ROBUST SPEED CONTROL OF PMSM USING
Mixed NONLINEAR $H_\infty$/SMC Techniques

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Abstract: A new technique for high performance and robust speed control of permanent magnet synchronous motor (PMSM) using a mixed non-linear $H_\infty$ and Sliding Mode Control (SMC) is applied. In spite of non-linear modes, motor parameters variation and uncertainty of load torque, non-linear robust techniques introduce a precise speed control. Since the load torque is an external disturbance and its variation and type is not generally available, $H_\infty$ technique is a suitable approach to minimize its influence on the output. However, motor parameters variations throw away the response from $H_\infty$ response and influence its response, therefore SMC technique is used to conduct the response towards $H_\infty$ response. Combination of these two techniques gives a suitable technique to robust speed control of PMSM.

Keywords: PMSM, Robust, Nonlinear Control, Nonlinear $H_\infty$, SMC.

1. INTRODUCTION

Many advantages of permanent magnet synchronous motors (PMSM) lead to their wide application in industry. They are used in robots and similar industry equipment as well as in many control systems. Quick and precise response, eliminating disturbance, approaching reference speed and low-sensitivity against parameters variation are the criteria of desirable performance of these motors. These characteristics make possible to use them in robotic and instrumentations. Application of vector control technique in these motors improves their performance. see (Senjyu, et al., 1996; Senjyu, et al., 1997).

However, system non-linearity, motor parameters variation and load torque variation make difficult to control the motor precisely. There are various methods to solve these problems. Some methods use adaptive or neural network techniques (Tengfei and Wang, 2006; Yu, et al., 2006; Zhou and Wang, 2002; Elbuluk, et al., 2002; Mingji, et al., 2004) and others are suitable due to slow change of some parameters such as stator resistance. However, some parameters of motor such as inductance, flux or load torque can quickly change and the above-mentioned methods do not provide a suitable response, see (Soo, et al., 2001); therefore, robust control techniques are used for this motor. One of most important techniques is SMC which overcomes the parameters variation well (Lee and Shtessel, 1996; Golea, et al., 1999).

However, the problem is the load torque variation that is identified by observer in some methods (Baik, et al., 2000); this has own problems such as dependency on the model, noise effect etc. In some methods of Sliding Mode Control (SMC) it is treated similar to parameters (Zhang and Panda, 1999), but the value and type of the load as an external disturbance is not defined and such action cannot be true.

Other problem with SMC technique is its chattering which may be solve if boundary layer is employed with SMC technique. Unfortunately variation of some motor parameters leads to a steady-state error with this solution.

The $H_\infty$ technique is an efficient method for robustness of non-linear against disturbances. In addition to guarantee the stability, this technique minimizes the disturbance influence at the output. Nonlinear $H_\infty$ has been described by (Isidori and Astopffi, 1992; Van der Shaft,
1992). The technique has no assumption on the disturbance except the energy limitation. In the introduced method, H-infinity is used for robustness of the response against load variation, since load torque variation is an external factor which can be any type, H-infinity technique has no assumption on the disturbance and is a suitable technique for dealing with the proposed problem. In such a case, the response is somehow robust against parameters variation. However, parameters variation throws away the response from H-infinity technique, so SMC technique can be used in order to control this variation and conduct the response to the H-infinity response. Therefore, considering different natures of load and parameters variation, two techniques have been employed to solve the problems. In section II of the paper the mathematical and vector model of PMSM is introduced. In section III, application of H-infinity in PMSM is proposed and it combination with SMC is given in section IV. Simulation results and their comparison with other methods are discussed in section V. Finally section VI concludes the paper.

2. MATHEMATICAL MODEL OF PMSM

The d-q Equations of PMSM, which are in fact the vector equations of the motor, are as follows:

\[
\begin{align*}
\frac{d\omega_d}{dt} &= \frac{3P}{J}\omega_d - \frac{D}{J}\omega_d - \frac{T}{J} \\
\frac{dl_d}{dt} &= -\frac{\lambda_d}{L_d}\omega_d + \frac{P}{2\lambda_d}i_d - \frac{R}{L_d}i_d + \frac{V_d}{L_d} \\
\frac{dl_q}{dt} &= -\frac{\lambda_q}{L_q}\omega_d + \frac{P}{2\lambda_q}i_q + \frac{V_q}{L_q}
\end{align*}
\] (1)

where \(\omega_d\) is the angular speed of the motor, \(i_d\) and \(i_q\) are the d and q currents, \(V_d\) and \(V_q\) are the input d and q voltages in vector control domain. \(R\) and \(L\) are the stator winding resistance and inductance respectively, \(P\) is the number of poles, \(\lambda\) is the flux of PM, \(J\) is the motor inertia, \(D\) is the damping factor, \(T\) is the load torque that is considered as disturbance. Since H∞ solution for the above-mentioned system of equations does not provide a tracking response and the aim is to obtain a robust tracking response, a state variable which is the difference between the speed and the reference speed is added to the system. This variable is called \(q\). Therefore, the following equation is added to (1) in order to complete the model of the system:

\[
\frac{dq}{dt} = \omega_d - \omega_r
\] (2)

where \(\omega_r\) is the reference speed.

3. DESIGN OF H∞ CONTROLLER

The reason for using the non-linear H-infinity in place of the linear H-infinity is that the motor is non-linear. If the linear H-infinity is used, there will be no guarantee for stability and it may be unstable when the load torque quickly changes. However, use of the non-linear H-infinity can guarantee the stability over any condition.

In the design of H-infinity controller, the following model for PMSM is defined:

\[
\begin{align*}
\dot{x} &= f(x) + B_w w + B_u u \\
z &= C_x x + D_u u \\
C^T_i D_i &= 0
\end{align*}
\] (3)

where \(x = [i_d, i_q, q, \omega]\), \(w = [v_d, v_q]\) and \(w\) is the disturbance or load torque (\(T\)) and \(z\) is the output, and also \(B_w = [-\frac{1}{J} 0 0 0]\) and \(B_u = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\). \(z\) is expressed as a combination of inputs and outputs. In H∞ technique the aim is to minimize the influence of disturbance (\(u\)) on output (\(z\)). A larger \(C_i\) compared to \(D_i\) has more disturbing influence in the inputs and less in the states. Since the goal is the tracking speed control, variables \(\omega_r\) and \(q\) are more important, therefore their corresponding elements in matrix \(C_i\) will be non-zero. It is noted that \(i_d\) has no influence on the torque directly, and it raises the amplitude of the current, so it is controlled toward zero in the H-infinity response. Therefore, considering different disturbance or load torque (\(l\)) and \(z\) is the output, and

\[
\begin{align*}
J\gamma \leq \frac{\partial^2 y(x)}{\partial x^2}
\end{align*}
\] (6)

where \(\gamma\) is attenuation level of the effect of \(w\) on \(z\). Since (5) in general does not have a close form solution, an approximate solution is considered. Let \(V(x)\) be approximated as follows:

\[
V(x) = V^{(1)}(x) + V^{(2)}(x) + V^{(3)}(x) + \ldots
\] (7)

where \(V^{(i)}(x)\) is an \(i\)-th order polynomial of \(x\). In this way, one has:

\[
f(x) = f^{(1)}(x) + f^{(2)}(x)
\] (8)

Other coefficients in PMSM are linear and there is no need to split-up them.

Considering \(V^{(2)}(x) = x^TPx\) in which \(P\) is symmetric and positive definite and \(f^{(1)}(x) = Fx\), \(P\) is obtained by solving the following Riccati equation:

\[
F^TP + PF + \frac{1}{\gamma} B_r B_r^T - B_w [D^T_i D_i]^{-1} B_w^T P + C^T_i C_i = 0
\] (9)

Since the model is linear in respect to the inputs and disturbance, higher-order norms of \(V\) are obtained as follows:

\[
\begin{align*}
V^{(n)}(x,F_w) &= \sum_{i=1}^{n} \sum_{j=1}^{m} \left( \begin{bmatrix} \lambda_i \omega_i x - i_i \omega_i x + j_i \omega_i x - d_i \omega_i x \end{bmatrix} + B_w u(x) \right) \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{m} \left( \begin{bmatrix} 1 \omega_i x + j_i \omega_i x - d_i \omega_i x \end{bmatrix} \right) \\
&- \sum_{i=1}^{n} \sum_{j=1}^{m} \left( \begin{bmatrix} \lambda_i \omega_i x + i_i \omega_i x + j_i \omega_i x - d_i \omega_i x \end{bmatrix} \right)
\end{align*}
\] (10)

where:

\[
\begin{align*}
\omega &= [i_d, i_q, q, \omega] \\
w &= [v_d, v_q]
\end{align*}
\]
\[ F_o = F + \frac{1}{\tau} B_i B_f P - B_i \left[ D_i D_f \right]^{\gamma} B_f^\gamma \] (11)

\[ u(t) = -(D_i D_f)^{\gamma} B_f^\gamma (V_{eq}^\gamma(x))^\gamma \] (12)

\[ w(t) = \frac{1}{\tau} B_f^\gamma (V_{eq}^\gamma(x))^\gamma \] (13)

4. APPLICATION OF SMC FOR CONDUCTING RESPONSE TOWARDS H∞

In H∞ technique, the effect of motor parameters variations has not been included. Although this technique is somehow robust against motor parameters variations, the variations throw away the response from H-infinity response. Therefore, the effect of motor parameters variations can be eliminated using SMC. In this case, the SMC solution conducts the response of the system towards solution of H-infinity response. Therefore this control rule of SMC technique response. In this case, the boundary layer will be as follows:

\[ S = C x - C \left[ f(x) - B_i \left[ D_i D_f \right]^{\gamma} B_f^\gamma V_{eq}^\gamma \right] I = 0 \] (14)

To obtain the control rule \( S = 0 \):

\[ \dot{S} = C x - C \left[ f(x) - B_i \left[ D_i D_f \right]^{\gamma} B_f^\gamma V_{eq}^\gamma \right] \] (15)

Considering \( C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \):

\[ \dot{S} = \dot{x}_1 + \dot{x}_2 - f_1(x) - f_2(x) + C \left[ B_i \left[ D_i D_f \right]^{\gamma} B_f^\gamma V_{eq}^\gamma \right] = 0 \] (16)

Substituting \( \dot{x}_1 \) and \( \dot{x}_2 \) leads to:

\[ \dot{S} = \frac{1}{L} V_{eq} - C \left[ B_i \left[ D_i D_f \right]^{\gamma} B_f^\gamma \right] V_{eq}^\gamma = 0 \] (17)

and finally:

\[ V_{eq} = \left[ 1 \ 0 \ 0 \right] \left[ D_i D_f \right]^{\gamma} B_f^\gamma V_{eq}^\gamma \] (18)

which in fact is the same as the result of control obtained through H∞ technique. Therefore this control rule of SMC will be as follows:

\[ V_q = V_{eq} - \beta \ sgn(S) \] (19)

Where \( V_{eq} \) and \( S \) are determined by (12) and (14), \( \beta \) is a positive integer and \( sgn \) is the sign function. Also to improve the response and reduce the chattering, the SMC with boundary layer can be used.

\[ V_q = V_{eq} - \beta \ sat(S/\varphi) \] (20)

where \( \varphi \) is the boundary layer width and \( sat \) is the saturation function. It is noted that the effect of SMC with boundary layer leads to a steady-state error in the speed response. The general block diagram of the control system has been shown in Fig. 1.

5. SIMULATION RESULTS

Simulations have been carried out using MATLAB software. Parameters of the simulated motor have been given in Table 1. To present the quality of combined controller, This technique is compared with non-linear H∞ and SMC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>R</td>
<td>5.26 (Ω)</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>Ls</td>
<td>0.46×10⁻³ (H)</td>
</tr>
<tr>
<td>PM flux</td>
<td>λf</td>
<td>24.9×10⁻³ (Wb)</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>J</td>
<td>10⁻³ (kg.m²)</td>
</tr>
<tr>
<td>Damping factor</td>
<td>D</td>
<td>1.32×10⁻⁶ (Nms)</td>
</tr>
<tr>
<td>No. of poles</td>
<td>P</td>
<td>4</td>
</tr>
</tbody>
</table>

First the influence of the load upon the output is studied. The non linear H∞ control for \( C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \) , up to the third mode is designed. For a nonlinear system, the maximum attenuation of the disturbance that corresponds to the smallest \( \gamma \), has the same value as for its linearization (Van der Shaft, 1992). Therefore, the smallest \( \gamma \) is selected such that the Riccati Eqn. (9) has a positive definite solution for \( P \). The minimum value for \( \gamma \) in this simulation was obtained as \( \gamma = 500 \). The value of \( \gamma \) is large because the disturbance coefficient is very small. Reference speed is chosen to be 100 rad/s. Fig. 2a shows the speed response of the system with the non linear \( H_∞ \) when the load \( T = 0 \) Nm is applied at \( t = 0.5 \) s. As shown, the response is fully tracked and this tracking continues after applying the load. Fig. 2b shows the SMC technique response. In this case, the boundary layer width is taken to be \( \varphi = 1000 \), \( λ = 10000 \) and \( \beta = 10 \). Fig. 2c presents the mixed response ( \( H_∞ \) and SMC with boundary layer) for \( C_1 \) and \( D_1 \) the same as \( H_∞ \) and \( \gamma = 500 \) and for \( H_∞ \), \( \varphi = 100 \) and \( \beta = -5000 \) for SMC with boundary layer.

![General block diagram of the control system](image-url)
As seen in the Fig. 2.c, SMC also help $H_{m}$ in the control of load disturbance and the mixed method response does better job for load variation. Fig. 2d shows the integral of the square of speed error from the reference speed at the instant of applying the load. As seen, the response of the mixed method has a sensible difference with other two techniques. In this section the influence of the parameter variation is shown. At $t=0.5$ s, the stator resistance is increased 50%. Fig. 3 shows the responses of different techniques. In this case load $T_{l}=0.2$ Nm is applied to the system at $t=0$. As seen in the figure, stator resistance change leads to a steady-state error in the SMC response. The integral of the square of error is highly rising. Also it is clear that the response of the mixed method is better than that of the $H_{m}$ and the error is lower.

![Graph showing speed response by applying the load](a)

![Graph showing speed response by applying the load](b)

![Graph showing speed response by applying the load](c)

![Graph showing comparison of errors](d)

Fig. 2. Speed response by applying the load: (a) $H_{m}$, (b) SMC, (c) Mixed and (d) comparison of errors
Now if the flux of the motor is reduced by 50% and the load $T_l=0.2$ Nm is applied to the system, the response shown in Fig. 4 is obtained. In this case it is observed in the steady-state error of SMC technique. Fig. 4b indicates that the mixed technique leads to a more suitable response compared with other techniques and the integral of the error is smaller than that of the other techniques.

Now all parameters are changed simultaneously, it means that the stator resistance is increased and motor flux is decreased 50% at $t=0.5$ s and at the same time the load $T_l=0.2$ Nm is applied to the system. Fig. 5 presents the response of the system.
Fig. 5a shows the response of $H_\infty$. It shows that the control and tracking has been carried out. The SMC technique is unstable in this case. Of course if $\beta$ increases, the system becomes stable. However, increase of $\beta$ leads to increase of chattering. In order to reduce the chattering the width of the boundary layer must be increased which leads to the increase of the steady-state error. Fig. 5b presents the system response using mixed control. Fig. 5c shows that the response of the mixed system is better than that of the $H_\infty$.

6. CONCLUSIONS

A novel method using non-linear $H_\infty$ was used to minimize the effect of the varying load in the output with SMC method to minimize the influence of motor parameters variation and system output. Considering the non-linearity of the motor and load torque variation as an external disturbance, $H_\infty$ method is a suitable one for system control. Since the design of $H_\infty$ controller leads to the solution of inequality HJI, its precise solution is very difficult; therefore an approximate solution was used. Also in order to reduce the effect of the parameters, the non-linear controller SMC was employed. Finally, simulations showed the high quality of the control method.

REFERENCES

