Real time control of hybrid electric vehicle
on a prescribed road

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Abstract: Hybrid vehicles use two energy sources for their propelling. In order to investigate an optimal splitting of the power flows between the engine and the electric machine, an optimal control algorithm is recalled. A new formulation using maps allows a very fast solving this problem which then may be used in real time control. In the particular case of a hybrid vehicle on a prescribed route, a new control strategy is proposed. It consists in computing an optimal control on the prescribed route and then updating the computed values in order to cope with the difference between the actual and prescribed vehicle route.

1. INTRODUCTION

Hybrid vehicle uses at least two energy sources for its propelling and at least one of them must be reversible. Usually an internal combustion engine is coupled with one or more electric machine. The objective of the hybridization is to lower both the fuel consumption and the emissions of the vehicle. This problem may be addressed in different ways: by optimizing the vehicle design, by using more efficient powertrain components, etc. In this paper, we will focus only on the energy split between the engine and the motor.

Control strategies are algorithms that choose at each sampling time the power split between the engine and the motors to minimize a criterion, usually the fuel consumption and/or the pollutant emissions. Two classes of algorithms may be distinguished: If the vehicle driving conditions are a priori known, it is possible to compute in simulation an optimal control using for example dynamic programming technics /Scordia & al. 2005/ or Lagrangian approaches /Delprat & al. 2004/. Theses algorithms provide an expertise of the energy flow splitting but are not suitable for real time control since they require the knowledge of the whole driving cycle.

Real time algorithms are not optimal but are suitable for real time driving conditions. Several approaches have been investigated, for example, energetic efficiency analysis of the powertrain /Seiler & al. 1998//Paganeli & al. 2000/. At last, some recent investigations propose to use the knowledge contained in the results provided by the global optimization algorithms to formulate real time ones /Brahma & al. 2000//Scordial & al. 2005/. In the particular case of a vehicle following a prescribed route, to take some benefits of this knowledge we propose to use in real time the previously proposed optimal control algorithm. Results are not optimal anymore since the prescribed route is not perfectly followed in real-time.

In the first part, a hybrid vehicle energetic model is presented and the fuel consumption minimization problem is derived. Then a global optimization algorithm is recalled in the second part. The third paragraph presents an analysis of the obtained optimality condition and how to derive an efficient real time control strategy. At last, some simulation results of the proposed algorithm are presented.

2. HYBRID VEHICLE ENERGY MANAGEMENT AS AN OPTIMIZATION PROBLEM

2.1. Hybrid vehicle modelling

The considered vehicle, figure 1, is a single shaft parallel hybrid vehicle designed by G. Paganelli /Paganelli & al. 2000/. It was part of a project between the LAMIH (Laboratoire d’Automatique, de Mécanique et d’Informatique industrielles et Humaines) and PSA Peugeot Citroën with the financial support of the ADEME (Agence de l’Environnement et de la Maîtrise de l’énergie), the FEDER (Fond Européen pour le Développement Régional), et la Région Nord Pas de Calais.

Fig. 1. The considered parallel single shaft arrangement

The IC engine is a 1.4l petrol engine which is able to develop 55 kW at 5500 tr/min. It is coupled through set a gears to a 43kW DC motor powered by 20 modules of 26Ah 12V pure lead acid batteries. The gearbox has been especially design for this vehicle. Due to the total torque available, it has only two gears $n_g = 2$, corresponding approximately to a 2nd and 5th gear of a conventional car.

Notations: Letter $T$ is used for torque, $P$ for powers, $\omega$ for powers, $\mu$ for speeds, $R(k)$ is the gearbox ratio for the kth gear (including the differential), subscript $w$ is used for wheel, $m$ for electric machine, $e$ for IC engine and $gb$ for
the gear box. \( \rho \) is the gear ratio between the electric machine and the IC engine. \( x \) is the battery state of charge. \( \theta \) is the IC engine state (\( \theta = 1 \Rightarrow \text{on} / \theta = 0 \Rightarrow \text{off} \)). \( i \) and \( j \) are sample numbers. \( s \) is the constant sampling period. \( n_{gs} \) is the number of selectable gears in the gearbox. \( r_w \) is the wheel radius.

To lighten expression, the dependence to the sample number \( i \) or \( j \) is omitted when there is no ambiguity.

For this study, two energetic models are considered. The first, Model 1, one is a quite detailed model used for simulation purpose whereas the second one, Model 2, is a simplified one only used to derive the global optimization problem. Model 2 is described by the following equations:

- **Relation on speeds**: \( \omega_n = \frac{\omega_m}{R(k) \rho} = \frac{\omega_j}{R(k)} \) (1)

- **Relation on torque**: \( T_e = R(k) \eta_{\phi} (T_m + \rho \eta \omega_m) \) (2)

With \( T_e \) the power to be produced at the wheel, and \( T_m \) the torque produced by the IC engine and electric machine on their output shaft. During real time experiment the wheel speed \( \omega_n \) is measured using sensors and \( T_e \) is the torque at the wheel set point provided by the driver. During simulation \( \omega_n \) is given by a driving cycle and \( T_e \) is computed by a controller.

- **Constraint on speeds**: \( \omega_{v_n} < \omega_e < \omega_{v_m} \) (3)

- **Constraint on torque**: \( T_{e_{\min}}(\omega_e) \leq T_e \leq T_{e_{\max}}(\omega_e) \) (4)

- **Relation on speed**:\( T_{e_{\min}}(\omega_e), T_{e_{\max}}(\omega_e), T_{m_{\min}}(\omega_m), T_{m_{\max}}(\omega_m) \) are usually maps over speed that may be either computed using analytic models or simply measured on a test bench.

- **Fuel consumption**: \( C = \sum_{i=0}^{N} q(T_e(i), \omega_e(i)) \theta(i) \) (5)

For the considered finite horizon \( N \), the total fuel consumption \( C \) is computed using \( q(T_e, \omega_e) \) the fuel mass flow (in g/s) required to produce the torque \( T_e \) at the speed \( \omega_e \). \( q(T_e, \omega_e) \) is usually given by a map.

- **Battery state of charge**: For the optimization problem, a very simple model of the electric machine and its associated DC/DC converter is considered:

\[
x(N) = x(0) - \sum_{i=0}^{N} P_{bat}(T_m(i), \omega_m(i)) s
\] (6)

With \( x(i) \) is the battery energy level at sample time \( i \), abusively called state of charge and \( P_{bat}(T_m, \omega_m) \) the power consumption of the whole electric chain (including internal battery losses) to produce the mechanical torque \( T_m \) at the speed \( \omega_m \). In practice \( P_{bat}(T_m, \omega_m) \) is also a map.

2.2. **Global optimization problem**

Without taking the battery state of charge into account, there exists a trivial solution to the optimization problem: \( \theta(i) = 0 \) \( \forall i = 0..N-1 \) which may lead to the full battery discharge. To avoid this solution, a constraint on the state of charge is needed:

\[
x(N) - x(0) = \Delta Soc
\] (7)

With \( \Delta Soc \) a prescribed state of charge variation at the end of the driving cycle. A particular value is \( \Delta Soc = 0 \) which ensures that the whole vehicle propelling is only due to the energy of the fuel. Then the obtained fuel consumption may be compared with the one obtained using a conventional vehicle.

In the particular case of simulations, the vehicle speed \( \omega_n \) is provided by a driving cycle and the required power at the wheel \( T_e \) is computed using the vehicle dynamics model. As a consequence \( \omega_n(i) \) and \( T_e(i) \) are known for \( i = 0..N-1 \).

Therefore, according to (1) and (2), it is obvious that the whole powertrain setpoint is defined if only a torque \( T_e \) or \( T_m \) and the IC engine state \( \theta \) and the gear number \( k \) are known. At each sample time, the decision vector is then \( (T_e(i), k(i), \theta(i)) \) \( \forall i = 0..N-1 \).

The other variables may be computed using equations (1)-(2).

For sake of convenience, the whole optimization problem may be rewritten using only the decision variables:

\[
J = \min \sum_{i=0}^{N-1} Q(i, T_e(i), k(i), \theta(i)) \theta(i) s
\] (8)

With \( Q(i, T_e(i), k(i), \theta(i)) = q(T_e(i), \omega_e(i)) R(k(i)) \).

The battery is considered as the dynamical system with:

\[
x(i+1) = x(i) - P_{bat}(T_e(i), k(i), \theta(i)) s
\] (9)

With \( P_{bat}(T_e, k, \theta) = \frac{1}{\rho \eta_{\phi}} \left( \frac{T_e}{R(k)} \frac{T_m}{\eta_{\phi}} \right) R(k) \) (10)

The torque limitations (5)-(6) may be combined into a single one: \( T_{e_{\max}}(\omega_e) \leq T_e \leq T_{e_{\min}}(\omega_e) \) (11)

\[
T_{e_{\max}}(\omega_e) = \max \left( \frac{T_e(\omega_e(i)) \rho R(k(i))}{\eta_{\phi}}, \frac{T_m(\omega_e(i)) \eta_{\phi} R(k(i))}{R_{\phi}} \right)
\] (12)

\[
T_{e_{\min}}(\omega_e) = \min \left( \frac{T_e(\omega_e(i)) \rho R(k(i))}{\eta_{\phi}}, \frac{T_m(\omega_e(i)) \eta_{\phi} R(k(i))}{R_{\phi}} \right)
\] (13)

At last the constraints on speed may also be combined:

\[
k(i) \in K(i)
\] (14)

With \( K(i) \) the set, at sample time \( i \), of the admissible gear numbers according to the speed constraints (3) and (4).
3. GLOBAL OPTIMIZATION

3.1. Principle

Several approaches have been proposed to the optimisation problem. If the state variable \( x \) is quantified, it is possible to formulate the optimization problem as a shortest path algorithm /Scordia & al. 2005/ and so a global optimum may be computed. The main drawback of this approach is the computational time required to obtain a single solution that varies exponentially with the quantification step.

The approach presented in this paper is based on a Lagrangian method derived from classical optimal control theory. The control law is computed on Model 2 but is directly applied to Model 1, therefore the controls are coherent with Model 1 in particular without any SOC correction. The main benefits of this approach are the very small computational effort required. For more information about this algorithm, the lecturer may refer to /Chen & al. 2005//Delprat & al. 2004/.

For sake of convenience, we assume that all the discrete variables \( k(i) \) and \( \vartheta(i) \) \( i = 0..N-1 \) are a priori fixed according, for example, to driving comfort rules.

Then the Hamiltonian is:

\[
H = \sum_{i=0}^{N-1} Q(i,T_e(i),k(i),\vartheta(i))\vartheta(i)s
\]

\[
+ \lambda(i)[x(i) - P_e(i,T_e(i),k(i),\vartheta(i))s]
\]

Lagrangian parameters \( \lambda_i \), \( i = 0..N-1 \), the optimal control should minimize the Hamiltonian and therefore the first and second order conditions are given by:

\[
\frac{\partial H}{\partial \lambda(i+1)} = x(i+1)
\]

\[
\frac{\partial H}{\partial x(i)} = \lambda(i) \Rightarrow \lambda(i) = \lambda(0)
\]

\[
\frac{\partial H}{\partial u(i)} = 0
\]

\[
\frac{\partial^2 H}{\partial u(i)^2} > 0
\]

Condition (17) allows reducing the computation of the value of the \( N \) Lagrangian parameters to a single scalar \( \lambda(0) \in \Re \). Condition (16) provides the system dynamics, (17) is the Adjoint Equation, (18) and (19) are often called Hamiltonian Minimization equation. At each sample time \( i \), the first order conditions (16)-(18) allow computing the control values \( T_e(i) \) for each subproblem:

\[
\frac{\partial Q(i,T_e(i),k)}{\partial T_e} \vartheta(i) - \frac{\partial P_e(i,T_e(i),k,\vartheta)}{\partial T_e} = 0
\]

\[
\frac{\partial^2 Q(i,T_e(i),k)}{\partial T_e^2} \vartheta(i) - \frac{\partial^2 P_e(i,T_e(i),k,\vartheta)}{\partial T_e^2} = 0
\]

The Hamiltonian (15) may be written into a more convenient form:

\[
H = \sum_{i=0}^{N-1} h(i,T_e(i),k(i),\vartheta(i))s + \lambda(0) \sum_{i=0}^{N-1} x(i)
\]

With \( h(i,T_e(i),k(i),\vartheta(i)) = Q(i,T_e(i),k(i))\vartheta(i) \)

\[
-\lambda(0)P_e(i,T_e(i),k(i),\vartheta(i))
\]

Finding the control value \( T_e(i) \) that minimises the Hamiltonian \( H \) is equivalent of minimizing \( h(\cdot) \).

As the criterion \( q(T_e,\vartheta) \) and the electric machine power consumption \( P_e(T_e,\omega_e) \) are given by maps, a second order piecewise continuous polynomial approximation of \( h(T_e,k,\vartheta) \) allows finding efficiently its minimum using conditions (20) and (21).

So for a given \( \lambda(0) \in \Re \), at each sample time, it is possible to compute a solution \( (T_e(i), k(i), \vartheta(i)) \) of the optimization problem. Therefore, for any given \( \lambda(0) \in \Re \), by summing (11) over the time horizon \( i = 0..N-1 \), a final state of charge \( x(N) \) is obtained. So the effective state of charge variation over the speed cycle \( \Delta x(\lambda(0)) \) only depends on \( \lambda(0) \):

\[
\Delta x(\lambda(0)) = \sum_{i=0}^{N-1} P_e(i,T_e(i),k(i),\vartheta(i))s
\]

The last step is to compute \( \lambda(0) \) with respect state of charge constraint (9).

Therefore, the whole optimization problem is reduced to find the zero of a function:

\[
f(\lambda_0) = \Delta SOC - \Delta x(\lambda(0))
\]

Many numerical experiments have been conducted on many different driving cycles, and this function is always a monotonic, so a simple dichotic search is used to find the value of \( \lambda(0) \) that ensures \( |\Delta SOC - \Delta x| < \varepsilon_{\Delta soc} \) with \( \varepsilon_{\Delta soc} \) a desired tolerance.

Nb: For all the subproblems with \( \vartheta_j(i) = 0 \), there is no optimization possible since the IC engine is turned off, \( T_e(i) = 0 \), the hybrid vehicle is on an pure electric mode. The necessary control \( T_e(i) \) is then computed using (2). If the discrete variable \( k(i) \) and \( \vartheta(i) \) are also optimised a sub-optimal solution is obtained by considering all their combination and choosing the one that minimises the Hamiltonian.

3.2. Simulation results

The controls are computed every \( s = 0.1 \) second and are applied to the more accurate vehicle model Model 1 that is simulated using a fixed step solver with 0.01 step size. The considered speed cycle is a trip of 12 km fig. 2.

In this configuration, the following figure illustrates the shape of the obtained state of charge variation over the whole driving cycle as a function of \( \lambda(0) \), fig. 3.
Predictive control has been already investigated for Hybrid vehicle control [Beck & al. 2007]. But the particular case of a vehicle following a predefined route is quite interesting because it may be possible to use the optimal controls computed off line. Many professional vehicles are working on this kind of conditions: bus, delivery truck, cleaning machines, etc.

Of course, the main drawback of this approach is that the actual driving conditions must be close enough to the predefined route. The word “route” will not only refer to the vehicle itinerary but also the variation of its speed along this itinerary. For a given geographical itinerary, significantly different traffic conditions (fluid, traffic jam, etc.) will lead to different route.

We assume that a prescribed route is a known set of wheel speed and wheel torque $(\widehat{\omega}_n(j),\widehat{T}_w(j)) \forall j=0..\bar{N}-1$ with $\bar{N}$ the prescribed number of sample.

In order to investigate the variations from one run to the other on the same route, several experiments have been conducted in Lyon with real traffic conditions, fig. 5, on a 12km long route.

Of course the vehicle speed may differ due to the traffic, but if when the vehicle speed is considered as a function of the distance, the runs exhibits quite similar patterns. Therefore it seems possible to take some benefits of this knowledge.

### 4.1. Problem statement

Let us recall that if the route was perfectly followed by the vehicle, it would be possible to predict the necessary value of $\lambda(0)$ to bring the battery state of charge from its initial value $x(0)$ to a final value $x_f = x(\bar{N})$. Moreover, from (20)-(24) it is obvious that if the route is perfectly known, at each sample time $i$, the control is only a function of $\lambda(i)$. Therefore, the real time control is reduced to the value of $\lambda(0)$ in order to compensate for difference between the actual route followed by the vehicle and the prescribed one.

So the proposed control strategy consists at some sampling time $i$ to update the value of $\lambda(i)$ according to:

- the battery state of charge constraint: The battery state of charge should evolve from its actual value $x(i)$ to a

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**Fig. 2:** The considered driving cycle

**Fig. 3:** $\Delta x$ as a function of $\lambda(0)$

**Fig. 4:** Results provided by the global optimization algorithm

**Fig. 5:** Vehicle speed recorded on 4 runs on the same route

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### 4. REAL TIME ENERGY MANAGEMENT OF A VEHICLE ON A PRESCRIBED ROUTE

Notations: $j$ will refer to a sample number related to the prescribed route whereas $i$ will refer as the actual sample time of the system.
desired state of charge $x_f$ at the end of the prescribed driving cycle, so we define:

$$\Delta \text{Soc} = x_f - x(i)$$  \hspace{1cm} (26)

- The prescribed route. At sample time $i$, we assume that the vehicle has already covered a distance $d(i)$ on the prescribed itinerary:

$$d(i) = \sum_{j=0}^{i} o_w(j) \cdot r_w \cdot s$$  \hspace{1cm} (27)

Due to disturbances, at each sample time $i$, the actual speed may differ from the prescribed one $o_w(i) \neq o_w(i)$ so the actual covered distance $d(i)$ may differ from the prescribed one:

$$d(i) \neq \hat{d}(i)$$  \hspace{1cm} (28)

With the “prescribed” distance at sample time $i$:

$$\hat{d}(i) = \sum_{j=0}^{i} o_w(j) \cdot r_w \cdot s$$

So, at sample time $i$, the uncovered part of the prescribed route is defined by $(\tilde{o}_w(j), \tilde{T}_w(j)) \quad j = j_i \cdot \tilde{N}$. $j_i$ is given by:

$$d(i) = \hat{d}(j_i)$$  \hspace{1cm} (29)

To update the value of $\lambda(0)$, the following optimization problem needs to be solved:

System: $\dot{x}(j+1) = x(j) - P_b (j, \tilde{T}_w, \tilde{k}, \tilde{\vartheta}) s$

$$J = \min_{\tilde{\vartheta}, \tilde{k}, \tilde{\vartheta}} \sum_{j=0}^{j_i \cdot \tilde{N} - 1} Q(j, \tilde{T}_w, \tilde{k}) \dot{s}$$

Criterion: $+\lambda(0)(x(j) - P_b (j, \tilde{T}_w, \tilde{k})) s$

Under constraints:

$$T_w'(j) \leq \tilde{T}_w(j) \leq T_w'(j) \quad j \in K(j)$$

$$\hat{x}(\tilde{N}) = x_f$$

With $\tilde{T}_w(j), j = j_i \cdot \tilde{N}$, the chosen IC engine torque for the prescribed route and $\hat{x}(j) = j_i \cdot \tilde{N}$ the state trajectory on the prescribed route.

4.3. Real time update of $\lambda(0)$

Even if the proposed global optimization algorithm does not require a lot of computation compared to dynamic programming approaches, it may not be suitable for real time solving of (30). To overcome this, it should be noticed that to solve the optimization problem, only the important value is $\lambda(0)$ that is computed using only $P_b (i, T_r, k, \vartheta)$. Since $T_r$, $k$ and $\vartheta$ depend only $\lambda(0)$, and $o_w$ and $T_v$ (cf (12)), it is possible to approximate $P_b (i, T_r, k, \vartheta)$ by a map, fig. 6:

$$P_b = M(\lambda(0), o_w, P_r)$$  \hspace{1cm} (31)

Let us recall the state of charge constraint:

$$x(i) + \sum_{j=0}^{i} P_r (j, T_r, k, \vartheta) s = x_f$$  \hspace{1cm} (32)

Finally, the optimization problem (30) is then reduced to solve

$$\Delta x(\lambda(0)) = \Delta \text{Soc}$$

With $\Delta x(\lambda(0)) = \sum_{j=0}^{j_i \cdot \tilde{N}} M(\lambda(0), o_w(j), P_r(j)) s$

(33) may be solved by approximating $\Delta x(\lambda(0))$ by a cubic spline. So for a given route $(o_w(j), P_r(j)) \forall j = 0, \tilde{N} - 1$, the state trajectory, (33) allows finding the values of $\lambda(0)$ such as $\tilde{x}(\tilde{N}) = x_f$. As no complex computation is involved, this calculation can be processed online.

At each sampling time $i$, $\lambda(0)$ is updated if $|x(i) - \hat{x}(j_i)| > \varepsilon$, that is if the actual value of the state of charge $x(i)$ differ significantly from the value of the state trajectory over the prescribed route $\hat{x}(j_i)$. If $\lambda(0)$ and the vehicle operating point $(o_w, T_v)$ is known, $P_b (j, \tilde{T}_w, \tilde{k}, \tilde{\vartheta})$ is also known. The map $P_{\text{bat}}(T_v, o_w)$ being invertible, $T_v$ can be obtained and (2) allows computing the control $T_r$.

4.4. Simulation results

All the simulations are performed using the most detailed model, Model 2. The reference cycle is the trial n°1 of fig. 5. The control strategies performances will be investigated for another trial, trial n°2. Let us recall that both cycle were recorded using a real vehicle. The initial state of charge was $x(0) = 80\%$ and the targeted final one was $x_f = 80\%$. The only parameter to be tuned is the tolerance on the state trajectory $\varepsilon$, that has been tuned after several trials and errors to $\varepsilon = 1\%$. The obtained results are shown fig. 7 and the corresponding control are shown fig. 8. The fuel consumption was 5.29 l/100km and the final state of charge was 79.38%. The number of $\lambda(0)$ update remained low (only 4). One of the main differences with classical control strategies is that it is not a strictly sustaining control strategy since, in real time, the state of charge has to follow non constant trajectory. The prescribed state trajectory is the state trajectory computed by the optimal control for the applied $\lambda(0)$.  

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig6.png}
\caption{Map of $P_b$ as a function of $o_w$ and $P_r$ for $\lambda(0)$}
\end{figure}
As there are two energy sources, it is not possible to evaluate the performance of the proposed simply on a single run. Fig. 9 propose a complete view of the control strategy performances. It represents the fuel consumption as a function of the overall state of charge variation $\Delta\text{Soc} = x(N) - x(0)$ on the whole driving schedule. The control strategy results are compared with the result provided by the optimal control.

5. CONCLUSION

A global optimization algorithm has been recalled and the computation process has been significantly speeded up by building maps. Considering the particular case of a vehicle driving on a prescribed path, experimental data shows that the driving cycles recorded on the same route exhibits similar patterns. The proposed real time algorithm consists in computing the value of the Lagrangian parameters of the optimisation problem that bring the state of charge from its currant values to the desired final value assuming that the prescribed route will be perfectly followed. As this hypothesis is never fulfilled in real time driving conditions, several updates are needed. The first simulation results illustrate the benefits of the proposed approach. Compared with other real time control strategies, it allows the state of charge varying from a nominal set point.

Further work will be devoted to robustness analysis and in particular to find a bound on the actual final state of charge with respect to the driving cycle variation.

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6. REFERENCES


¹ AUTORIS: AUTOmatique pour la Route Intelligente et Sûre, project supported by Région Nord-Pas-de-Calais and FEDER (European Funds)
² GRAISyHM: Regional Research Group in Automation and Man Machine Systems