

where \( N \) is arbitrary positive integer, \( C_0(s) \in R(s) \), \( C_i(s) \neq 0 \in R(s)(i = 1, \ldots, N) \), \( q_i(s) \in R(s)(i = 1, \ldots, N) \) are low-pass filter satisfying \( \sum_{i=1}^{N} q_i(0) = 1 \) and \( T_i \in R(i = 1, \ldots, N) \).

Using the modified repetitive controller \( C(s) \) in (7), the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output \( y \) in (1) are written as
\[
\frac{y}{r} = \frac{C(s)G(s)}{1 + C(s)G(s)}
= \left\{ C_0(s)G_m(s)(1 + \Delta(s)) - \sum_{i=1}^{N} (C_0(s)q_i(s) - C_i(s)) e^{-sT_i} G_m(s)(1 + \Delta(s)) \right\}[1 + \sum_{i=1}^{N} q_i(s) \{1 + C_0(s)G_m(s)(1 + \Delta(s))\}]^{-1}
\]
respectively. Generally, the transfer function from the periodic reference input \( r \) to the output \( y \) in (8) and that from the disturbance \( d \) to the output \( y \) in (9) have an infinite number of poles, even if \( \Delta(s) = 0 \). When the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output \( y \) have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output \( y \) are desirable to have a finite number of poles.

From above practical requirement, we clarify the parametrization of all robust stabilizing simple multi-period repetitive controllers defined as follows:

**Definition 1. (robust stabilizing simple multi-period repetitive controller)**

We call the controller \( C(s) \) the robust stabilizing simple multi-period repetitive controller, if following expressions hold true:

1. The controller \( C(s) \) works as a multi-period repetitive controller. That is, the controller \( C(s) \) is written by (7), where \( C_0(s) \in R(s) \), \( C_i(s) \neq 0 \in R(s)(i = 1, \ldots, N) \) and \( q_i(s) \in R(s)(i = 1, \ldots, N) \) satisfies \( \sum_{i=1}^{N} q_i(0) = 1 \).
2. When \( \Delta(s) = 0 \), the controller \( C(s) \) makes the transfer function from the periodic reference input \( r \) to the output \( y \) in (1) and that from the disturbance \( d \) to the output \( y \) in (1) have a finite number of poles.
3. The controller \( C(s) \) satisfies the robust stability condition in (5).

### 3. THE PARAMETRIZATION OF ALL ROBUST STABILIZING SIMPLE MULTI-PERIOD REPEATED CONTROLLERS

In this section, we give the parametrization of all robust stabilizing simple multi-period repetitive controllers defined in Definition 1.

In order to obtain the parametrization of all robust stabilizing simple multi-period repetitive controllers, we must see that the robust stabilizing controllers hold (5). The problem of obtaining the controller \( C(s) \), which is not necessarily a multi-period repetitive controller, satisfying (5) is equivalent to the following \( H_\infty \) problem. In order to obtain the controller \( C(s) \) satisfying (5), we consider the control system shown in Fig. 1. \( P(s) \) is selected such that the transfer function from \( w \) to \( z \) in Fig. 1 is equal to \( T(s)W_T(s) \). The state space description of \( P(s) \) is, in general,

\[
\dot{x}(t) = Ax(t) + B_1u(t) + B_2w(t)
\]
\[
z(t) = C_1x(t) + D_{11}u(t) + D_{12}w(t)
\]
where \( A \in R^{n \times n}, B_1 \in R^n, B_2 \in R^n, C_1 \in R^{1 \times n}, C_2 \in R^{1 \times n}, D_{12} \in R, D_{21} \in R \). \( P(s) \) is called the generalized plant. \( P(s) \) is assumed to satisfy the standard assumption in (Doyle et al., 1989). Under these assumptions, according to (Doyle et al., 1989), following lemma holds true.

**Lemma 1.** If controllers satisfying (5) exist, both

\[
X \left( A - B_2D_{12}^T C_1 \right) + \left( A - B_2D_{12}^T C_1 \right)^TX + X \left( B_1B_1^T - B_2 \left( D_{12}^T D_{12} \right) \right)^{-1} B_2^T X + \left( D_{12}^T C_1 \right)^T D_{12}^T C_1 = 0
\]
and

\[
Y \left( A - B_1D_{21}^T C_2 \right)^T + \left( A - B_1D_{21}^T C_2 \right) Y + Y \left( C_1^T C_1 - C_2^T \left( D_{21}D_{21}^T \right)^{-1} C_2 \right) Y + B_1D_{21}^T \left( B_1D_{21} \right)^T = 0
\]

have solutions \( X \geq 0 \) and \( Y \geq 0 \) such that

\[
\rho(XY) < 1
\]
and both

\[
A - B_2D_{12}^T C_1 + \left( B_1B_1^T - B_2 \left( D_{12}^T D_{12} \right) \right)^{-1} B_2^T X
\]
\[ A - B_1 D_1^T C_2 + Y \left( C_1^T C_1 - C_2 \left( D_{21} D_{21}^T \right)^{-1} C_2 \right) \]

have no eigenvalue in the closed right half plane. Using \( X \) and \( Y \), the parametrization of all controllers satisfying (5) is given by

\[
C(s) = C_{11}(s) + C_{12}(s) Q(s) (I - C_{22}(s) Q(s))^{-1} C_{21}(s),
\]

where

\[
\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \begin{bmatrix} A_c & B_{c1} & B_{c2} \\ C_{11} & D_{c11} & D_{c12} \\ C_{21} & D_{c21} & D_{c22} \end{bmatrix}
\]

\[
A_c = A + B_1 B_1^T X - B_2 \left( D_{12}^T C_1 + E_{12} B_2^T X \right) - (I - XY)^{-1} \left( B_1 D_{21}^T + Y C_2 E_{21}^{-1} \right) \left( C_2 + D_{21} B_2^T X \right),
\]

\[
B_{c1} = (I - XY)^{-1} \left( B_1 D_{12}^T + Y C_2 E_{21}^{-1} \right),
\]

\[
B_{c2} = (I - XY)^{-1} \left( B_2 + Y C_2^T D_{12} \right) E_{12}^{-1/2},
\]

\[
C_{c1} = -D_{12}^T C_1 - E_{12} B_2^T X,
\]

\[
C_{c2} = -E_{21}^{-1/2} \left( C_2 + D_{21} B_2^T X \right),
\]

\[
D_{c11} = 0, \quad D_{c12} = E_{12}^{-1/2}, \quad D_{c21} = E_{21}^{-1/2},
\]

\[
D_{c22} = 0, \quad E_{12} = D_{12}^T D_{12}, \quad E_{21} = D_{21} D_{21}^T
\]

and the free parameter \( Q(s) \in H_{\infty} \) is any function satisfying \( \| Q(s) \|_{\infty} < 1 \) (Doyle et al., 1989).

Using Lemma 1, the parametrization of all robust stabilizing simple multi-period repetitive controllers is given by following theorem.

**Theorem 1.** If simple multi-period repetitive controllers satisfying (5) exist, both (11) and (12) have solutions \( X \geq 0 \) and \( Y \geq 0 \) satisfying (13) and both

\[
A - B_2 D_1^T C_1 + \left( B_1 B_1^T - B_2 \left( D_{12} D_{12}^T \right)^{-1} B_2^T \right) X
\]

and

\[
A - B_1 D_2^T C_2 + Y \left( C_1^T C_1 - C_2 \left( D_{21} D_{21}^T \right)^{-1} C_2 \right)
\]

have no eigenvalue in the closed right half plane. Using \( X \) and \( Y \), the parametrization of all robust stabilizing simple repetitive controllers satisfying (5) is given by

\[
C(s) = C_{11}(s) + C_{12}(s) Q(s) (I - C_{22}(s) Q(s))^{-1} C_{21}(s),
\]

where \( C_{ij}(i = 1, 2; j = 1, 2) \) are given by (15) and the free parameter \( Q(s) \in H_{\infty} \) is any function satisfying \( \| Q(s) \|_{\infty} < 1 \) and written by

\[
Q(s) = Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s) e^{-sT_i},
\]

\[
Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) e^{-sT_i},
\]

and

\[
Q_{ni}(s) = G_d(s) Q_i(s) \quad (i = 1, \ldots, N)
\]

\[
Q_{di}(s) = -\frac{1}{1 + C_{11}(s) G_m(s)} G_n(s) Q_i(s) 
\]

\[
(i = 1, \ldots, N),
\]

where \( Q_{n0}(s) \in RH_{\infty}, Q_{d0}(s) \in RH_{\infty}, G_n(s) \in RH_{\infty} \) and \( G_d(s) \in RH_{\infty} \) are coprime factors of \(-C_{22}(s) + (-C_{11}(s) C_{22}(s) + C_{12}(s) C_{21}(s)) G_m(s) \) on \( RH_{\infty} \) satisfying

\[
G_n(s) = Q_{d0}(0) - Q_{d0}(0) Q_{n0}(0) \]

\[
Q_{d0}(0) - Q_{d0}(0) Q_{n0}(0) = 1
\]

holds true.

(Proof) First, necessity is shown. That is, if the multi-period repetitive controller written by (7) stabilizes the control system in (1) robustly and makes the transfer function from the periodic reference input \( r \) to the output \( y \) in (8) and that from the disturbance \( d \) to the output \( y \) in (9) have a finite number of poles when \( \Delta(s) = 0 \), then \( C(s) \) and \( Q(s) \) are written by (16) and (17), respectively. From Lemma 1, the parametrization of all robust stabilizing controllers \( C(s) \) for \( G(s) \) is written by (16), where \( \| Q(s) \|_{\infty} < 1 \). In order to prove the necessity, we will show that if \( C(s) \) written by (7) stabilizes the control system in (1) robustly and makes the transfer function from the periodic reference input \( r \) to the output \( y \) in (8) and that from the disturbance \( d \) to the output \( y \) in (9) have a finite number of poles when \( \Delta(s) = 0 \), then the free parameter \( Q(s) \) in (16) is written by (17). Substituting \( C(s) \) in (7) into (16), we have (17), where

\[
Q_{n0}(s) = (C_{11d}(s) C_{0d}(s) - C_{11d}(s) C_{0n}(s)) C_{d}(s)
\]

\[
q_d(s) C_{12d}(s) C_{22d}(s) C_{21d}(s),
\]

\[
Q_{n1}(s) = \{ C_{0n}(s) C_d(s) C_{11d}(s) - C_{0d}(s) C_d(s) C_{11d}(s) \}
\]

\[
q_n(s) - C_{0d}(s) C_{0n}(s) q_d(s) C_{11d}(s) C_{22d}(s) C_{21d}(s)
\]

\[
(i = 1, \ldots, N),
\]

\[
Q_{d0}(s) = (-C_{0n}(s) C_{11d}(s) C_{12d}(s) C_{22n}(s) C_{21d}(s)
\]

\[
+ C_{0d}(s) C_{11n}(s) C_{12d}(s) C_{22n}(s) C_{21d}(s)
\]

\[
- C_{0d}(s) C_{11d}(s) C_{12n}(s) C_{22d}(s) C_{21n}(s))
\]

\[
C_{d}(s) q_d(s)
\]
and

\[ Q_{di}(s) = (Q_{0n}(s)Q_{0d}(s)Q_{11d}(s)Q_{12d}(s)Q_{22n}(s) \]
\[ Q_{21d}(s) + C_{0d}(s)Q_{0d}(s)Q_{11n}(s)Q_{12d}(s) \]
\[ Q_{22n}(s)Q_{21d}(s))q_{in}(s) - C_{0d}(s)Q_{0d}(s)Q_{11n}(s)Q_{12d}(s) \]
\[ C_{1n}(s)q_{d}(s)Q_{11d}(s)Q_{12d}(s)Q_{22n}(s)Q_{21d}(s)i = 1, \ldots , N \]

Here, \( Q_{0n}(s) \in RH_{\infty} \) and \( C_{0d}(s) \in RH_{\infty} \) are coprime factors of \( C_{0}(s) \) on \( RH_{\infty} \) satisfying

\[ C_{0}(s) = Q_{0n}(s)C_{0d}(s), \]
\[ q_{in}(s) \in RH_{\infty}(i = 1, \ldots , N), q_{d}(s) \in RH_{\infty}, C_{in}(s) \in RH_{\infty}(i = 1, \ldots , N), C_{id}(s) \in RH_{\infty}, C_{ij}(s) \in RH_{\infty}(i = 1, 2; j = 1, 2) \] and \( C_{ijd}(s) \in RH_{\infty}(i = 1, 2; j = 1, 2) \) are coprime factors satisfying

\[ q_{i}(s) = q_{in}(s)q_{d}(s)(i = 1, \ldots , N), \]
\[ C_{1}(s) = \frac{Q_{0n}(s)}{Q_{0d}(s)}(i = 1, \ldots , N) \]
\[ C_{ij}(s) = C_{ijn}(s)C_{ijd}(s)(i = 1, 2; j = 1, 2). \]

From (22)–(25), all of \( Q_{0n}(s), Q_{in}(s)i = 1, \ldots , N, Q_{0d}(s) \) and \( Q_{di}(s)i = 1, \ldots , N \) are included in \( RH_{\infty} \). Thus, we have shown that if \( C(s) \) written by (7) stabilize the control system in (1) robustly, \( Q(s) \) in (16) is written by (17). Since \( \sum_{i=1}^{N} q_{i}(0) = 1 \) \( i = 1, \ldots , N \), (21) holds true.

The rest to prove necessity is to show that when \( \Delta(s) = 0 \), if \( C(s) \) in (7) make the transfer function from the periodic reference input \( r \) to the output \( y \) and the disturbance \( d \) to the output \( y \) have a finite number of poles, then \( Q_{in}(s) \) and \( Q_{di}(s) \) (i = 1, \ldots , N) are included in (18) and (19), respectively. From (17), when \( \Delta(s) = 0 \), the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output \( y \) are written by

\[ y = \frac{G_{r,y}(s)}{G_{r,y}(s)}(30) \]
\[ y = \frac{G_{d,y}(s)}{G_{d,y}(s)}, \]

respectively, where

\[ G_{r,y}(s) = \left[ (C_{11}(s)Q_{0d}(s) + (-C_{11}(s)C_{22}(s) + C_{12}(s)) \]
\[ C_{21}(s)Q_{22}(s) + \sum_{i=1}^{N} \{ C_{11}(s)Q_{di}(s) \}
\[ + (-C_{11}(s)C_{22}(s) + C_{12}(s)C_{21}(s))Q_{ni}(s) \}
\[ e^{-sT_{i}}G_{m}(s), \]

From the assumption that the transfer function from the periodic reference input \( r \) to the output \( y \) in (30) and that from the disturbance \( d \) to the output \( y \) in (31) have a finite number of poles, (33) and (35),

\[ Q_{di}(s) = -C_{22}(s)Q_{ni}(s) + \{ C_{11}(s)Q_{di}(s) \}
\[ + (-C_{11}(s)C_{22}(s) + C_{12}(s)C_{21}(s))Q_{ni}(s) \} \]
\[ = 0 \quad (i = 1, \ldots , N) \]

is satisfied. Using (20), this equation is rewritten by

\[ Q_{di}(s) = \frac{1}{1 + C_{11}(s)G_{m}(s)} \]
\[ Q_{di}(s) \]

Since \( Q_{ni}(s) \in RH_{\infty}(i = 1, \ldots , N) \) and \( Q_{di}(s) \in RH_{\infty}(i = 1, \ldots , N) \), \( Q_{ni}(s) \) and \( Q_{di}(s) \) are written by (18) and (19), respectively, where \( Q_{d}(s) \in RH_{\infty}(i = 1, \ldots , N) \). From the assumption that \( C_{1}(s) \neq 0 \) \( i = 1, \ldots , N \) and from (23) and (25), \( Q_{d}(s) \neq 0 \) \( i = 1, \ldots , N \) hold true. We have thus proved necessity.

Next, sufficiency is shown. That is, if \( C(s) \) and \( Q(s) \) \( H_{\infty} \) are settled by (16) and (17), respectively, then the controller \( C(s) \) is written by the form in (7), \( \sum_{i=1}^{N} q_{i}(0) = 1 \) holds true and the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output \( y \) have a finite number of poles. Substituting (17) into (16), we have (7), where, \( C_{0}(s), C_{1}(s) \) \( i = 1, \ldots , N \) and \( q_{i}(s) \) \( i = 1, \ldots , N \) are denoted by

\[ C_{0}(s) = \frac{Q_{0}(s) - C_{22}(s)Q_{22}(s)}{Q_{0d}(s)}, \]
\[ C_i(s) = \frac{C_{11}(s)Q_{di}(s) + (-C_{11}(s)C_{22}(s)Q_{d0}(s)
+ C_{12}(s)C_{21}(s))Q_n0(s) + C_0(i)q_i(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} (i = 1, \ldots, N) \]

(39)

We find that if \( C(s) \) and \( Q(s) \) are settled by (16) and (17), respectively, then the controller \( C(s) \) is written by the form in (7). From \( Q_i(s) \neq 0(i = 1, \ldots, N) \) and (39), \( C_i(s) \neq 0(i = 1, \ldots, N) \) holds true. Substituting (21) into (40), we have \( \sum_{i=1}^{N} q_i(0) = 1 \). In addition, from (18) and (19) and easy manipulation, we can confirm that when \( \Delta(s) = 0 \), the transfer function from the periodic reference input \( r \) to the output \( y \) and that from the disturbance \( d \) to the output have a finite number of poles.

We have thus proved Theorem 1.

4. CONCLUSION

In this paper, we proposed the parametrization of all robust stabilizing simple multi-period repetitive control systems such that the controller works as a robust stabilizing multi-period repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles.

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