Stabilizing Nonlinear Adaptive PID State Feedback Control for Spacecraft Capturing

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Abstract: In the future space infrastructure, the missions of refueling and capturing of the inoperative spacecraft by the orbital servicing vehicle or the space robot are considered. To achieve them, the six degrees of freedom tracking control of the chaser spacecraft is required to approach the target spacecraft. Moreover, the stability of the connected system of the chaser and target must be ensured. In addition, it is also important to suppress the position and attitude error under the influence of the disturbance. In this paper, we derive the PID controller that satisfies the stability of the spacecraft system before and after capturing, and removes the states error caused by constant disturbance. The effectiveness of controller is verified by numerical simulations.

1. INTRODUCTION

In the future space infrastructure, the capturing of the inoperative spacecraft by the orbital servicing vehicle or the space robot is considered. To achieve the mission, the chaser spacecraft must fly around the target spacecraft so as to track its docking port whose position and attitude generally change with respect to the inertial frame, according to the motion or tumbling motion of the target spacecraft. When the relative errors of the position and attitude become sufficiently small, the chaser can safely capture the target. This is the first operation before capturing. Then, the connected spacecraft system of target and chaser must be stabilized by damping the energy, and carried to some place, say international space station (ISS), by taking another given trajectory if necessary. After the second operation, the mission is completed. These control actions must be performed using only the controller of the chaser since that of the target has been out of order.

From the viewpoint of control problem, the following two issues are raised. First is six degrees-of-freedom (6 d.o.f.) tracking control of spacecraft under the influence of external disturbance. The controller must be designed for the nonlinear translation and rotation dynamics and kinematics coupled with each other. Second is the stabilizing control during above operation including the connecting instance.

On the nonlinear tracking control under the influence of external disturbance, almost all researchers have concentrated on the nonlinear $\mathcal{H}_\infty$ controller that makes $\mathcal{L}_2$ gain of closed-loop system from disturbance to controlled output less than $\gamma > 0$ (Dalsmo and Egeland [1996, 1997], Luo et al. [2004] and so on). They also employ PD type state feedback control. However, although they generally require higher feedback gains to achieve higher disturbance attenuation capability, it is not realizable since the maximum level of control input is practically constrained. Therefore the authors consider it is not necessarily only an approach to the control purpose. Moreover, they discuss only the problem before chaser connects to target. For this problem, authors have proposed the control method which ensures the stability before and after spacecraft connection without changing controller by passivity based control (Ito [2005], Ito et al. [2006]). However, it does not consider the influence of external disturbance.

Viewing this, we propose PID state feedback controller that guarantees the asymptotic stability and tracking capability before and after capturing without changing the controller, and can effectively attenuate the constant secular signal in disturbance by using backstepping approach (Kristi et al. [1995]). In addition, since it is difficult to know all physical parameters exactly, we extend it to the adaptive controller which can estimate the physical parameters. The proposed method has the advantage of ensuring the stability of the system before and after capturing without changing controller as the literatures (Ito [2005], Ito et al. [2006]). Finally, numerical simulation results are shown.

Following notations are used throughout this paper.

\begin{align*}
\{o\} & : \text{inertial frame}, \\
\{t\}, \{c\} & : \text{target and chaser body fixed frame}, \\
\{s\}, \{R\} & : \text{connected system and reference trajectory fixed frame}, \\
m_i, J_i & : \text{mass and inertia matrix},
\end{align*}
fi, τi: control force and torque,
wf, wτ: disturbance force and torque,
ri, [εj ηj]T: position and attitude vector
of each frame,
vj, ωj: linear and angular velocity vector
of each frame,
p0, pR: constant vector fixed {t} and {R} frame
ρii, ρsi: position vector from a nominal fixed point
to force input point,
lii, lsi: position vector from a nominal fixed point
to center of mass of target and chaser,
l: distance of center of mass between target
and chaser,
I: 3 × 3 identity matrix,
a×: skew symmetric matrix determined by a ∈ R3,
C^∞ X: direction cosine matrix from the frame X to Y,
‖a‖ = √aT a: the norm of vector a,
diag{a, b, c, . . .}: diagonal matrix
R^n: linear space of real vectors of dimension n,
S^q: hyperspace of dimension q,
where subscripts i and j represent i = t, c and j = t, c, s, R.

2. MODELING AND PROBLEM DESCRIPTION

We consider the control problem that a nominal point A
fixed at {c} tracks a target point B fixed at {t} before
capturing as shown in Fig. 1 (Phase 1), and then a nominal
point A fixed at {s} tracks a target point B fixed at {R}
after connection as shown in Fig. 2 (Phase 2). In order
to simplify the discussion on the stability for both cases,
we only describe the Phase 2 problem, since they can be
unified as described in Remark 3.

Here, the following assumptions are made.
Assumption 1. The chaser and the target are rigid, and
they are connected rigidly by the docking port. Therefore,
the spacecraft system is rigid before and after the connection.
Assumption 2. The control input is not applied to the
target.

Then the translation and rotation dynamics equation of
the rigid body around a nominal point A, that is not
necessarily the center of mass, becomes in {s} frame,
\[ \dot{r}_s = v_s - \omega^s_x r_s, \]  
\[ \dot{\omega}_s = E(\theta_s)\omega_s, \]  
where \( \theta_s = [\epsilon^T_s \eta^T_s]^T \in S^3 \) satisfies the constraint \( \|\theta_s\| = 1 \).

Our tracking control problem is to find a controller such that
\[ r_s = r_{Rs}, \quad \epsilon_s = \epsilon_r, \quad \eta_s = \eta_{Rs}, \quad v_s = v_{Rs}, \quad \omega_s = \omega_R \]
when \( t \to \infty \). The position and velocity of the point B
fixed at {R} are given by
\[ r_{Rp} = r_{R} + v_{R}t, \quad v_{Rp} = v_{R} + \omega^R_x P_{Rs}. \]  
(4)

To this end, we describe an error system in {s}. Let the
direction cosine matrix from \{R\} to \{s\}
\[ C^s_R = (\eta^2_s - \epsilon^T_s \epsilon_s)I + 2\epsilon^s_s \epsilon^T_s \epsilon_s - 2\eta^s_s \epsilon^T_s \]  
(5)
using the quaternion of relative attitude $q_e = [q^T e_1 e_2 e_3]^T$, where $e_1$ and $e_2$ are defined as

$$
\varepsilon e = \eta R e_1 - \eta e R + e \times e_1 e_2 R + \varepsilon e \times R.
$$

(6)

The relative position, linear velocity and angular velocity are given in the same $\{s\}$ frame as

$$
\begin{align*}
\dot{r}_{se} &= r - C^s_R \dot{r}_p, \\
\dot{v}_{se} &= v - C^s_R \dot{v}_p, \\
\dot{\omega}_{se} &= \omega - C^s_R \dot{\omega}_p.
\end{align*}
$$

(7)

Hereafter, we simply represent $C^s_R$ as $C$ by dropping subscripts. Substitution of (7) into (1), (2) and (3), using the identity $\dot{C} = -\omega^x_C$, yields the relative equation of motion as

$$
\begin{align*}
\mathcal{M}_{se} \ddot{q}_{se} + \mathcal{C}_{se} \dot{q}_{se} + \Delta e &= \ddot{u} + w, \\
\dot{r}_{se} &= v_{se} - (\omega_{se} + C\omega_R)^x r_{se}, \\
\dot{\theta}_{se} &= \mathcal{E}(\theta_{se} \omega_{se}).
\end{align*}
$$

(8)

(9)

(10)

where

$$
\begin{align*}
q_{se} &= [q^T se]^T, \\
p_{se} &= [v^T se \omega_{se}]^T, \\
\mathcal{M} &= \mathcal{M}, \\
C_c &= \sum \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \\
\Delta e &= \sum [\Delta e_1^T \Delta e_2^T]^T, \\
c_{11} &= m_i(\omega_{se} + C\omega_R)^x, \\
c_{12} &= -m_i(\omega_{se} + C\omega_R)^x I^x_{s1}, \\
c_{21} &= m_i I^x_{s1}(\omega_{se} + C\omega_R)^x, \\
c_{22} &= \omega_{se} I^x_{s1} - m_i I^x_{s1}(\omega_{se} + C\omega_R)^x, \\
\Delta e_1 &= m_i [C(\omega_R^x p_r + \omega^x R p_R) - l^x_{s2}(C\omega_R - \omega_{se} C\omega_R)] \\
&- (\omega_{se} + C\omega_R)^x I^x_{s2} C\omega_R, \\
\Delta e_2 &= m_i I^x_{s1}[C(\omega_R^x p_r + \omega^x R p_R) - l^x_{s2}(C\omega_R - \omega_{se} C\omega_R)] \\
&- \omega_{se} I^x_{s1} C\omega_R + (C\omega_R)^x I^x_{s1}(\omega_{se} + C\omega_R) \\
&+ J_i(C\omega_R - \omega_{se} C\omega_R) + \omega_{se} J_i C\omega_R \\
&+(C\omega_R)^x J_i(\omega_{se} + C\omega_R).
\end{align*}
$$

By the transform, the tracking control problem is reduced to a regulation problem to design control input $u_f$ and $u_T$ such that

$$
\begin{align*}
\dot{r}_{se} = 0, \quad \varepsilon_{se} = 0, \quad \eta_{se} = 1, \quad v_{se} = 0, \quad \omega_{se} = 0
\end{align*}
$$

when $t \to \infty$ according to (8)-(10).

Remark 3. By setting the parameters with respect to the target to zero, and the frames $\{s\} \to \{e\}$, $\{R\} \to \{t\}$ in relative equation of motion of the connected system (8)-(10), this can be described as the equation of motion of the chaser with respect to the target before capturing. Therefore, if the control law which makes the relative error asymptotically stable can be found, then this control law can stabilize both cases.

3. ADAPTIVE CONTROLLER DESIGN

In this section, we derive the adaptive PID control law which makes the relative error asymptotically stable under unknown physical parameters, i.e.

$$
\begin{align*}
r_{se} &= 0, \quad \varepsilon_{se} = 0, \quad \eta_{se} = 1, \quad v_{se} = 0, \quad \omega_{se} = 0,
\end{align*}
$$

by applying the backstepping approach when $w = 0$. On the reference signals, we assume the followings.

Assumption 4. The reference signals $r_p, \varepsilon_R, \eta_I, v_p, \omega_R$ and $\dot{\omega}_R$ are uniformly continuous, bounded and known for all $t \in [0, \infty]$.

The concrete design procedure is given as follows.

Step 1:

We suppose that $v_{se}$ and $\omega_{se}$ are the virtual inputs to the subsystem (9) and (10), and define the stabilizing functions such that

$$
\alpha_1 = -K_{P1} r_{se} - K_{I1} \zeta_1, \quad \alpha_2 = -K_{P2} \varepsilon_{se} - K_{I2} \zeta_2,
$$

(11)

where $K_{Pn}$ and $K_{In}$ $(n = 1, 2)$ are the symmetric and positive definite matrices, $\zeta_n (n = 1, 2)$ is the integral variable defined as

$$
\zeta_1 = \int_0^t r_{se} \, dt, \quad \zeta_2 = \int_0^t \varepsilon_{se} \, dt.
$$

(12)

Now, we define the error variable between the state $(v_{se}, \omega_{se})$ and the desired control $(\alpha_1, \alpha_2)$ such as

$$
z_1 = v_{se} - \alpha_1, \quad z_2 = \omega_{se} - \alpha_2.
$$

(13)

From (13), the subsystem (9) and (10) become

$$
\dot{r}_{se} = (z_1 + \alpha_1) - (\omega_{se} + C\omega_R)^x r_{se}, \\
\dot{\theta}_{se} = \mathcal{E}(\theta_{se} z_2 + \alpha_2).
$$

(14)

(15)

Define the following candidate Lyapunov function such as

$$
V_1 = \frac{1}{2} ||r_{se}||^2 + \frac{1}{2} \zeta_1^2 K_{I1} \zeta_1 \\
+ ||\varepsilon_{se}||^2 + (\eta_{se} - 1)^2 + \frac{1}{2} \zeta_2^2 K_{I2} \zeta_2.
$$

(16)

By utilizing the following skew symmetric matrix properties

$$
a^T a^x = 0, \quad a^T b^x a = 0, \quad \forall a, b \in \mathbb{R}^3,
$$

the time derivative of (16) along the trajectories of the closed-loop system becomes

$$
\dot{V}_1 = r_{se}^T \{(z_1 + \alpha_1) - (\omega_{se} + C\omega_R)^x r_{se}\} + r_{se}^T K_{I1} \zeta_1 \\
+ \varepsilon_{se}^T \{(\eta_{se} l + \varepsilon_{se}) (z_2 + \alpha_2)\} - \varepsilon_{se}^T (\eta_{se} - 1) (z_2 + \alpha_2) \\
+ \varepsilon_{se}^T K_{I2} \zeta_2 \\
= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2} \varepsilon_{se} + r_{se}^T z_1 + \varepsilon_{se}^T z_2.
$$

(17)

From (17), obviously $r_{se} \to 0$, $\varepsilon_{se} \to 0$ and $\eta_{se} \to 1$ as $t \to \infty$ when $z_1 = z_2 = 0$.

Step 2:

We derive the adaptive controller which makes the state $z = [z_1^T z_2^T]^T$ asymptotically stable, i.e. $z = 0$, when $w = 0$, From (8), (11) and (13), the dynamics with respect to $z$ is as follows.

$$
\mathcal{M}_z \ddot{z} + \mathcal{C}_z z + \Delta_z = u,
$$

(18)

where
Here, by selecting control inputs and estimate laws as

$$
\begin{align*}
\dot{u}_f &= -r_{se} - k_{D1} z_1 + \sum \hat{m}_i \phi_i \\
\dot{u}_r &= -\varepsilon_{se} - k_{D2} z_2 + \sum \hat{m}_i \phi_i + \tilde{\psi} \hat{\beta}_s \\
\dot{\hat{m}}_i &= -\Gamma_{i1} [\phi_i^T \phi_i] z_i \\
\dot{\hat{\psi}} &= -\tilde{\Gamma}_2 \tilde{\psi}^T z_2
\end{align*}
$$

where $K_{Dh}$ ($n = 1, 2$) is the symmetric and positive definite matrix, $V_2$ becomes

$$
\dot{V}_2 = -v_{se} K_{P} p_{1r} - \varepsilon_{se} K_{P} p_{2} z_2 + r_{se} z_1 + \varepsilon_{se} z_2 \\
= -\sum m_i [\phi_i^T \phi_i] z_i - \tilde{\Gamma}_2 \tilde{\psi} T z_2 \\
+ \sum \hat{m}_i [\phi_i^T \phi_i] z_i + \sum \hat{m}_i [\phi_i^T \phi_i] z_i \\
= -\sum m_i [\phi_i^T \phi_i] z_i - \tilde{\Gamma}_2 \tilde{\psi} T z_2 \\
- \sum \hat{m}_i [\phi_i^T \phi_i] z_i + \sum \hat{m}_i [\phi_i^T \phi_i] z_2 \\
= -r_{se} K_{P} r_{se} - \varepsilon_{se} K_{P} p_{se} z_2 + r_{se} z_1 + \varepsilon_{se} z_2 \\
= -r_{se} K_{P} r_{se} - \varepsilon_{se} K_{P} p_{se} z_2 + r_{se} z_1 + \varepsilon_{se} z_2
\end{align*}
$$

where

$$
\tilde{\psi} = \psi_1 (\alpha_2 + C \omega_R - \omega_0^* C \omega_R) + \psi_2 (\omega_0^* + \alpha_2 + 2 \alpha + 2 \alpha_2 + C \omega_R)
$$

Obviously, $\dot{V}_2 \leq 0$ holds. Therefore, $x$ is bounded since

$$
\dot{V}_2 (x(t)) \leq V_2 (x(0)), \forall t \geq 0,
$$

where $x = [\xi_0^T \xi_1^T \xi_2^T \varepsilon_{se} \eta_{se} \nu_{se} \omega_{se} \omega_{se}]^T$, and $V_2$ is radially unbounded in state space $\mathbb{R}^{23} \times S^5$. Then $\dot{x}$ is also bounded since the control inputs (21) and the derivative of estimate variable are bounded by assumption 4. These follow that $\dot{V}_2$ is bounded. Therefore, it is shown that

$$
\dot{V}_2 \rightarrow 0 \iff r_{se} \rightarrow 0, \varepsilon_{se} \rightarrow 0, z_1 \rightarrow 0, z_2 \rightarrow 0
$$

as $t \rightarrow \infty$ from Lyapunov-Like Lemma (Slotine and Li [1991]), and then

$$
\zeta_1 \rightarrow 0, \zeta_2 \rightarrow 0, \eta_{se} \rightarrow 1, \nu_{se} \rightarrow 0, \omega_{se} \rightarrow 0
$$

as $t \rightarrow \infty$ from (11)-(13) and $V_2 = 0$. Furthermore, the closed loop system becomes

$$
\left\{ \begin{array}{l}
\sum \hat{m}_i \phi_i = 0 \\
\sum \hat{m}_i \phi_i \psi \hat{\beta}_s = 0
\end{array} \right.
$$

(25)
when \( t \to \infty \). It implies the estimate variables converging to some constant values, i.e. \( \hat{m}_i \to c_{i1}, \hat{\beta}_s \to c_2 \) as \( t \to \infty \). Especially, from the first equation of (25)

\[
\sum \hat{m}_i \phi_{i1} = \hat{m}_t \phi_{i1} + \hat{m}_c \phi_{c1} = 0,
\]

if \( \| \phi_{i1} \| \neq 0 \) for all \( t \geq 0 \) and \( \phi_{i1} \) is linear independent, then \( \hat{m}_t \) obviously becomes \( \hat{m}_t \to 0 \), i.e. \( \hat{m}_t \to m \), as \( t \to \infty \).

**Remark 5.** It is noted that the adaptive controller (21) and (22) can be stabilized before and after capturing by Remark 3.

### 4. NUMERICAL SIMULATION

#### 4.1 Mission setup

As a numerical simulation, we suppose the situation where the chaser tracks the docking port fixed at the target that performs the tumbling motion. After capturing, as the problem of transportation to the space station, the connected system tracks the reference trajectory. Both cases are controlled by adaptive controller (21) and (22).

#### 4.2 Capturing method of target spacecraft

In order to avoid the collision and excessive docking impact, the following scenario is considered in Phase 1. First, the chaser approach the target in a certain distance \( p_t \). Then, during the tracking control, the target position is gradually decreased by \( \delta p_t \). When \( p_t = 1.75 \) [m], the target is supposed to be captured by the chaser.

#### 4.3 Physical model

The physical parameters used in the simulation are as follows.

\[
\begin{align*}
\begin{array}{c}
  m_t = 300 \text{ [kg]},
  m_c = 200 \text{ [kg]},
  J_t = \text{diag}(50, 275, 275) \text{ [km}^2\text{],}
  J_c = \begin{bmatrix}
    75.0 & -28.125 & -28.125 \\
    -28.125 & 75.0 & -28.125 \\
    -28.125 & -28.125 & 75.0
  \end{bmatrix} \text{ [km}^2\text{],}
  l_{tt} = l_{cc} = l_{st} = [0 0 0]^T \text{ [m]},
  l_{sc} = [l 0 0]^T \text{ [m]},
  \rho_{tt} = \rho_{cc} = \rho_{st} = [0 0 0]^T \text{ [m]},
  \rho_{sc} = [l 0 0]^T \text{ [m]},
\end{array}
\end{align*}
\]

where \( l = 1.75 \) [m]. A nominal point \( A \) is the mass center of the chaser before capturing, and it is set as the mass center of the target after capturing. In the latter case, it mean that \( \{ t \} \) and \( \{ s \} \) are the same frame.

#### 4.4 Simulation results

Simulation results are shown in Figs. 3-6 where the initial values, the target constant vector fixed at \( \{ t \} \) and the controller gains are set as

\[
\begin{align*}
  r_t(0) &= [20 20 5]^T \text{ [m]},
  \varepsilon_t(0) = [0 0 0]^T,
  \eta_t(0) = 1,
  \nu_t(0) &= [0 0 0]^T \text{ [m/s]},
  \omega_t(0) = [0.05 0.0 0.1]^T \text{ [rad/s]},
  r_c(0) &= [15 15 5]^T \text{ [m]},
  \varepsilon_c(0) = [0.18 0.31 0.18]^T,
  \eta_c(0) = 0.92,
  \nu_c(0) &= [0 0 0]^T \text{ [m/s]},
  \omega_c(0) = [0 0 0]^T \text{ [rad/s]},
  \hat{\beta}_s(0) &= [45.0 -16.875 -16.875 45.0 -16.875 45.0]^T \\
  \hat{\eta}_s(0) &= \frac{2 \pi}{600},
  \rho_t = [2.75 0 0]^T \text{ [m]},
  \delta p_t = 0.1 \text{ [m]},
  K_{P1} = 0.2I,
  K_{D1} = 50I,
  K_{T1} = 0.04I,
  \Gamma_{T1} = \Gamma_{c1} = 10,
  K_{P2} = 0.2I,
  K_{D2} = 250I,
  K_{T2} = 0.04I,
  \Gamma_2 = 1000.
\end{align*}
\]

This reference trajectory is the circular orbit of radius of 30 [m] in \( x - y \) plane at \( \{ t \} \). Furthermore, following constant external disturbance

\[
\begin{align*}
  w_f &= [1 1 1]^T \text{ [N]},
  w_r = [0.5 0.5 0.5]^T \text{ [Nm]}
\end{align*}
\]

is added for all \( t \). These values are ten times of the air drag disturbance at the lower earth orbit.

Figures 3 and 4 show time histories of the relative position, velocity, attitude and angular velocity. Fig. 5 shows trajectory of connected system (solid) and reference (dashed) at \( \{ t \} \) after capturing. It is observed that chaser arrives at the target vector \( p_t \) and gradually approaches to target by changing the reference signal under the constant external disturbance. After capturing is achieved at 203.1 [s], the connected system tracks the reference trajectory under the constant external disturbance. Time responses of estimated parameters are shown Fig. 6. Although all estimated parameters not converge to real parameters, as described section 3, stability of the system is guaranteed under uncertain physical parameter.

### 5. CONCLUSION

In this paper, we have proposed the adaptive PID controller that ensures the stability before and after capturing an inoperative spacecraft in orbit. The effectiveness of controller is verified by numerical simulations. As the future works, collision avoidance of the chaser and the target before capturing, the extension to the case considering the dynamics of the docking port are considered.
REFERENCES


Fig. 3. Time response of relative position, velocity, attitude, and angular velocity before capturing.

Fig. 4. Time response of relative position, velocity, attitude, and angular velocity after capturing.

Fig. 5. Trajectory of connected system and reference at $\{i\}$ after capturing.

Fig. 6. Time response of estimated parameters $\hat{m}_t$, $\hat{m}_c$ and $\hat{\beta}_s$ (solid), and real parameters $m_t$, $m_c$ and $\beta_s$ (dashed-dotted).